Lyapunov exponent in entanglement renormalization

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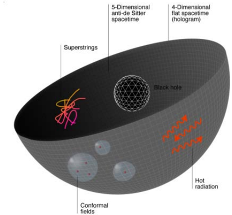
Outline

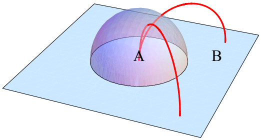
- Geometry from entanglement renormalization
- Maximal chaos, a signal of gravity
- Exponential growth of renormalized operator



CFT$_d$ $\leftrightarrow$ string theory in AdS$_{d+1}$

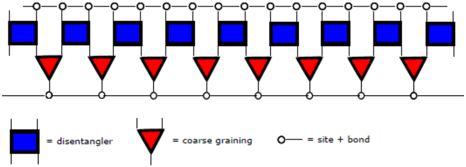
- RT formula

$$S_{\mathrm{EE}}(A) = \frac{\mathrm{Area}(\gamma_A)}{4G}, \quad \partial\gamma_A = \partial A$$

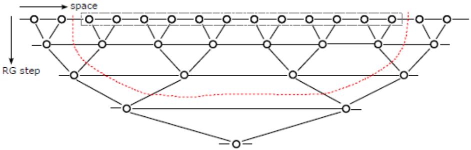


Question: What entanglement?

Multi-scale entanglement renormalization ansatz



Tensors (disentangler, isometry) modify the entanglement structure



MERA version of RT-formula

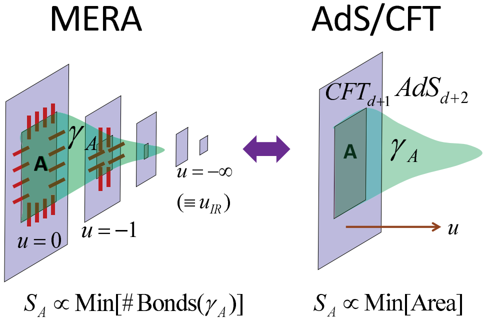
$$S_A \leq \#(\text{LUs cut})$$

figures courtesy of arXiv:1209.3304

AdS/MERA

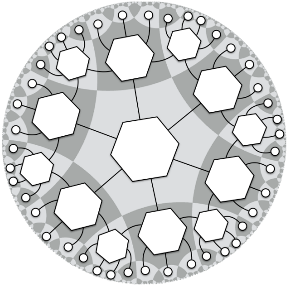
MERA = Discretized AdS   Swingle 2009

Spacetime generated by entanglement renormalization



figures courtesy of arXiv:1208.3469

Network of perfect tensors



Holographic hexagon state

- A toy model of (discretized) AdS using perfect tensors
  Pastawski etal arXiv:1503.06237
- A perfect tensor guarantees maximal entanglement for subsystems

$$T = \frac{1}{3}\left(|0000\rangle + |0111\rangle + |0222\rangle + |1012\rangle + |1120\rangle + |1201\rangle + |2021\rangle + |2102\rangle + |2210\rangle\right)$$

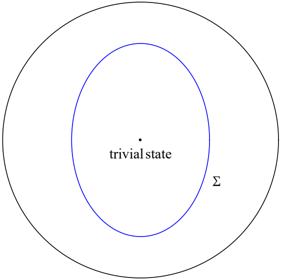
- spacetime emerges via an error correction code

figures courtesy of arXiv:1503.06237

Surface/state correspondence

Miyaji & Takayanagi arXiv:1503.03542

Surface in bulk $\Sigma \leftrightarrow$ State $|\Psi(\Sigma)\rangle$ in the CFT



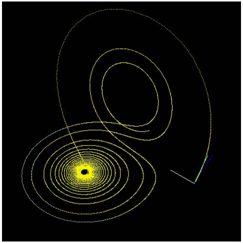
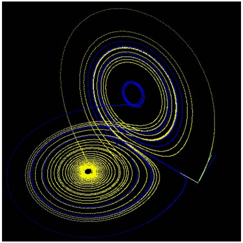
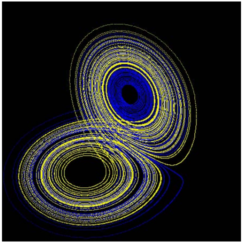
- Continuous version of MERA, *i.e.*, CMERA
- Pure states $|\Psi(\Sigma)\rangle$ related by unitary transformation
- Trivial state has no spatial entanglement

Chaos

- Chaotic behavior is very common. It tells the system is highly sensitive to the initial condition

$$|\delta x(t)| \sim e^{\lambda_L t}|\delta x(0)|$$

- Definition of quantum chaos is less clear. OTOC provides a way to diagnose chaos.

