NEC Violation in the Very Early Universe

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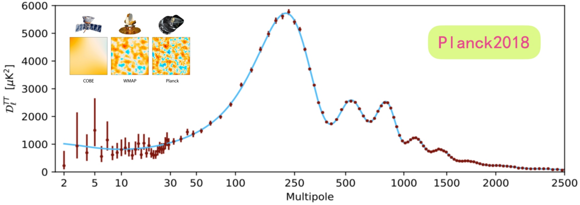
Outline

1. **Motivation**
2. **No-Go or feasible**
3. **Observable effects**
4. **Summary**

Phys.Rev.Lett. 133 (2024) 2, 021001;
Phys.Rev.D 103 (2021) 8, 083521;
Phys.Rev.D 107 (2023) 6, 063512;
JHEP 06 (2022) 067;
JHEP 02 (2024) 008;
JHEP 09 (2024) 067.

Outline

Motivations



- The choice of the **initial state** for **inflationary** primordial perturbations is troubled by the **cosmological singularity problem**.

$$\boxed{\lambda \propto a} \Rightarrow \lambda \simeq \mathcal{O}(1/\Lambda) \Rightarrow \textbf{higher-order derivative operators}$$

$$\boxed{u_k'' + \left(k^2 - \cancel{\frac{z''}{z}}\right) u_k = 0} \quad \rightarrow \quad \boxed{u_k'' + \left(\sim \sum_p \frac{k^{2p}}{\Lambda^{2p}}\right) k^2 u_k = 0}\,, \quad p \geqslant 0\,.$$

Bunch-Davis vacuum $\rightarrow$ **?**

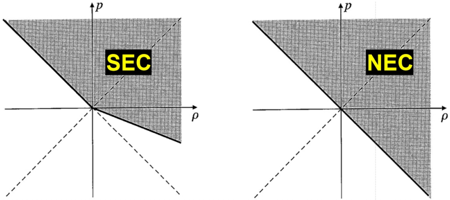
Motivations

- The **past-completion** requires that the null energy condition **(NEC) must be violated**, at least for some period, i.e.,

$$\boxed{\text{NEC violation}} \Rightarrow \boxed{R_{\mu\nu}k^{\mu}k^{\nu} < 0} \Rightarrow \boxed{\dot{H} > 0} \Rightarrow \boxed{\epsilon \equiv -\dot{H}/H^2 < 0},$$

since $R_{\mu\nu}k^{\mu}k^{\nu} = -2\dot{H}(k^0)^2$ for the *spatially flat FLRW metric.*

- One possible source of the **stochastic GW background** at the **PTA** scale: NEC violatioin during inflation.
- NEC is quite **robust** in nature.
- Can the NEC be **healthily violated**?
- What **observable effects** would arise if NEC violation occurred in the very early universe?



Outline

Why is it difficult to achieve NEC violation?

- A healthy NEC violation: **background** + **perturbations**.
- In **unitary gauge**, $S^{(2)}_{\zeta} = \int \mathrm{d}^4x\, a^3 Q_s \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right]$, $Q_s > 0$, $c_s^2 > 0$.

$$\boxed{Q_s < 0 \rightarrow \textbf{ghost instability}}, \quad \boxed{c_s^2 < 0 \rightarrow \textbf{gradient instability}}.$$

- $S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{M_p^2}{2} R + \mathbf{P}(\phi, \mathbf{X})\right]$, $X = \nabla_\mu\phi\nabla^\mu\phi = -\dot{\phi}^2$,

$$Q_s = c_1 = \epsilon M_p^2 + 2\frac{\dot{\phi}^4}{H^2} P_{XX}\,, \quad c_s^2 = \frac{2\dot{H}M_p^2}{2\dot{H}M_p^2 - 4\dot{\phi}^4 P_{XX}} = \boxed{\frac{\epsilon}{Q_s} M_p^2}.$$

When $Q_s > 0$, we will have $c_s^2 < 0$ during the NEC violation ($\epsilon < 0$).

