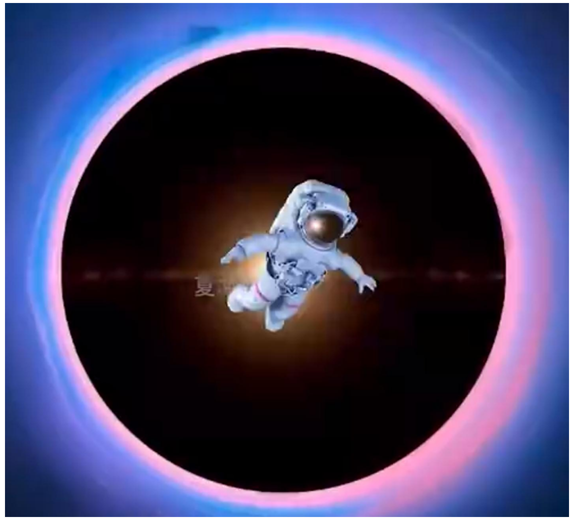
“2024引力与宇宙学”专题研讨会



黑洞的冻结星范式

王永强 兰州大学

2024. 11. 15

@ 彭桓武高能基础理论研究中心（合肥）

R. Oppenheimer and **H. Snyder** **1939**

恒星在耗尽核燃料之后的坍塌过程

（1）坍塌会毫不停滞地穿越视界， 并且在有限时间内产生奇点 （Black holes）

（2）当恒星坍塌到接近引力半径时，从恒星表面发出的光的波长会变得越来越长，坍塌过程会显得越来越慢，直至“冻结“ (frozen star)

朗道学派

栗弗席兹 卡拉特尼科夫

没有奇点 （对称性）

60年代中期一度写入朗道十卷

奇点定理 彭罗斯 （1965）

冻结星 Zel'dovich and Novikov 1964

SOVIET PHYSICS

USPEKHI

A Translation of Uspekhi Fizicheskikh Nauk

É. V. Shpol'skiĭ (Editor in Chief), S. G. Suvorov (Associate Editor), D. I. Blokhintsev, V. I. Veksler, S. T. Konobeevskiĭ (Editorial Board).

SOVIET PHYSICS USPEKHI (Russian Vol. 84, Nos. 3 and 4) MAY-JUNE, 1965

523 + 530.12:531.51

*RELATIVISTIC ASTROPHYSICS. I.**

Ya. B. ZEL'DOVICH and I. D. NOVIKOV

Usp. Fiz. Nauk 84, 377-417 (November, 1964)

Thus we have resolved the "paradox of large masses" (i.e., the conclusion that a large mass must unavoidably be catastrophically contracted), resulting from the work of Oppenheimer and his co-workers[2,3] and discussed in the literature (see the paper of Wheeler[14] and the review of Chiu[4]). At first glance

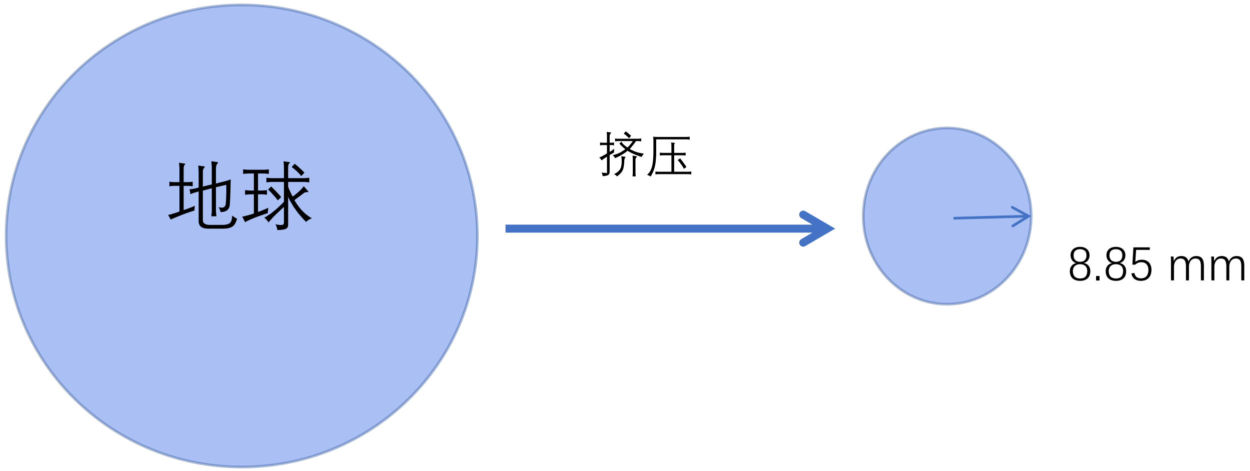
From our point of view there is no paradox whatever. For an external observer the collapse "stops" at $R \to R_g$, and there is no need for fabricating fantastic