A bound on Lyapunov exponent

Maldacena, Shenker & Stanford arXiv:1503.01409

Quantum mechanically, the dependence on initial condition is characterized by the commutator

$$C(t) = -Z^{-1}\mathrm{Tr}[e^{-\beta H}[W(t), V(0)]^2]$$

Roughly speaking $C(t) \sim c_0 - F(t - i\beta/4) - F(t + i\beta/4)$. The early time behavior of $F(t)$ goes like

$$F(t) = \mathrm{Tr}[yV(0)yW(t)yV(0)yW(t)] \sim f_0 - f_1 e^{\lambda_L t}$$

where $y^4 = Z^{-1}e^{-\beta H}$
A bound on the Lyapunov exponent is conjectured

$$\boxed{\lambda_L \leq \frac{2\pi}{\beta}}$$

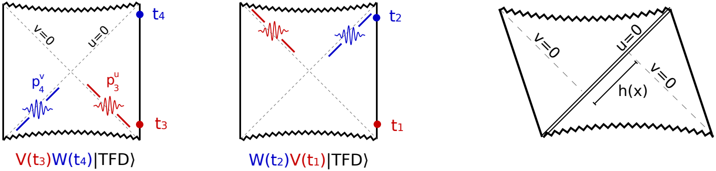
Holographic computation

The bound is saturated for holographic CFT

$$F(t) = f_0 - \frac{f_1}{N^2} e^{\frac{2\pi}{\beta} t} + \mathcal{O}(N^{-4})$$

OTOC can be checked gravitationally by the scattering of shock waves

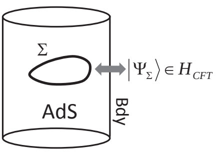
Shenker & Stanford arXiv:1306.0622, arXiv:1412.6087

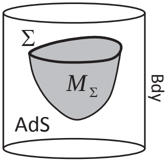


figures courtesy of arXiv:1412.6087

CMERA from path integral

Question: What if $W(t)$ is evolved radially





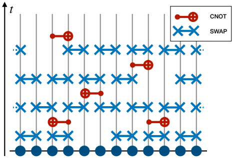
- CMERA is realized by Euclidean PI Takayanagi arXiv:1808.09072

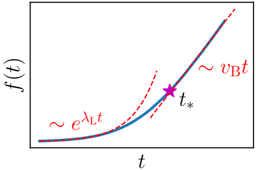
$$e^{C(M_\Sigma)} \cdot \Psi_\Sigma[\varphi_0(x)] = \int \left[\prod_{y\in M_\Sigma} D\varphi(y)\right] e^{-S^{CFT}_{M_\Sigma}[\varphi]} \prod_{x\in\Sigma} \delta(\varphi(x) - \varphi_0(x)),$$

- The evolution is generated by the same Hamiltonian and hence it is natural expect the same chaotic behavior

Random unitary circuit

Keselman, Nie and Berg arXiv:2009.10104
$V(0), W(0)$ are single-bit Pauli operators. $W(t)$ is evolved by randomly applying SWAP and CNOT gates





The integrated OTOC

$$f(t) = \sum_j C_{i,j}(t) = -\sum_j \langle [W_i(t), V_j(0)]^2 \rangle$$

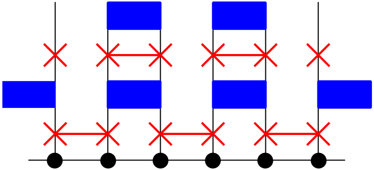
is found to grow exponentially

figures courtesy of arXiv:2009.10104

MERA network as a random circuit

XH w/ Binchao Zhang, work in progress

MERA tensor network is formulated in terms of random unitary circuit. The evolution from IR to UV

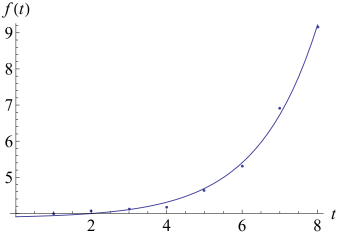


CNOT gates are replaced by the (dis)entanglers (blue boxes), whose number increases at every time step

$$|00\rangle \to \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\,, \quad |11\rangle \to \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|01\rangle \to \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)\,, \quad |10\rangle \to \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

OTOC of renormalized operators

ITensor package, Fishman, White, & Stoudenmire arXiv:2007.14822



$$N = 16,\ V(0) = \sigma_y, W(0) = \sigma_x,\ p = 0.9, r = 2^{t-9}$$

Exponential growth of iOTOC is obtained

$$f(t) \sim e^{\lambda_L t}, \quad \lambda_L \sim 0.6$$

Discussions

- Exponential growth could be trivial since it is quite universal
- Temperature is not well defined in the current scenario. It is not clear how the bound is imposed, if there is one
- Reproduce RT formula
- Other tensor networks like those made by perfect tensor

Summary

- Spacetime geometry emerges from entanglement renormalization
- Chaotic behavior shall leave its signature in the radial evolution of entanglement structure
- Preliminary numerical result supports the exponential growth of OTOC