- $S = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2} R + P(\phi, X) + \mathbf{G}(\phi, \mathbf{X})\Box\phi\right]$,

$$c_s^2 = \frac{H\gamma - \gamma^2 - \dot{\gamma}}{\gamma^2 Q_s} M_p^2\,, \quad \gamma = H - \frac{\dot{\phi}^3 G_X}{M_p^2} \Rightarrow \boxed{\text{~~NEC~~} \neq \textbf{instabilities}}$$

D. A. Easson, I. Sawicki and A. Vikman, **JCAP** 1111, 021 (2011);
A. Ijjas and P. J. Steinhardt, **Phys.Rev.Lett.** 117, no. 12, 121304 (2016).

From "no-go" to the EFT of fully stable nonsingular cosmology

"no-go" theorem

- "no-go" theorem for **cubic Galileon**: healthy nonsingular cosmological models based on the cubic Galileon does not exist.

  M. Libanov, S. Mironov and V. Rubakov, **JCAP** 1608, 08, 037 (2016)

- "no-go" theorem for full **Horndeski** theory.

  T. Kobayashi, **Phys.Rev.D** 94, no. 4, 043511 (2016)

- loophole: **strong gravity** in the past? Y. Ageeva, P. Petrov, V. Rubakov, **Phys.Rev.D** 104 (2021) 6, 063530

The EFT of fully stable nonsingular cosmology

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K\delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}\Big]$$

The **covariant** Lagrangian belongs to the **beyond Horndeski** theory (**GLPV**).

YC, Y. Wan, H. G. Li, T. Qiu and Y. S. Piao, **JHEP** 1701, 090 (2017), arXiv:1610.03400;
YC, H.-G. Li, T. Qiu, and Y.-S. Piao, **Eur.Phys.J.C** 77, 369 (2017), arXiv:1701.04330;
YC and Y. S. Piao, **JHEP** 1709, 027 (2017), arXiv:1705.03401.

- Do some fundamental properties of **UV-complete** theories or the **consistency requirements of EFT** (such as unitarity, causality, or local UV completion) **forbid** a violation of the NEC?

Bound of tree-level perturbative unitarity

$$|\mathcal{M}\left(A \to \{q_i\}\right)| \leq E^{4-(n_A+n_{q_i})} \simeq s^{2-(n_A+n_{q_i})/2} \,.$$

C. de Rham and S. Melville, **Phys.Rev.D** 95 (2017) 123523, arXiv: 1703.00025;

S. Kim, T. Noumi, K. Takeuchi and S. Zhou, **JHEP** 07 (2021) 018, arXiv: 2102.04101.

- We worked in the contexts of both Galileon and "beyond Horndeski" genesis cosmology. [**YC**, J. Xu, S. Zhao, S. Zhou, **JHEP** 10 (2022) 140, arXiv:2207.11772.]
- This direction requires further study.

Outline

Primordial perturbations

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)\,, \quad h_{ij} = a^2 e^{2\zeta}\left(e^{\gamma}\right)_{ij}\,, \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij}$$

Tensor perturbation (GWs)

$$S_\gamma^{(2)} = \frac{M_p^2}{8}\int d^4x\, a^3 Q_T \left[\dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial_k \gamma_{ij})^2}{a^2}\right],$$

$$\boxed{u_T'' + \left(c_T^2 k^2 - \frac{z_T''}{z_T}\right) u_T = 0}\,, \quad u_T = z_T \gamma_{\mathbf{k}}\,, \quad z_T = \frac{M_p\sqrt{Q_T}\,a}{2}\,, \quad \boxed{P_T \equiv \frac{k^3}{2\pi^2}|\gamma_{\mathbf{k}}|^2}\,.$$

Scalar perturbation

$$S_\zeta^{(2)} = \int d^4x\, a^3 Q_s \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial\zeta)^2}{a^2}\right].$$

$$\boxed{u_s'' + \left(c_s^2 k^2 - \frac{z_s''}{z_s}\right) u_s = 0}\,, \quad u_s = z_s \zeta_{\mathbf{k}}\,, \quad z_s = \sqrt{2a^2 Q_s}\,, \quad \boxed{P_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta_{\mathbf{k}}|^2}\,.$$

- The power spectrum depends on: (1) the evolution of background at **horizon crossing**, (2) the evolution of perturbations on **super-horizon** scales.

