彭桓武高能基础理论研究中心（合肥）
Peng Huanwu Center for Fundamental Theory

2024引力与宇宙学专题研讨会

Quasinormal modes and scalar clouds of a rotating BTZ-like black hole in Einstein-bumblebee gravity

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Based on: Wang*, Chen, Tong, **Pan*** and Jing, *PRD* 103, 064079 (2021)

Wang, Chen, **Pan*** and Jing, *EPJC* 81, 469 (2021)

Chen, **Pan*** and Jing*, *PLB* 846, 138186 (2023)

Quan, **Pan*** and Jing, to appear

安徽·合肥 2024年11月15日

Outline

1. **Brief introduction to quasinormal modes (QNMs) and clouds**

2. **Scalar perturbation around rotating Einstein-bumblebee BTZ black holes under Robin boundary conditions: quasinormal modes and clouds**

3. **Conclusions and discussions**

1. Brief introduction to QNMs and clouds

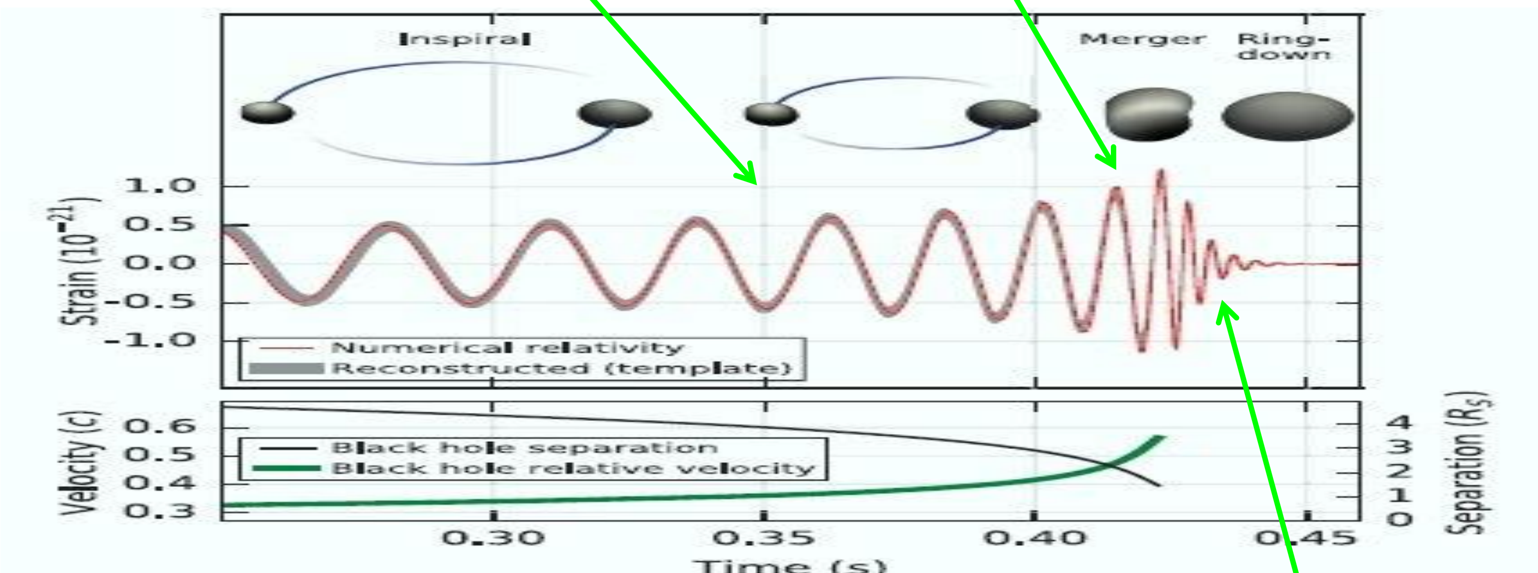
GW150914： 并合将大约3倍太阳质量转换成引力波能量

B. P. Abbott, et.al. PRL 116, 061102 (2016)

并合双黑洞演化过程主要分为三个阶段

旋近(Inspiral)

并合(Merger)



Cai, Cao, Guo, Wang and Yang, *NSR* 4, 687 (2017)

Gong, Luo, and Wang, *Nat. Astron.* 5, 881-889 (2021)

铃宕(Ringdown)

黑洞微扰理论

R.A. Konoplya and A. Zhidenko, *Rev. Mod. Phys.* 83, 793 (2011)

1957年，Regge和Wheeler采用线性微扰理论研究了Schwarzschild黑洞时空自身扰动的情况

$$g_{\mu\nu} = g_{\mu\nu}^{background} + h_{\mu\nu}$$

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_l^{(s)}(r)\right]\psi_l(r_*, \omega) = 0$$

$$\omega = \omega_R + i\omega_I$$

The perturbations: $\Psi \sim e^{-i\omega t} = e^{\omega_I t} e^{-i\omega_R t}$

$\omega_I < 0$ Quasinormal modes (QNMs)