冻结星概念

1. 黑洞

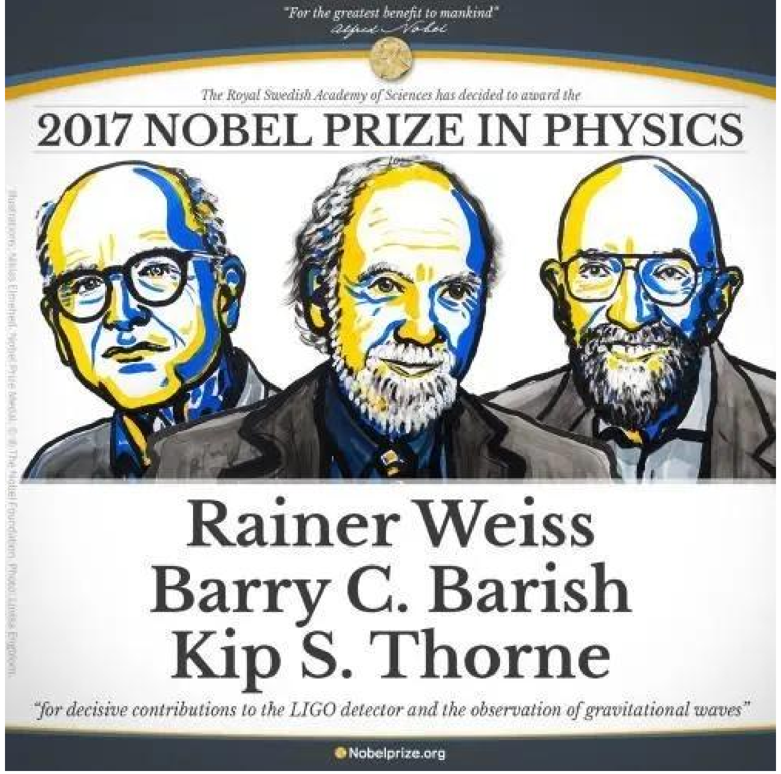
冻结星悖论

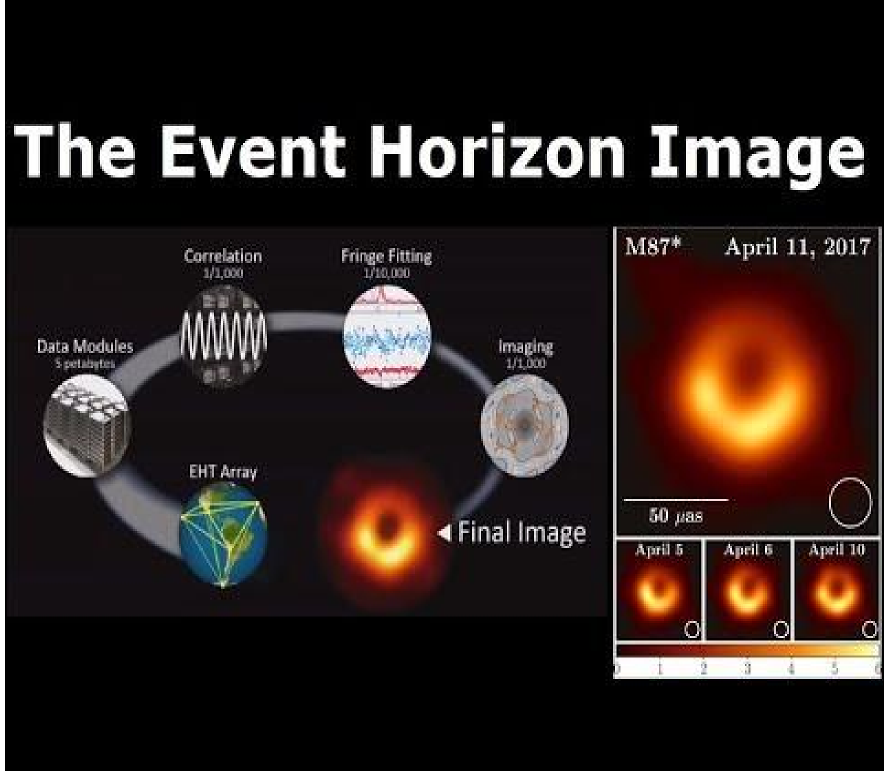
2. 无限接近引力半径的致密星

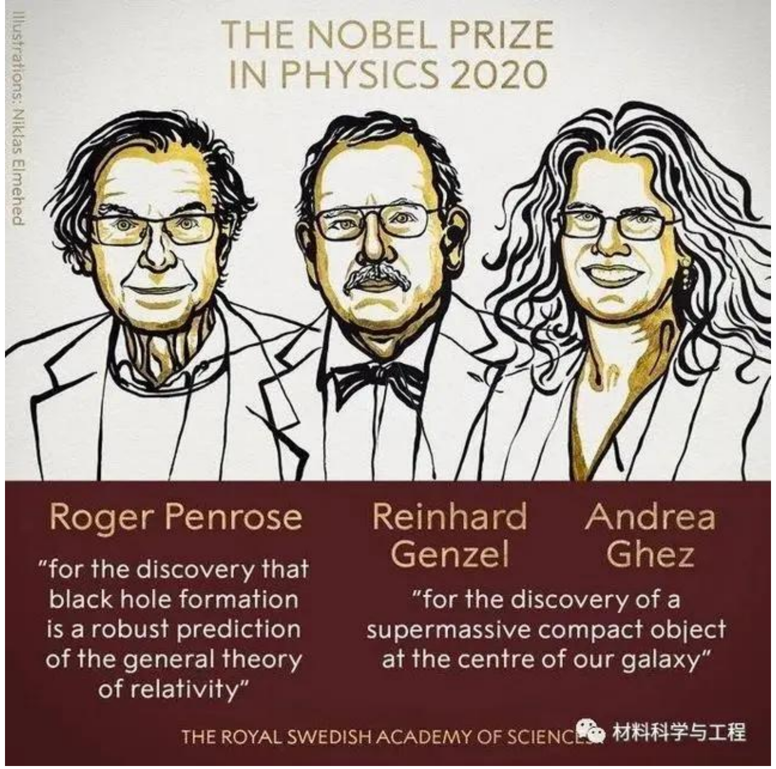


球面无限接近引力半径
（形成事件视界）

星体表面时
间几乎停止





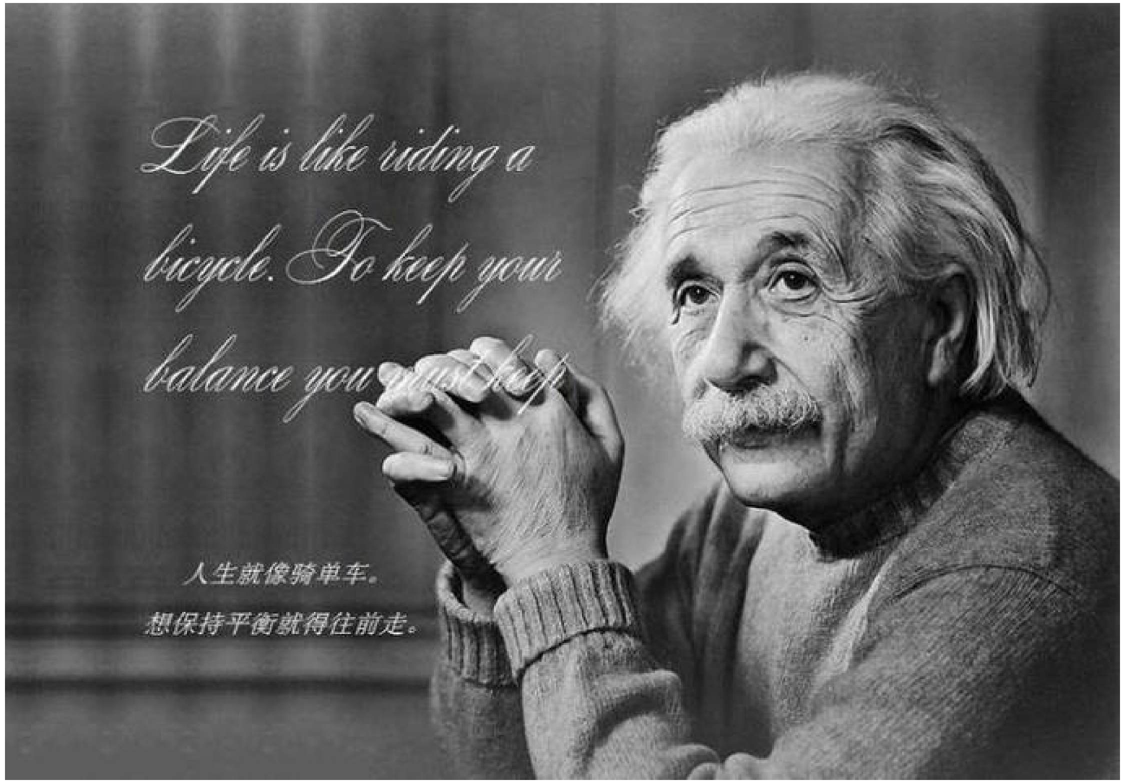


目前**黑洞解**研究现状

- 中心存在奇点（可以带毛）
- 中心没有奇点 （目前没有找到带毛解）

The View From Singularity

ANNALS OF MATHEMATICS
Vol. 40, No. 4, October, 1939



ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES

BY ALBERT EINSTEIN

(Received May 10, 1939)