Primordial GWs

- **Sub-horizon** (Initial condition): BD vacuum or Minkowskian vacuum.
- **Horizon-crossing**: $c_T^2 k^2 \simeq z_T''/z_T$.
- **Super-horizon**: $u_T''/u_T \approx z_T''/z_T \Rightarrow \boxed{\gamma_k = \gamma_{\rm const} + \gamma_{\rm evol}}$,

$$\gamma_{\rm const} = C_k\,, \quad \boxed{\gamma_{\rm evol} = D_k \int z_T^{-2} \mathrm{d}\tau}, \quad z_T = \frac{M_p \sqrt{Q_T} a}{2}\,,$$

$$\boxed{z_T^2 \sim |\tau|^n}, \tau < 0 \;\to\; |\gamma_{\rm evol}| \sim |\tau|^{1-n} \;\to\; \begin{cases} n < 1\,, & |\gamma_{\rm evol}| \textit{ is a } \textbf{decaying} \textit{ mode.} \\ n > 1\,, & |\gamma_{\rm evol}| \textit{ is a } \textbf{growing} \textit{ mode.} \end{cases}$$

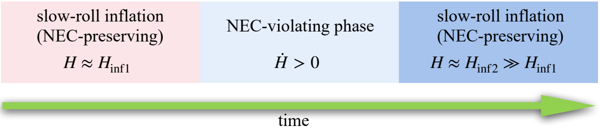
$$S = \int d^4x \sqrt{-g} \Big[\frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) \left(\delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu} \right) + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \Big]$$

$$f(t) = Q_T c_T^2\,, \qquad m_4^2 = \frac{M_p^2}{2} f(t) \left(\frac{1}{c_T^2} - 1 \right)$$

- For canonical **slow-roll inflation**, $Q_T = 1$, $a \propto |\tau|^{-1} \Rightarrow n = -2$, $|\gamma_{\rm evol}|$ is the decaying mode, $\gamma_k \approx \gamma_{\rm const}$ on super-horizon scales.
- For an **expanding universe**, in order for $|\gamma_{\rm evol}|$ to grow, $Q_T \equiv f + 2\frac{m_4^2}{M_p^2}$ must decrease over time, which can only happen if f decreases or c_T^2 increases with time.

  $f(t) \downarrow \Rightarrow$ **strong gravity**? $c_T^2 \uparrow \Rightarrow$ **superluminal**?
- Additionally, an increasing c_T would alter the horizon-crossing condition, leading to more suppression at horizon crossing than the growth outside the horizon. Overall, $P_T \simeq c_T^{-1}$ [**YC**, Y.-T. Wang, Y.-S. Piao, **Phys.Rev.D** 93 (2016) 6, 063005; **Phys.Rev.D** 94 (2016) 4, 043002].
- Considering $f = 1$, $m_4^2 = 0$, i.e., $\boxed{Q_T = 1}$, $\boxed{c_T^2 = 1}$, we enhance P_T by modifying the **background** evolution (i.e., H).
- How about bounce-inflation or Genesis-inflation scenarios? (focusing on CMB scale)

- A scenario motivated by PTA data: **inf 1 + NEC violation + inf 2**, $P_T \propto H^2$



$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}\mathrm{R} + P(\phi, X) + L_{\delta g^{00}\mathrm{R}^{(3)}}\right],$$

where $X = \nabla_\mu\phi\nabla^\mu\phi$, $L_{\delta g^{00}\mathrm{R}^{(3)}} = \frac{f(\phi)}{2}\delta g^{00}\mathrm{R}^{(3)}$, $P(\phi, X) = -\frac{g_1(\phi)}{2}M_{\rm P}^2 X + \frac{g_2(\phi)}{4}X^2 - M_{\rm P}^4 V(\phi)$.