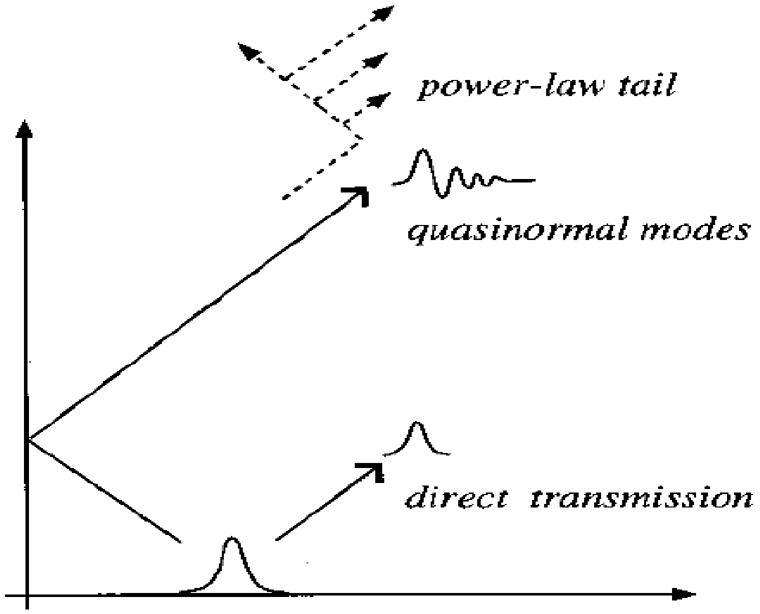
$\omega_I > 0$ Superradiance

$\omega_I = 0,\ \omega = \omega_R$ Clouds

1.1 Quasinormal modes (QNMs)

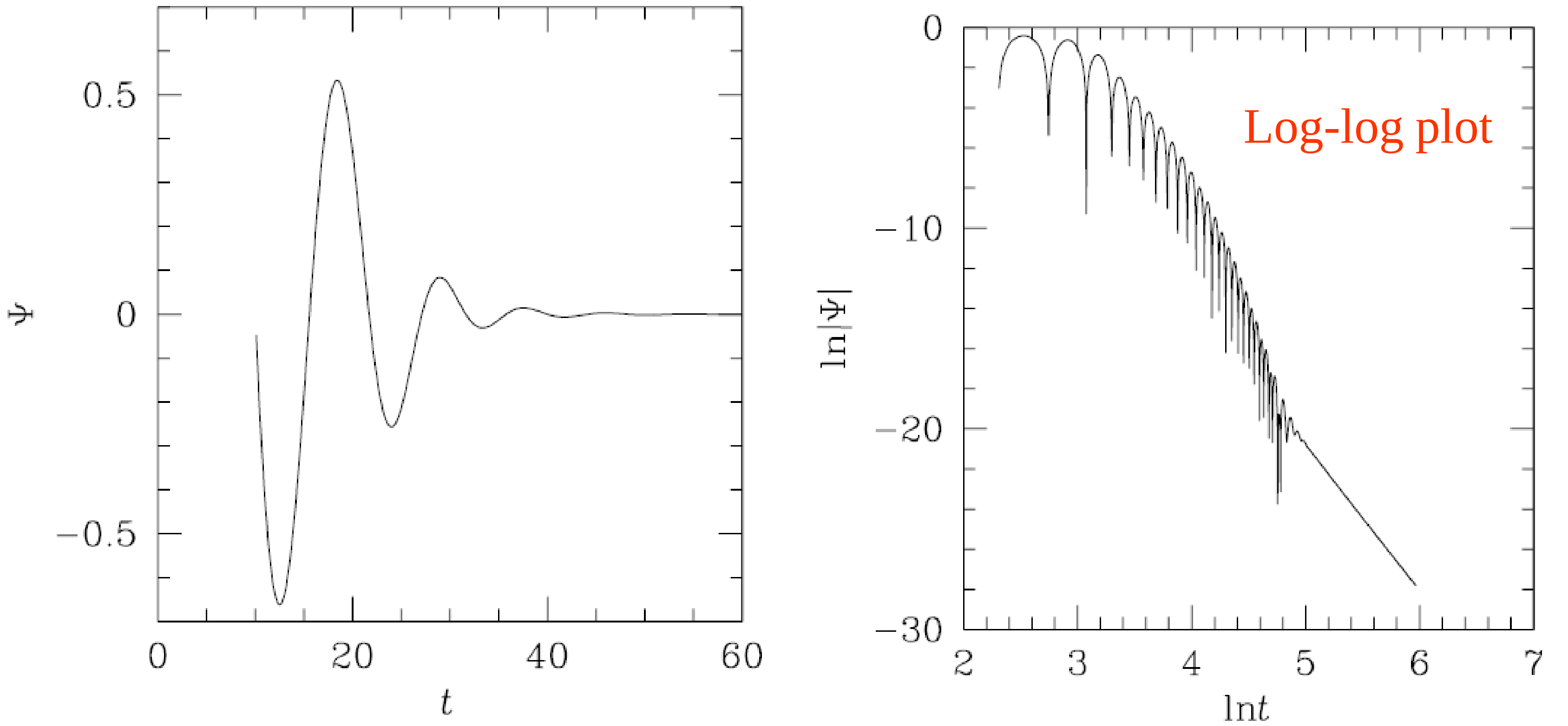
微扰场在黑洞时空中的演化可以分为三个阶段

- ★ 初始波爆发阶段
- ★ 似正模(quasinormal modes-QNMs)阶段
- ★ 幂率拖尾(power-law tail)阶段



Adapted from Fig. 1 of **N. Andersson, *Phys. Rev. D* 55, 468 (1997)**

Schwarzschild黑洞时空中微扰场演化示意图



$$\omega = \omega_R + i\omega_I$$ characteristic sound of black holes

Adapted from Figs. 1.1 and 1.2 of **V. Cardoso, arXiv:gr-qc/0404093**

QNMs is interesting:

- **asymptotically flat spacetimes**

  (test gravity in strong field regime by gravitational waves ...)

- **asymptotically dS spacetimes**

  (test strong cosmic censorship ...)

- **asymptotically AdS spacetimes**

  (AdS/CFT correspondence ...)

Asymptotically AdS (AAdS) spacetime is interesting:

- holography (entanglement entropy, complexity, condensed matter, ...)
- extended phase space
- various nonlinear solutions
- instabilities (superradiance, …)
- boundary conditions (implications to other aspects)
- ....

Boundary conditions in asymptotically flat spacetimes:

a concrete example (for massive scalar fields)

quasinormal modes: ingoing at the horizon while outgoing boundary conditions at infinity

quasi-bound states: ingoing at the horizon while decaying boundary conditions at infinity

scattering states: ingoing at the horizon while both ingoing and outgoing boundary conditions at infinity

Boundary conditions in AAdS spacetimes:

(1) at the horizon: an ingoing wave boundary condition

(2) At infinity: energy flux should be vanished

The idea: the AdS boundary may be regarded as a perfectly reflecting mirror, in the sense that NO energy flux can cross it.

Bifurcation of the Maxwell quasinormal spectrum on asymptotically anti-de Sitter black holes

Wang*, Chen, Tong, **Pan*** and Jing, *PRD* 103, 064079 (2021)
Wang, Chen, **Pan*** and Jing, *EPJC* 81, 469 (2021)

The background geometry of Schwarzschild–AdS BHs both without and with a global monopole

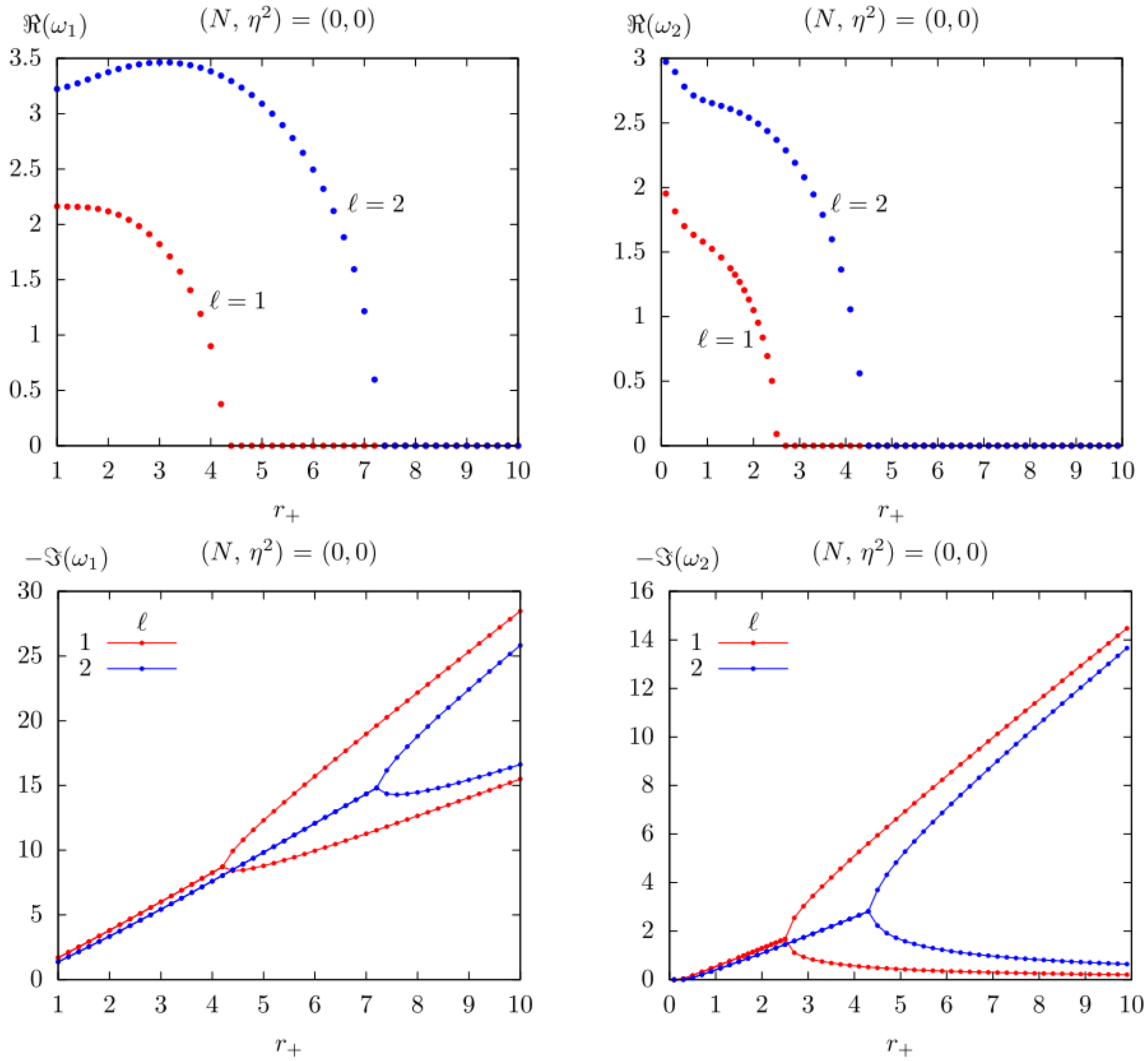
$$ds^2 = \frac{\Delta_r}{r^2}dt^2 - \frac{r^2}{\Delta_r}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