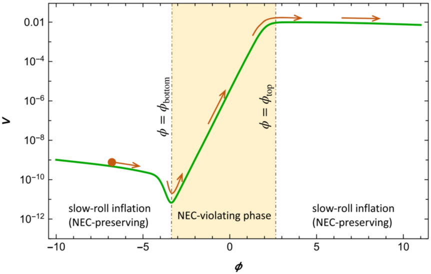
$$3H^2M_p^2 = \frac{M_p^2}{2}g_1\dot{\phi}^2 + \frac{3}{4}g_2\dot{\phi}^4 + M_p^4 V\,,$$

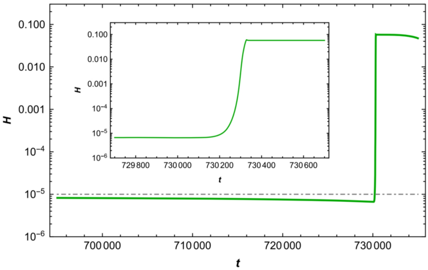
$$0 = \left(g_1 + \frac{3g_2\dot{\phi}^2}{M_p^2}\right)\ddot{\phi} + 3g_1 H\dot{\phi} + \frac{1}{2}g_{1,\phi}\dot{\phi}^2 + \frac{3g_2 H\dot{\phi}^3}{M_p^2} + \frac{3g_{2,\phi}\dot{\phi}^4}{4M_p^2} + M_p^2 V_{,\phi}\,,$$

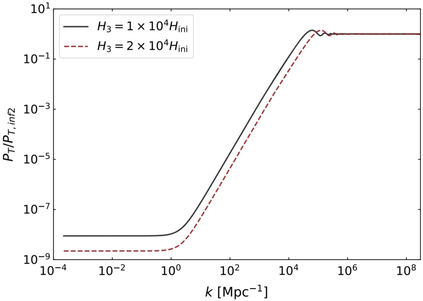
We set $g_1 < 0$ and $g_2 > 0$ around NEC violation, so that $3g_1 H\dot{\phi}$ acts as an **anti-friction** force, thereby **accelerating** the ϕ field.

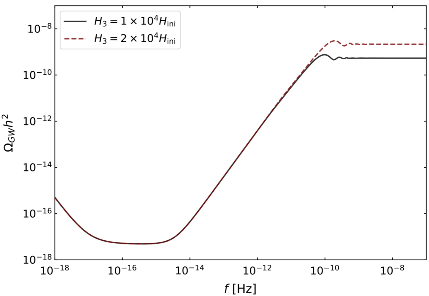
YC, Y.-S. Piao, **Phys.Rev.D** 103 (2021) 8, 083521.

Primordial GWs









- **inf 1 + NEC violation + inf 2**: $Q_T = 1,\ c_T = 1,\ \boxed{P_T \propto H^2}$.

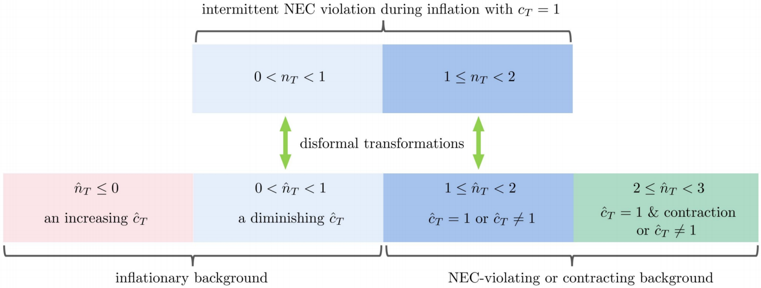
$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}\right],$$

- **slow-roll inflation with $c_T(\tau)$**, $\hat{Q}_T = \hat{c}_T^{-2},\ \hat{c}_T^2 = \left(1 + \frac{2\hat{m}_4^2}{M_p^2}\right)^{-1},\ \boxed{P_T \propto H^2/c_T}$.

$$S = \int d\hat{t}d^3\hat{x}\sqrt{-\hat{g}}\left[\frac{M_p^2}{2}\hat{R} - \hat{\Lambda}(t) - \hat{c}(t)\hat{g}^{00} - \hat{m}_4^2(t)\left(\delta\hat{K}^2 - \delta\hat{K}_{\mu\nu}\delta\hat{K}^{\mu\nu}\right)\right],$$

YC, Y.-S. Piao, **JHEP** 06 (2022) 067.

$$\boxed{\hat{g}_{\mu\nu} \to \hat{c}_T^{-1}\left[\hat{g}_{\mu\nu} + \left(1 - \hat{c}_T^2\right)\hat{n}_\mu\hat{n}_\nu\right]}$$

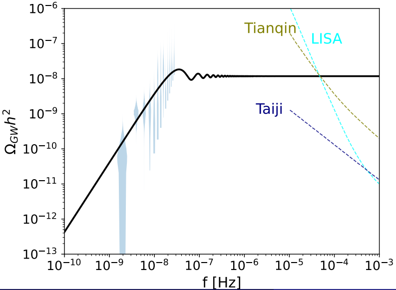


Parameterization of the power spectrum

$$P_T = P_{T,1} + \frac{\pi}{4}(2-n_T)\frac{k}{k_2}|g(k)|^2 P_{T,2}\,, \quad P_{T,2} \gg P_{T,1}\,, \quad f_c \equiv 2\pi k_2$$

$$g = H^{(1)}_{\frac{3-n_T}{2}}\left[\frac{2-n_T}{2}\frac{k}{k_2}\right]\sin\frac{k}{k_2} + H^{(1)}_{\frac{1-n_T}{2}}\left[\frac{2-n_T}{2}\frac{k}{k_2}\right]\left(\cos\frac{k}{k_2} - \frac{k_2}{k}\sin\frac{k}{k_2}\right).$$

$$\Omega_{\rm GW} = \frac{k^2}{12a_0^2H_0^2}\left[\frac{3\Omega_m j_l(k\tau_0)}{k\tau_0}\sqrt{1.0+1.36\frac{k}{k_{eq}}+2.50\left(\frac{k}{k_{eq}}\right)^2}\right]^2 P_T$$



Parameter	Prior
$\log_{10} P_{T,2}$	[-5,0]
n_T	[0, 2]
$\log_{10} f_c$	[-9,-5]

G. Ye, M. Zhu, **YC**, **JHEP** 02 (2024) 008.

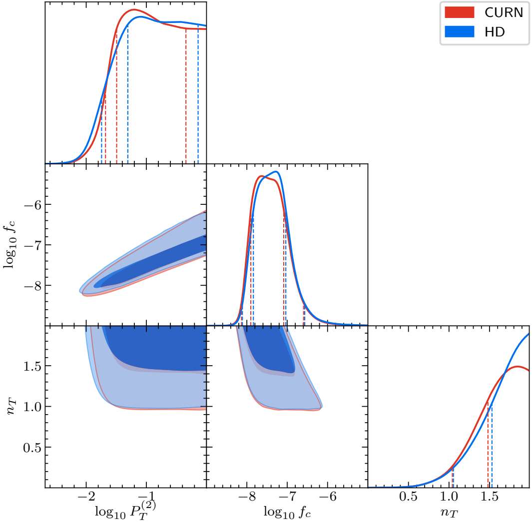


Fig: 68% and 95% posterior distributions of model parameters.

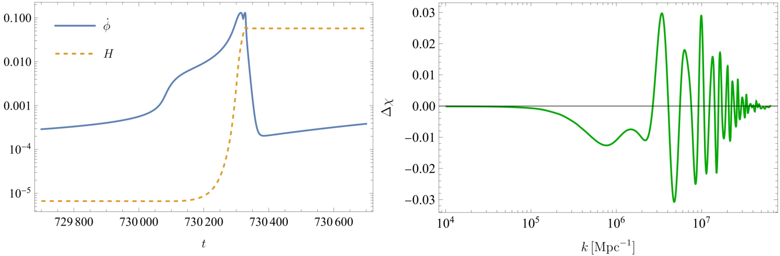
Parity-violating primordial GWs from NEC violation