$$\tilde{\eta}^2 \equiv 1 - 8\pi\eta^2$$

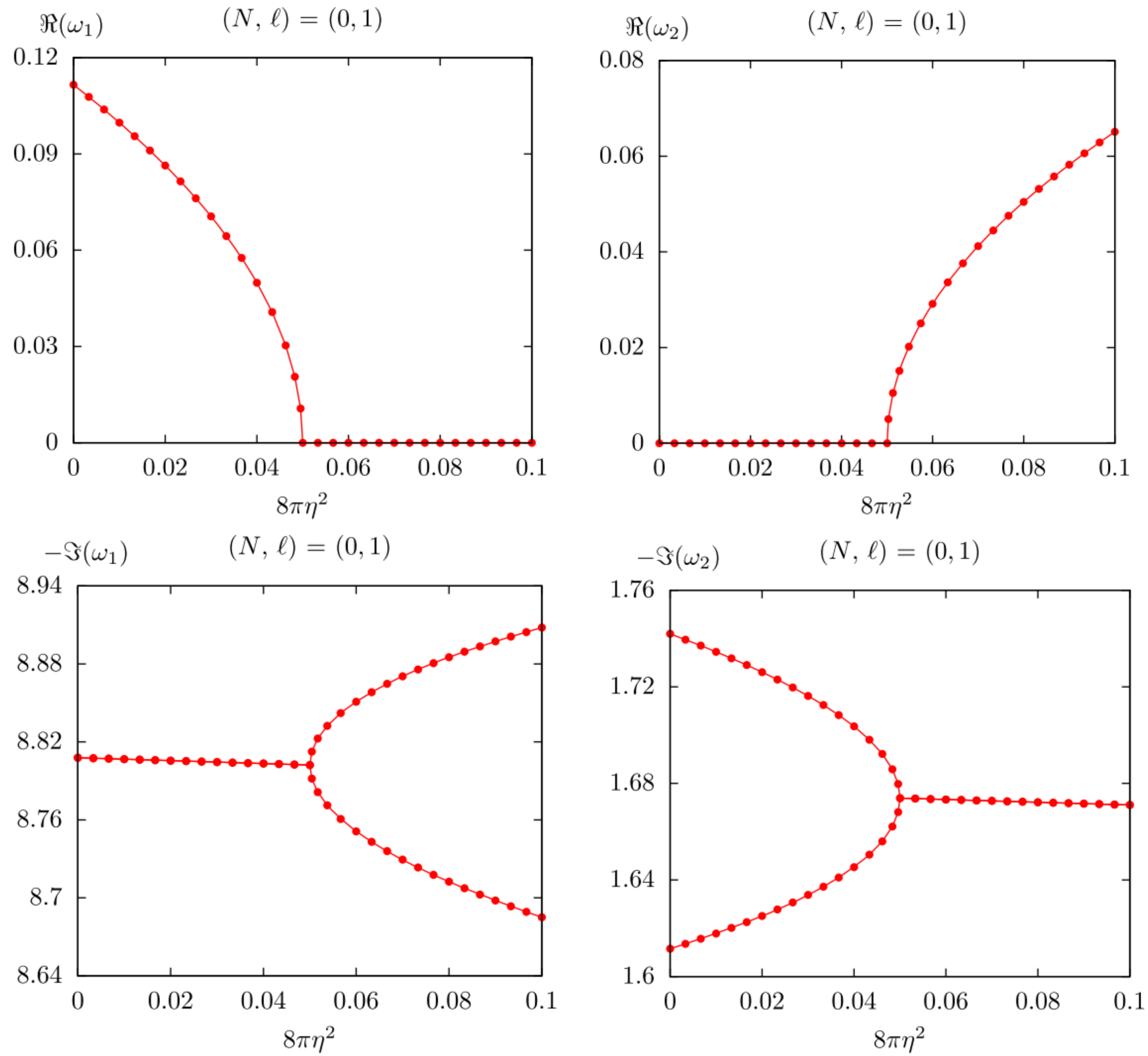
with the metric function $\Delta_r \equiv r^2\left(\tilde{\eta}^2 + \frac{r^2}{L^2}\right) - 2Mr$

The Maxwell equations $\nabla_\nu F^{\mu\nu} = 0$

the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$



When the BH is larger than the critical BH radius, the real part of QNMs turns into zero while the imaginary part branches off into two sets of modes---**mode split effect**



By fixing a proper BH radius, **the first (second) boundary condition may trigger (terminate) the mode split effect**

1.2 Clouds

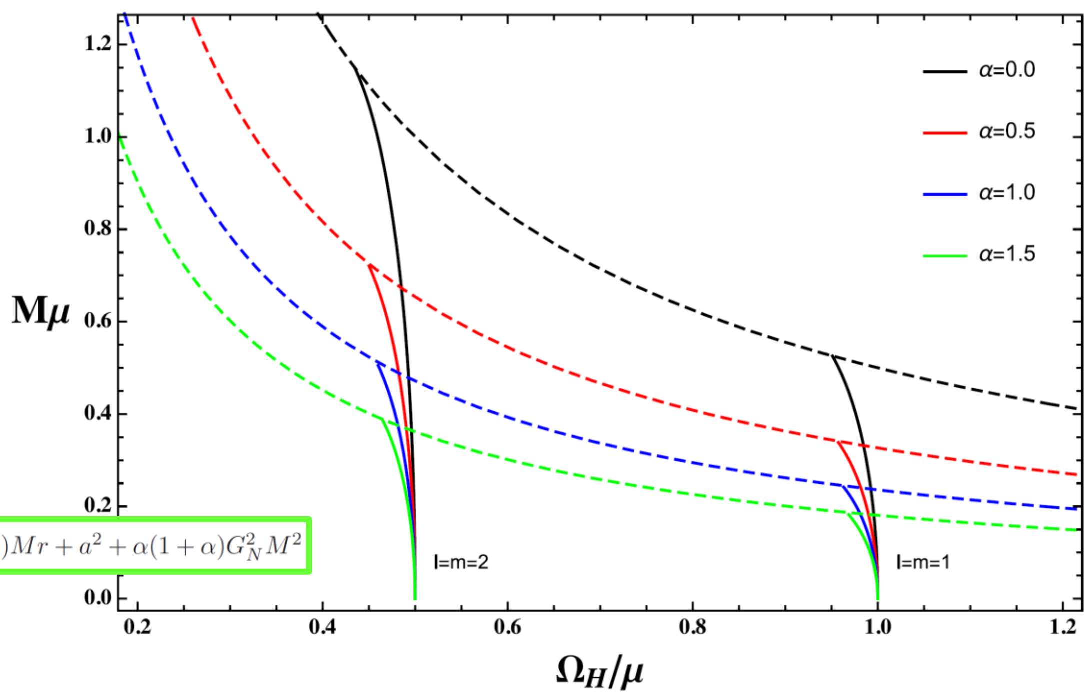
The field's frequency equals the critical frequency for superradiant scattering:

$$\omega = \omega_c = k\Omega_H$$

which allows the existence of stationary scalar configurations.

Kerr-MOG black holes with stationary scalar clouds

Qiao, Wang, **Pan*** and Jing, *EPJC* 80, 509 (2020)



$$\Delta \equiv r^2 - 2G_N(1+\alpha)Mr + a^2 + \alpha(1+\alpha)G_N^2 M^2$$

- 标量云的频谱中含有黑洞的信息，可以用来鉴别不同引力理论和黑洞时空
- 标量云是构造带“毛”黑洞解的预研

2. Scalar perturbation around rotating Einstein-bumblebee BTZ black holes under Robin boundary conditions: quasinormal modes and clouds

Chen, **Pan*** and Jing*, *PLB* 846, 138186 (2023)
Quan, **Pan*** and Jing, to appear

The rotating BTZ-like black hole in Einstein-bumblebee gravity

$$ds^2 = -f(r)dt^2 + \frac{(s+1)}{4rf(r)}dr^2 + r\left(d\theta - \frac{j}{2r}dt\right)^2$$

where $f(r) = \frac{r}{l^2} - M + \frac{j^2}{4r}$

$$S = \int d^3x\sqrt{-g}\left[\frac{1}{2\kappa}\left(R + \frac{2}{l^2}\right) + \frac{\xi}{2\kappa}B^{\mu\nu}R_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - V(B_\mu B^\mu \mp b^2)\right]$$