$$S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R-M_p^2\frac{g_1(\phi)}{2}X+\frac{g_2(\phi)}{4}X^2-M_p^4V(\phi)+\frac{g_3(\phi)}{8}R\wedge R+L_{\rm HD}\right],$$

$$u_{\mathbf{k}}^{(s)\prime\prime}+\left[\left(c_{\rm T\mathbf{k}}^{(s)}\right)^2k^2-\frac{z_{\rm T}^{(s)\prime\prime}}{z_{\rm T}^{(s)}}\right]u_{\mathbf{k}}^{(s)}=0\,,\quad u_{\mathbf{k}}^{(s)}\equiv z_{\rm T}^{(s)}\gamma_{\mathbf{k}}^{(s)}\,,$$

$$c_{\rm T\mathbf{k}}^{(s)}=1\,,\quad z_{\rm T}^{(s)}=\frac{a}{2}\sqrt{1-\lambda^{(s)}\frac{k}{a^2}\frac{g_3'}{M_p^2}}\,,\quad \frac{k}{a^2}\frac{g_3'}{M_p^2}<1\,,\quad g_3(\phi)=\phi\,,$$

$$P_{\rm T}^{(s)}=\frac{k^3}{2\pi^2}\left|\gamma_{\mathbf{k}}^{(s)}\right|^2\,,\qquad \boxed{\Delta\chi=\frac{P_{\rm T}^{(\rm L)}-P_{\rm T}^{(\rm R)}}{P_{\rm T}^{(\rm L)}+P_{\rm T}^{(\rm R)}}}\,.$$



Parity-violating primordial GWs from NEC violation

- NEC violation during inflation naturally **enhances both P_T and $\Delta\chi$**, and the scales at which they reach their maximum values are nearly identical.
  YC, **Phys.Rev.D**, 107 (2023) 6, 063512.
- What would happen if the **spectator** couples to the parity-violating term? The Chern-Simons term poses a ghost risk, so we consider the Nieh-Yan term instead.

$$\mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{NY} = \mathcal{L}_\phi - \nabla_\mu\psi\nabla^\mu\psi/2 - U(\psi) + \frac{\xi F(\psi)}{4}\mathcal{T}_{A\mu\nu}\tilde{\mathcal{T}}^{A\mu\nu},$$

$$u_k^{(s)\prime\prime} + \left[c_T^{(s)2}k^2 - \frac{a''}{a}\right]u_k^{(s)} = 0, \quad c_T^{(s)2} \equiv 1 + \mu^{(s)}, \quad \mu^{(s)} = \lambda^{(s)}\xi F'/k.$$

$|\dot{\psi}| \ll |\dot{\phi}|$, ψ does not have the acceleration mechanism that ϕ has.

$$\int_{t_i}^{t} \frac{c_T^{(s)}(\tilde{t})}{a(\tilde{t})} d\tilde{t} \simeq \frac{2\pi}{k}$$

If $\xi\dot{\psi}$ reaches a level comparable to the maximum value of $\xi\dot{\phi}$, $k_{\rm max}^{(s)}$ will be much smaller. As a result, the scales at which $\Delta\chi$ and P_T reach their maximum values will be offset.

Z.-W. Jiang, **YC**, F. Wang, Y.-S. Piao, **JHEP** 09 (2024) 067.

Primordial curvature perturbations

- **Sub-horizon** (Initial condition): BD vacuum or Minkowskian vacuum.
- **Horizon-crossing**: $c_s^2 k^2 \simeq z_s''/z_s$.
- **Super-horizon**: $u_s''/u_s \approx z_s''/z_s$,

$$\boxed{\zeta \equiv u_s/z_s = \zeta_c + \zeta_e}\,, \quad \zeta_c = \text{const}\,, \quad \boxed{\zeta_e \propto \int \frac{d\tau}{z_s^2}} \to |\dot{\zeta}| = \frac{1}{a}|\zeta'| = \frac{D_k}{a z_s^2}\,,$$

where $z_s = \sqrt{2a^2 Q_s}$, D_k is a k-dependent integration constant.

$$\boxed{z_s^2 \sim |\tau|^n}, \tau < 0 \to |\zeta_e| \sim |\tau|^{1-n} \to \begin{cases} n < 1\,, & |\zeta_e| \textit{ is a } \textbf{decaying} \textit{ mode.} \\ n > 1\,, & |\zeta_e| \textit{ is a } \textbf{growing} \textit{ mode.} \end{cases}$$

- Canonical **slow-roll inflation**: $z_s^2 = 2a^2 Q_s$, $Q_s = \epsilon M_p^2$, $a \sim |\tau|^{-1}$, $z_s^2 \sim \epsilon|\tau|^{-2}$, if $\epsilon =$ const., then $n = -2$, ζ_e is the decaying mode, $\zeta \approx \zeta_c$ on super-horizon scales.
- **Ultra slow-roll inflation**: $\epsilon \sim |\tau|^m$, $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H} = -m$, $\eta = -6 \to m = 6$, $\epsilon \sim |\tau|^6 \searrow$, $z_s^2 \sim \epsilon|\tau|^{-2} \sim |\tau|^4$. Therefore, ζ_e is a growing mode, $\zeta \approx \zeta_e$ on super-horizon scales.

- **slow-roll inflation:** $P_\zeta \propto H^2/\epsilon$
- **inf 1 + NEC violation ($\dot{H} > 0$) + inf 2**:

$$\boxed{\dot{H} > 0 \Rightarrow H \uparrow} \quad \text{and} \quad \boxed{|\epsilon| \equiv |\dot{H}/H^2| \downarrow} \quad (|\epsilon| \gg 1)$$

During NEC violation, $Q_s \approx 3|\epsilon|$.

background at **horizon-crossing** $(H \uparrow)$ growth of ζ_e on **super horizon** scales $(|\epsilon| \downarrow)$	$\Rightarrow$ **enhancement of P_ζ** .

$\Downarrow$

- ✓ **Primordial black holes**
- ✓ **Scalar-induced GWs**
- ✓ **Primordial GWs**

YC, M. Zhu, Y.-S. Piao, **Phys.Rev.Lett.** 133 (2024) 2, 021001. arXiv: 2305.10933.

- **Primordial curvature perturbations:**

$$S=\int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}{\rm R}+P(\phi,X)+L_{\delta g^{00}{\rm R}^{(3)}}\right],\quad L_{\delta g^{00}{\rm R}^{(3)}}=\frac{f(\phi)}{2}\delta g^{00}{\rm R}^{(3)},$$

$$Q_s=\frac{2\dot\phi^4 P_{XX}-M_{\rm P}^2\dot H}{H^2},\quad c_s^2=\frac{M_{\rm P}^2}{Q_s}\left(\frac{\dot c_3}{a}-1\right)=1,\quad c_3=a\left(1+\frac{2f}{M_{\rm P}^2}\right)/H$$

$$\boxed{v_k''+\left(k^2-\frac{z_s''}{z_s}\right)v_k=0},\quad v_k=z_s\zeta\,,\quad z_s=\sqrt{2a^2Q_s}\quad\Rightarrow\quad\boxed{\mathbf{P}_\zeta}$$

- **Primordial black holes:**

$$\beta(R)\simeq\frac{\sigma_R}{\sqrt{2\pi}\delta_c}e^{-\frac{\delta_c^2}{2\sigma_R^2}},\quad \sigma_R^2\equiv\langle\delta_R^2\rangle=\int_0^\infty\frac{dk}{k}W^2\frac{16}{81}(kR)^4T^2(k,\tau=R)\mathbf{P}_\zeta(\mathbf{k}),$$

$$T(k,\tau)\equiv\frac{9\sqrt3}{(k\tau)^3}\left[\sin\left(\frac{k\tau}{\sqrt3}\right)-\frac{k\tau}{\sqrt3}\cos\left(\frac{k\tau}{\sqrt3}\right)\right],$$

$$R\equiv k^{-1},\quad \frac{M}{M_\odot}\simeq\left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-\frac16}\left(\frac{k}{1.9\times10^6{\rm Mpc}^{-1}}\right)^{-2},$$

$$\mathbf{f}_{\rm PBH}(\mathbf{M})=\frac{\beta(M)}{2.70\times10^{-8}}\left(\frac{k}{1.9\times10^6{\rm Mpc}^{-1}}\right)^{-2}.$$