The scalar field equation of motion

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi\right) - \mu_0^2\psi = 0$$

The following factorized ansatz for the massive scalar field

$$\psi = e^{-i\omega t + im\theta} R(r)$$

leads to the second order differential equation for the radial part

$$z(1-z)\frac{d^2R(z)}{dz^2} + (1-z)\frac{dR(z)}{dz} + \left(\frac{A}{z} - \frac{B}{1-z} - C\right)R(z) = 0$$

with

$$z = \frac{r - r_+}{r - r_-}$$

$$A = \frac{l^4 r_+ (1+s)(\omega - \frac{jm}{2r_+})^2}{4(r_+ - r_-)^2}, \qquad B = \frac{l^2(1+s)\mu_0^2}{4}, \qquad C = \frac{l^4 r_- (1+s)(\omega - \frac{jm}{2r_-})^2}{4(r_+ - r_-)^2}.$$

The asymptotic behavior for radial function

$$R(z) = A_I(1-z)^{\beta} F(a, b, a+b-c+1; 1-z) + A_{II}(1-z)^{1-\beta} F(c-a, c-b, c-a-b+1; 1-z)$$

The new constants are given by

$$A_I = \frac{C_1 \Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \qquad A_{II} = \frac{C_1 \Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}$$

The vanishing energy flux leads to

$$\frac{A_{II}}{A_I} = \frac{\Gamma(c-a)\Gamma(c-b)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)\Gamma(c-a-b)} = \kappa$$

Different types of boundary conditions:

Dirichlet $-\kappa \to \infty$ (i.e., $A_I = 0$)

Newmann $-\kappa = 0$ (i.e., $A_{II} = 0$)

Robin $A_{II}/A_I = \kappa$

2.1 Quasinormal modes

(1) Dirichlet boundary condition $-\kappa \to \infty$ (i.e., $A_I = 0$)

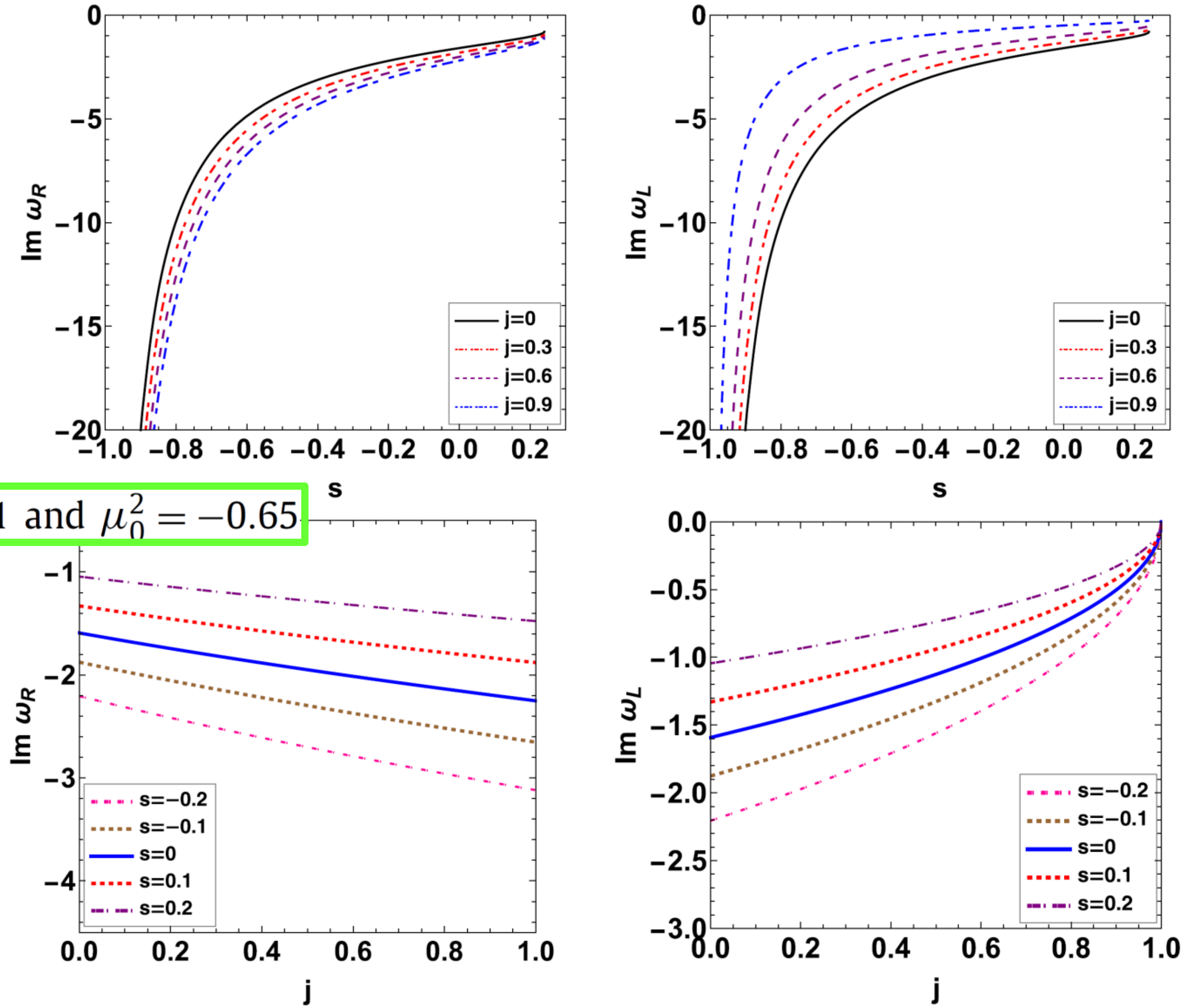
Right-moving QNMs:

$$\omega_R = -\frac{m}{l} - i\frac{2(\sqrt{r_+} + \sqrt{r_-})}{l^2\sqrt{1+s}}\left[n + \frac{1}{2} + \frac{1}{2}\sqrt{1 + l^2(1+s)\mu_0^2}\right]$$

Left-moving QNMs:

$$\omega_L = \frac{m}{l} - i\frac{2(\sqrt{r_+} - \sqrt{r_-})}{l^2\sqrt{1+s}}\left[n + \frac{1}{2} + \frac{1}{2}\sqrt{1 + l^2(1+s)\mu_0^2}\right]$$

$M = 1,\ l = 1$ and $\mu_0^2 = -0.65$



The **positive** symmetry breaking parameter smakes the absolute values of the imaginary parts **decrease**, but the **negative** smakes them **increase**

(2) Neumann boundary condition $-\kappa = 0$ (i.e., $A_{II} = 0$)

Right-moving QNMs:

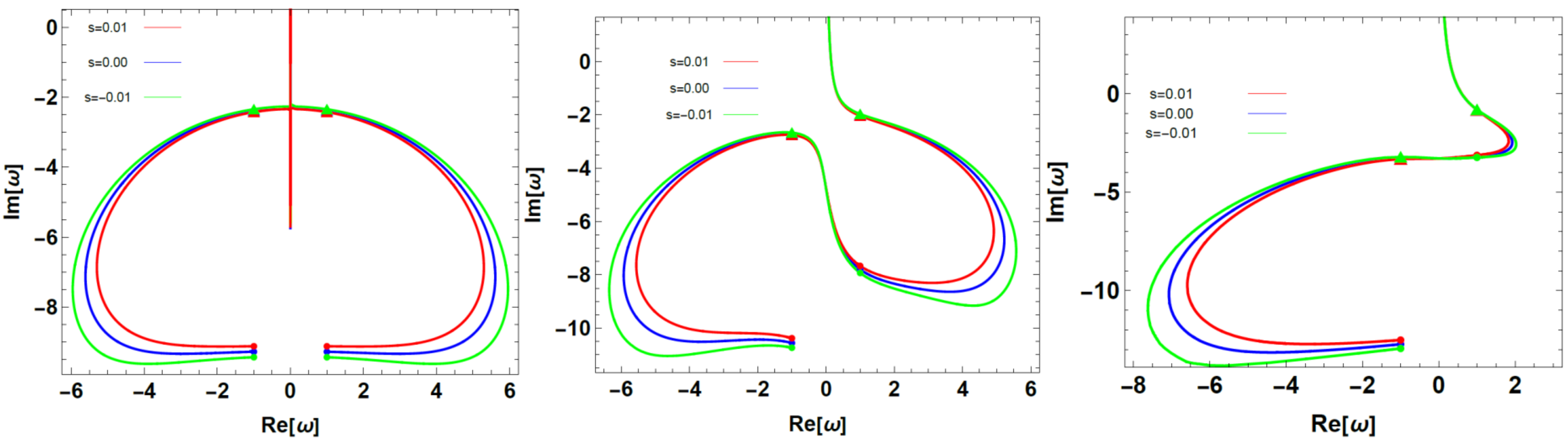
$$\omega_R = -\frac{m}{l} - i\frac{2(\sqrt{r_+} + \sqrt{r_-})}{l^2\sqrt{1+s}}\left[n + \frac{1}{2} - \frac{1}{2}\sqrt{1 + l^2(1+s)\mu_0^2}\right]$$

Left-moving QNMs:

$$\omega_L = \frac{m}{l} - i\frac{2(\sqrt{r_+} - \sqrt{r_-})}{l^2\sqrt{1+s}}\left[n + \frac{1}{2} - \frac{1}{2}\sqrt{1 + l^2(1+s)\mu_0^2}\right]$$

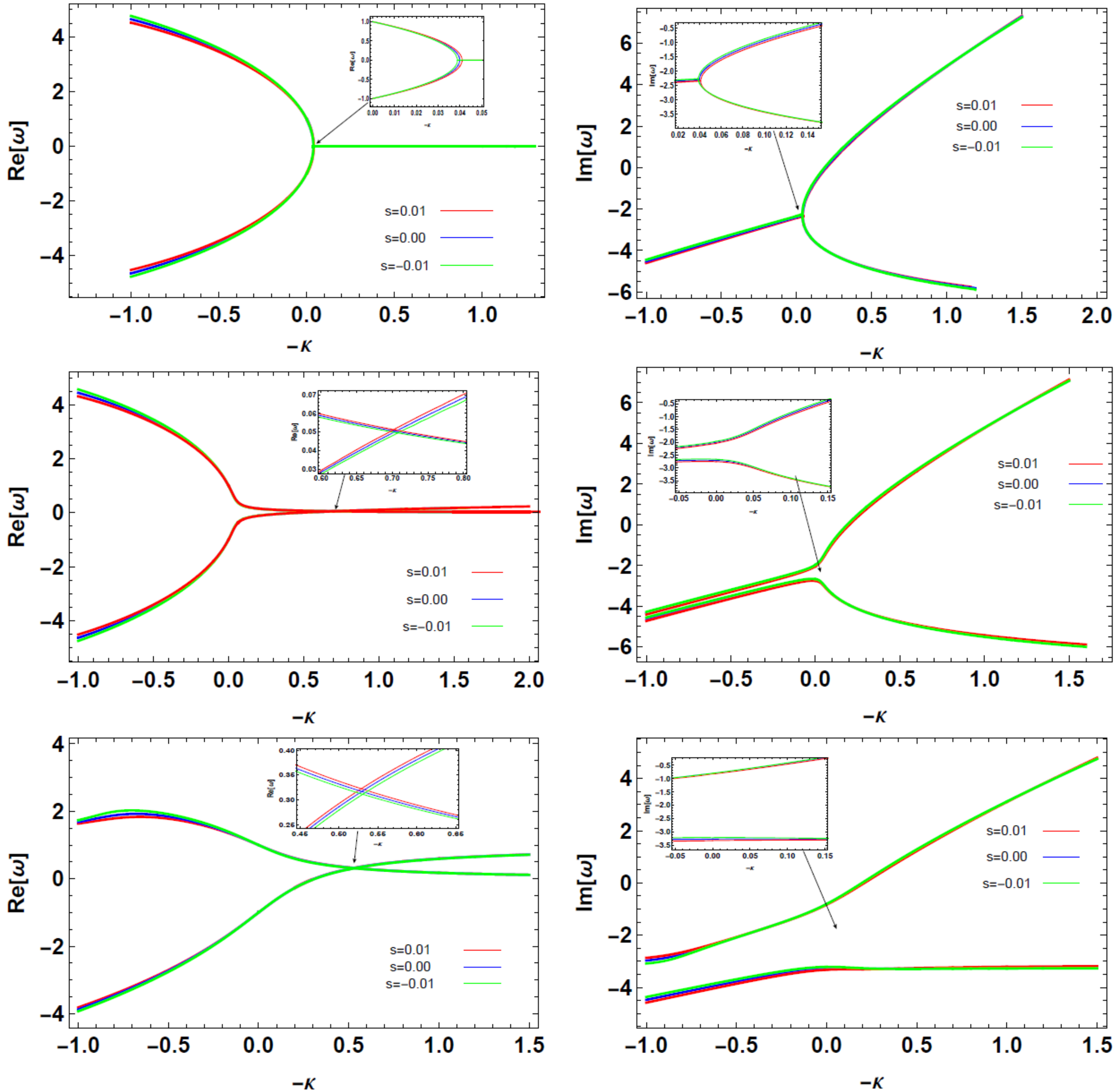
(3) Robin boundary condition $A_{II}/A_{I} = \kappa \;\; (\kappa \in \mathbb{R})$

j=0 *j*=10 *j*=30



$$M = 34,\; l = 1,\; n = 0 \text{ and } \mu_0^2 = -0.65.$$

The change of QNM frequencies with the Lorentz symmetry breaking parameter *s* **depends on the parameter of Robin boundary conditions**

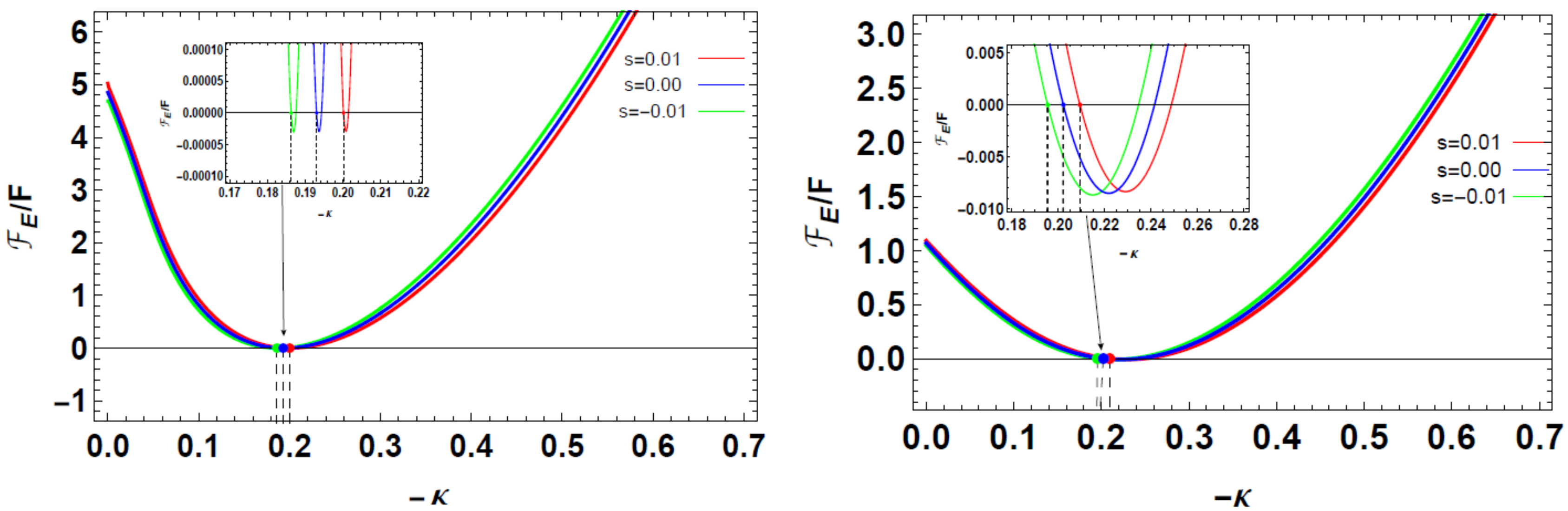


Re[ω]
−κ
s=0.01
s=0.00
s=−0.01
Im[ω]
−κ
s=0.01
s=0.00
s=−0.01
Re[ω]
−κ
s=0.01
s=0.00
s=−0.01
Im[ω]
−κ
s=0.01
s=0.00
s=−0.01
Re[ω]
−κ
s=0.01
s=0.00
s=−0.01
Im[ω]
−κ
s=0.01
s=0.00
s=−0.01