Primordial curvature perturbations

- **Scalar-induced GWs:**

$$P_h(\tau,k) = 576\int_0^\infty dt\int_{-1}^{1} ds P_\zeta\left(k\frac{t+s+1}{2}\right)$$
$$\times P_\zeta\left(k\frac{t-s+1}{2}\right)\frac{[-5+s^2+t(2+t)]^4}{(1-s+t)^6(1+s+t)^6}$$
$$\times\left\{\left[\frac{s^2-(t+1)^2}{-5+s^2+t(2+t)}+\frac{1}{2}\ln\left|\frac{-2+t(2+t)}{3-s^2}\right|\right]^2+\frac{\pi^2}{4}\Theta(t-\sqrt{3}+1)\right\}.$$

- **Primordial tensor perturbations (GWs):**

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t)\,,\quad h_{ij} = a^2e^{2\zeta}\left(e^{\gamma}\right)_{ij}\,,\quad \gamma_{ii} = 0 = \partial_i\gamma_{ij}$$

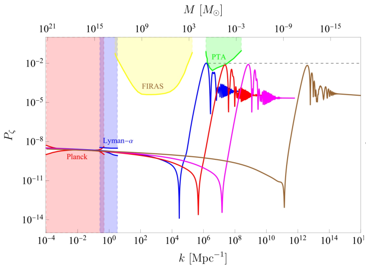
$$\boxed{P_T \equiv \frac{k^3}{2\pi^2}|\gamma_{\mathbf{k}}|^2}.$$

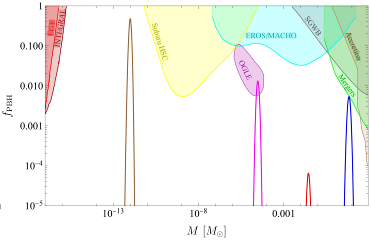
- **Distinctive features:**

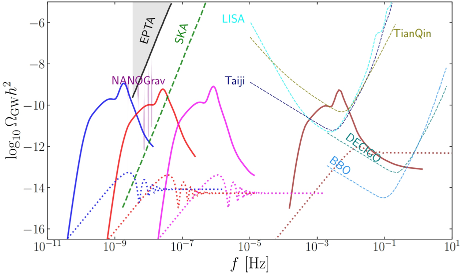
NEC violation ($\dot{H} > 0$) **during inflation** $\Rightarrow$ **enhancement of** $\begin{cases} P_\zeta \\ P_T \end{cases}$ $\Rightarrow$

- ✓ **Primordial black holes**
- ✓ **Scalar-induced GWs**
- ✓ **Primordial GWs**

YC, M. Zhu, Y.-S. Piao, **Phys.Rev.Lett.** 133 (2024) 2, 021001. arXiv: 2305.10933.







YC, M. Zhu, Y.-S. Piao, **Phys.Rev.Lett.** 133 (2024) 2, 021001. arXiv: 2305.10933.

Outline

Summary

- NEC is crucial to the proof of the Penrose's singularity theorem.
- NEC is quite robust in nature.
- A fully stable NEC violation can be realized in the "beyond Horndeski" (GLPV) theory.
- A combination of PBHs, SIGW signals, and primordial GWs can serve as a valuable probe for exploring the NEC violation during inflation.

NEC violation ($\dot{H} > 0$) **during inflation** $\Rightarrow$ **enhancement of** $\begin{cases} P_\zeta \\ P_T \end{cases}$ $\Rightarrow$

- ✓ **Primordial black holes**
- ✓ **Scalar-induced GWs**
- ✓ **Primordial GWs**

Thanks!