2.2 Superradiance

The energy flux across the event horizon of the black hole:

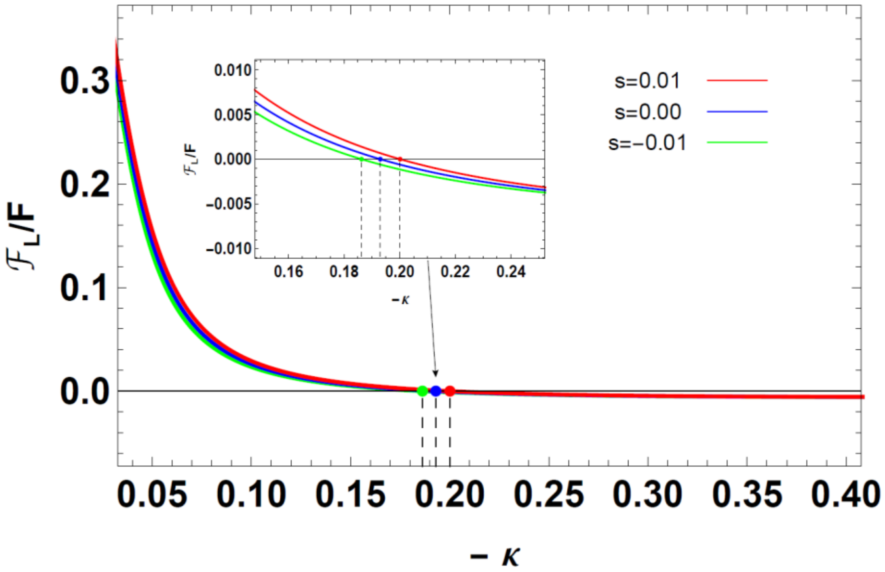
$$\mathcal{F}_E(v) = \int_0^{2\pi} \mathrm{d}\hat{\varphi}\sqrt{r_+}\chi_\mu T^\mu_\nu k^\nu = F[Im[\omega]^2 + Re[\omega](Re[\omega] - m\Omega_{\mathcal{H}})]$$

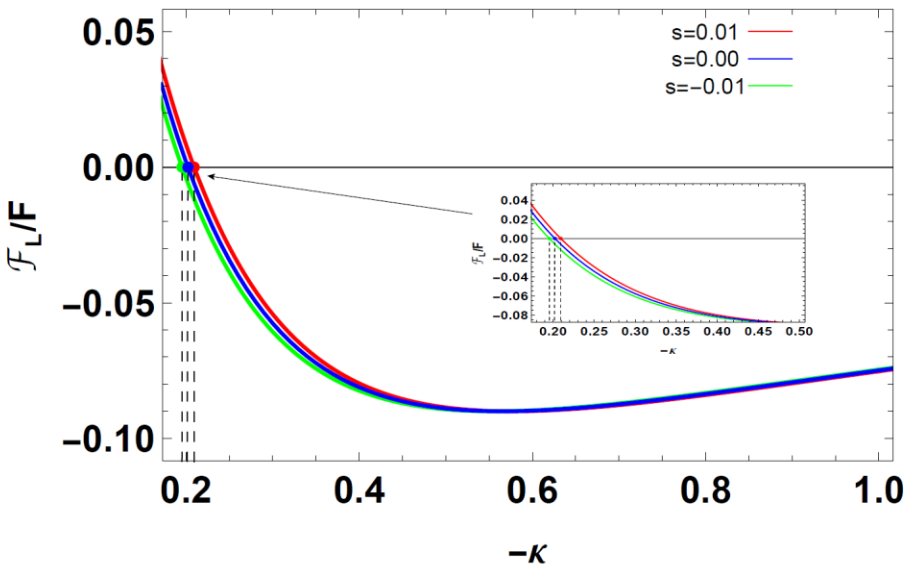


The energy flux increases with the increase of the symmetry breaking parameter *s* for the small parameter of Robin boundary conditions and the decrease for the large one

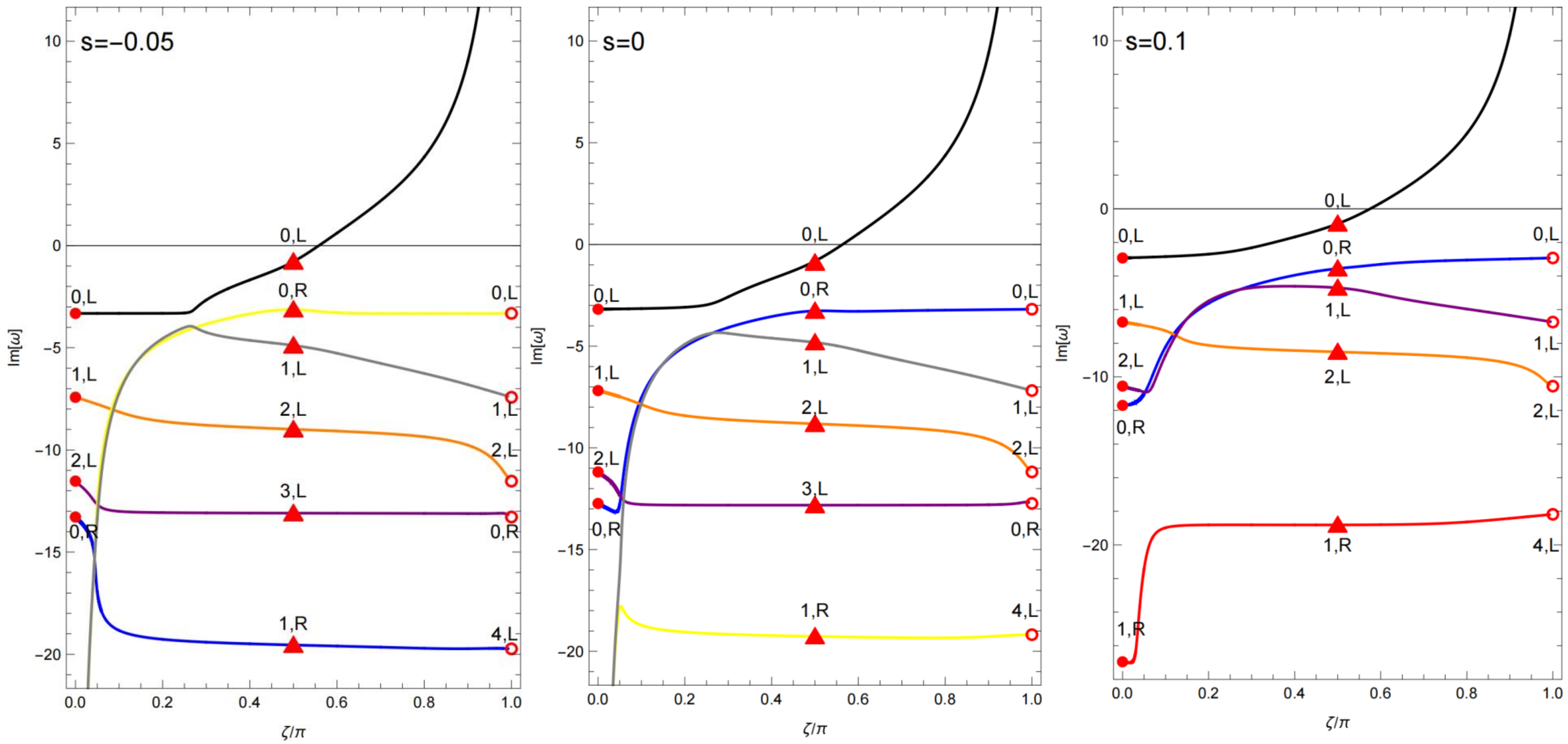
The flux of the angular momentum across the horizon:

$$\mathcal{F}_L(v) = -\int_0^{2\pi} \mathrm{d}\hat{\varphi}\sqrt{r_+}\chi_\mu T^\mu_\nu p^v = Fm\left(Re[\omega] - m\Omega_{\mathcal{H}}\right)$$





Both the energy flux and the angular momentum flux increase with the increase of the symmetry breaking parameter *s* for the small parameter of Robin boundary conditions and the decrease for the large one.



$$\Phi = \cos(\zeta)\Phi^{(D)} + \sin(\zeta)\Phi^{(N)}$$

$$\begin{aligned}\phi^{(D)}(z) =& z^{\alpha}(1-z)^{\beta}F(a,b;a+b-c+1;1-z),\\ \phi^{(N)}(z) =& z^{\alpha}(1-z)^{1-\beta}F(c-a,c-b;c-a-b+1;1-z)\end{aligned}$$

The superradiance only exists in the nodeless (n = 0) modes in BTZ-like BH whenever *s* is positive or negative.

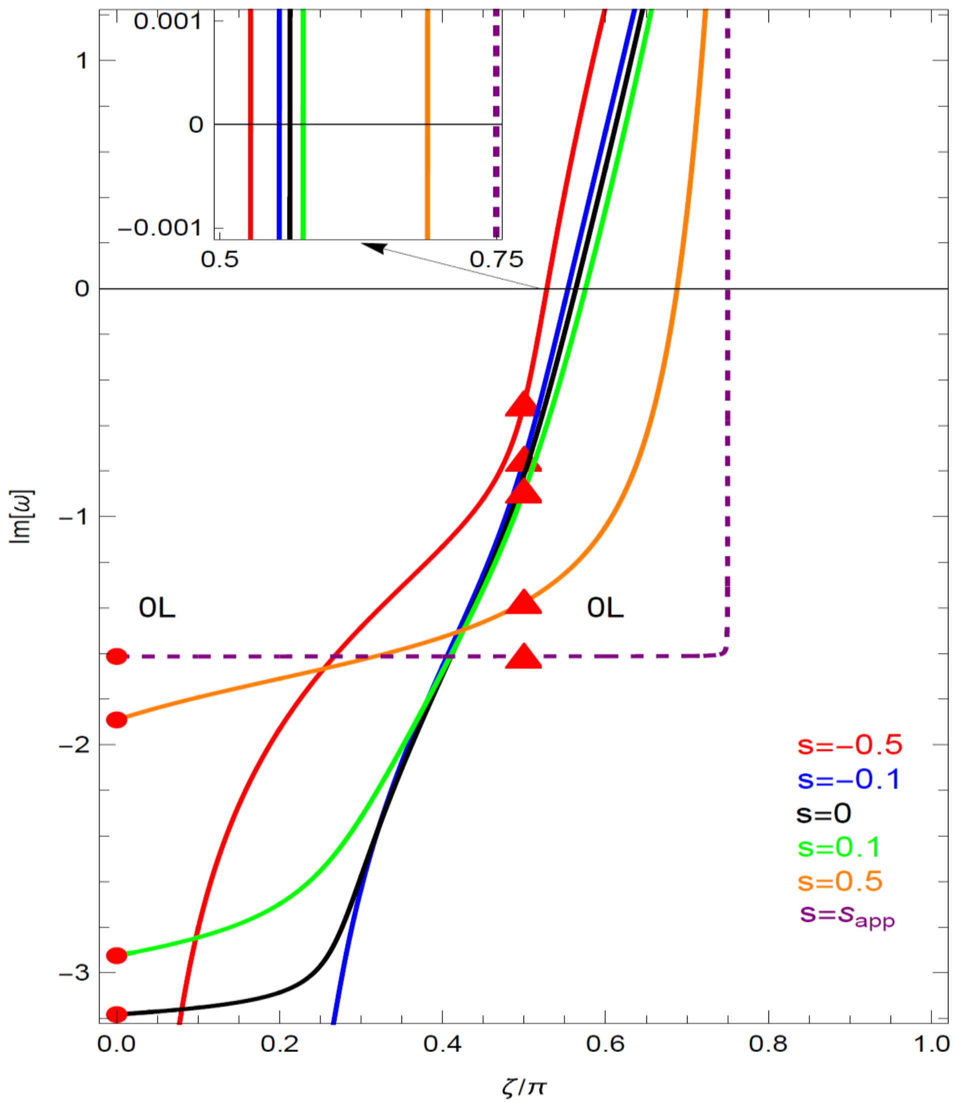
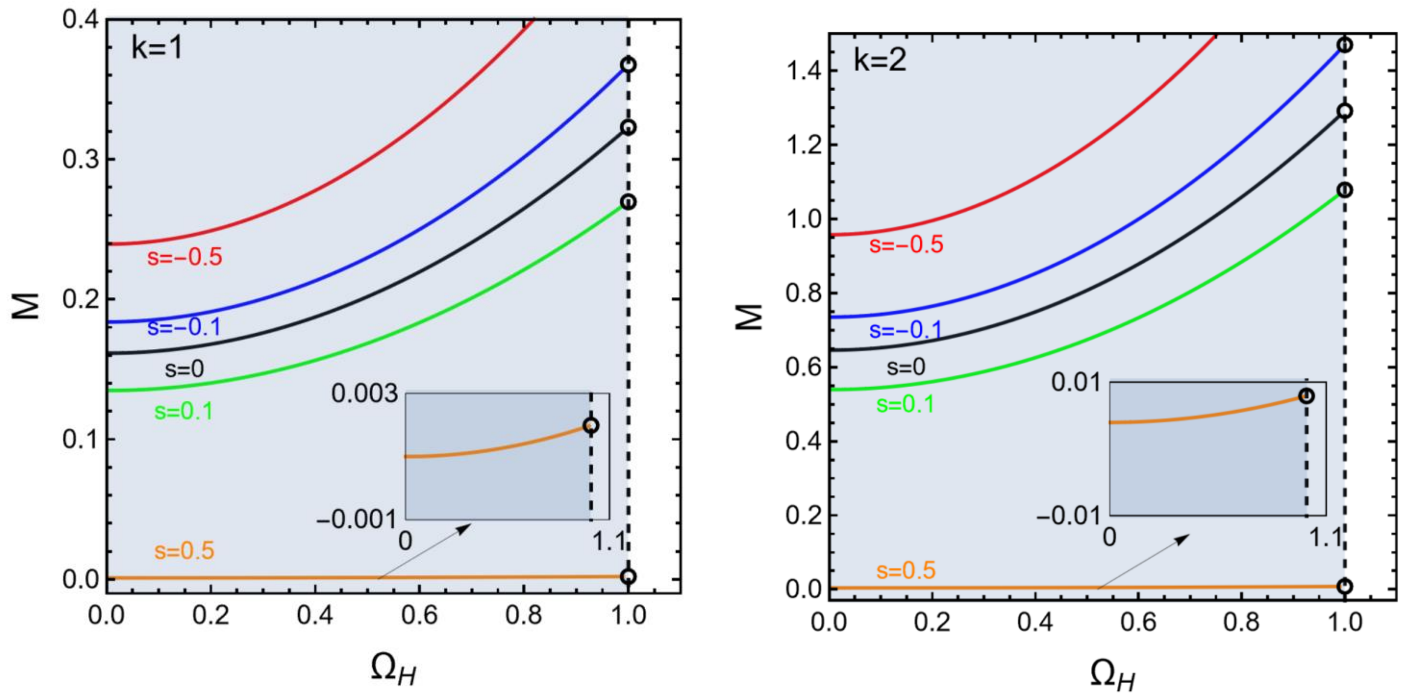


FIG. 7. Imaginary parts of some quasi-bound frequencies of nodeless modes ($n = 0$) as a function of ζ/π with different Lorentz symmetry breaking parameters s for BTZ-like black holes and scalar field with $\mu^2 = -0.65$, $r_+ = 5$, $r_- = 3$, $\ell = 1$ and $k = 1$. The dashed purple curve corresponds to the case of $s_{app} == 0.53846$.

2.3 Stationary scalar clouds

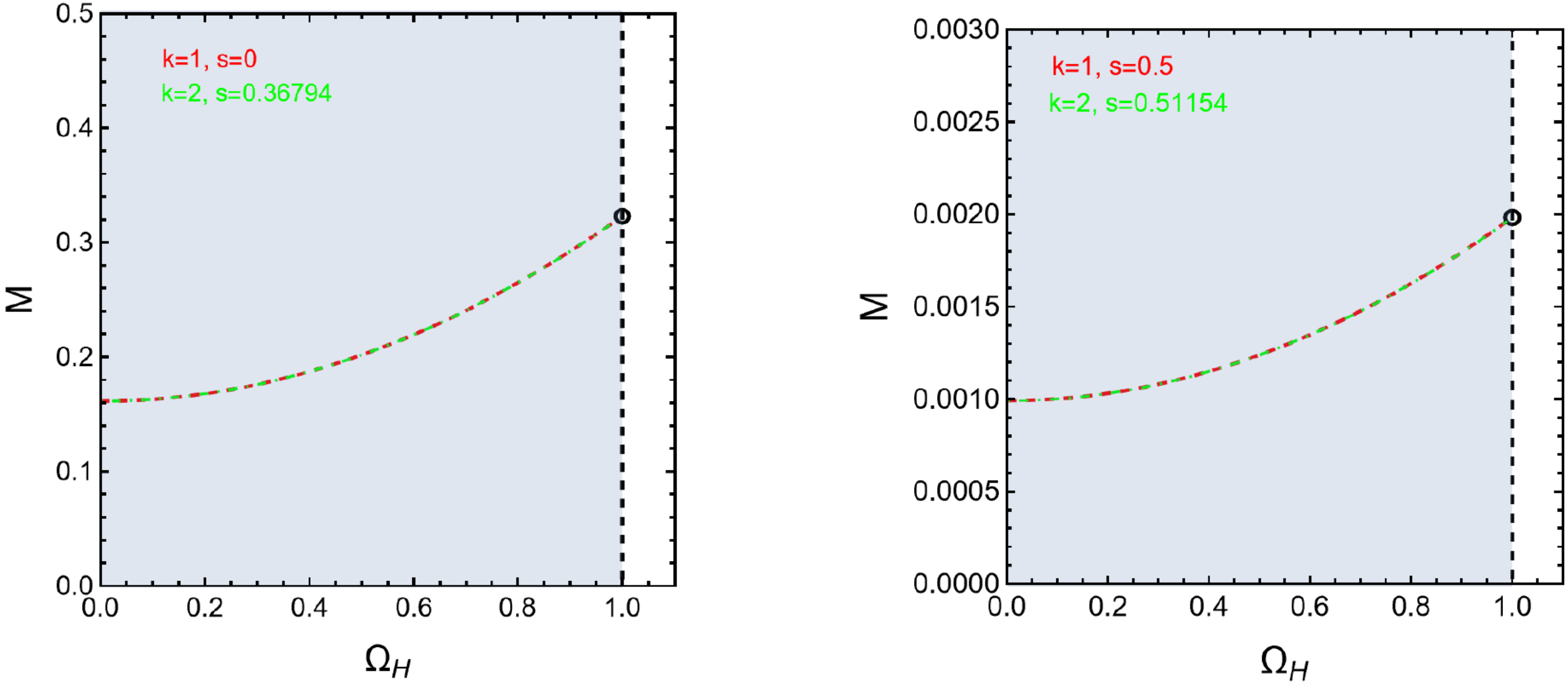
The angular velocity of the horizon:

$$\Omega_H = \frac{r_-}{\ell r_+} \qquad r_\pm^2 = \frac{\ell^2}{2}\left(M \pm \sqrt{M^2 - \frac{j^2}{\ell^2}}\right)$$



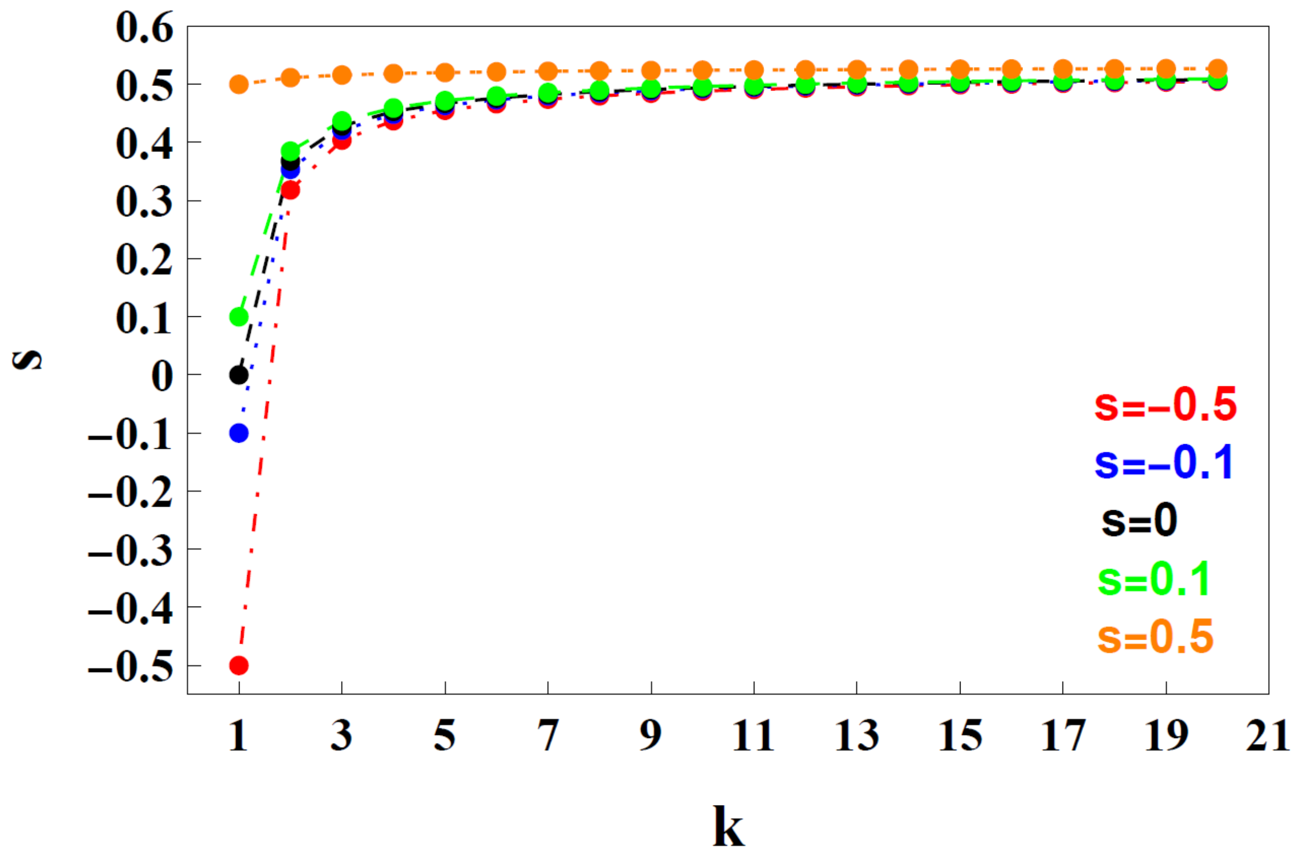
Existence lines of nodeless scalar clouds ($n = 0$) with $\mu^2 = -0.65$, $\zeta = 0.9\pi$

The higher Lorentz symmetry breaking parameter make it easier for the emergence of scalar clouds



Degenerate existence lines of nodeless scalar clouds ($n = 0$) with $\mu^2 = -0.65$, $\zeta = 0.9\pi$ and $\ell = 1$

Fixing the initial parameter *s* and quantum number *k*, one always obtains the same existence line---the so-called degenerate clouds



Degenerate clouds with $\mu^2 = -0.65$, $\zeta = 0.9\pi$ and $k_0 = 1$

There are infinite degenerate clouds for any initial values of Lorentz symmetry breaking parameter s

3. Conclusions and discussions

- Quasinormal spectrum in rotating BTZ-like Black holes bifurcates when the Robin boundary condition parameter is varying (**the *mode split effect***), and the quasinormal spectrum in the complex ω plane can reflect the symmetry of the BTZ-like spacetime.

- The superradiance only exists in the nodeless (n = 0) modes in BTZ-like BH whenever s is positive or negative, which leads to the unique existence of **nodeless stationary scalar clouds** (n = 0).


Thanks for your attention!