The essential result of this investigation is a clear understanding as to why the "Schwarzschild singularities" do not exist in physical reality. Although the theory given here treats only clusters whose particles move along circular paths it does not seem to be subject to reasonable doubt that more general cases will have analogous results. The "Schwarzschild singularity" does not appear for the reason that matter cannot be concentrated arbitrarily. And this is due to the fact that otherwise the constituting particles would reach the velocity of light.

The View From Event Horizon

UC SANTA BARBARA
Kavli Institute for Theoretical Physics

HOME DIRECTORY ACTIVITIES PROPOSE ACTIVITY APPLY FOR VISITORS ONLINE TALKS

KITP Rapid Response Workshop: Black Holes: Complementarity, Fuzz, or Fire?
(Aug 19-30, 2013)
Coordinators: Raphael Bousso (UCB), Samir Mathur (OSU), Rob Myers (PI), Joe Polchinski (KITP), Lenny Susskind (Stanford)
Scientific Advisor: Don Marolf (UCSB)

Information Preservation and Weather Forecasting for Black Holes
S.W. Hawking (Cambridge U., DAMTP) (Jan 22, 2014)
e-Print: 1401.5761 [hep-th]
pdf cite claim ref

nature

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nature > news > article

Published: 24 January 2014

Stephen Hawking: 'There are no black holes'

Zeeya Merali

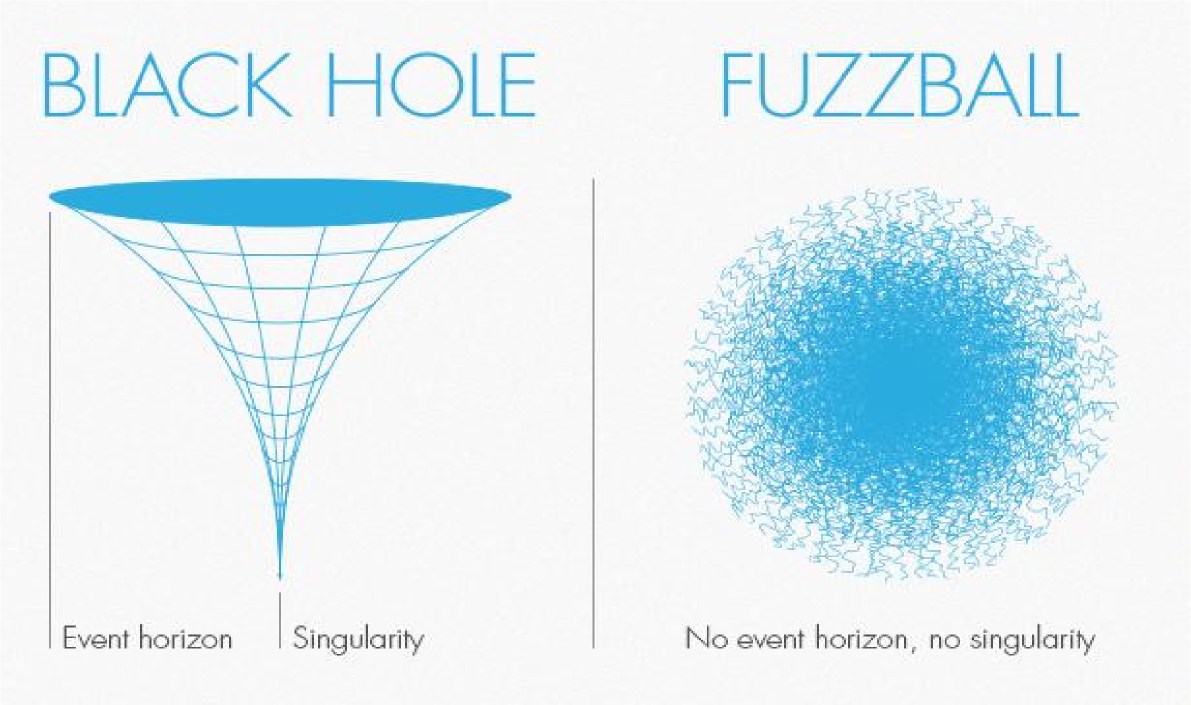
Nature (2014) | Cite this article

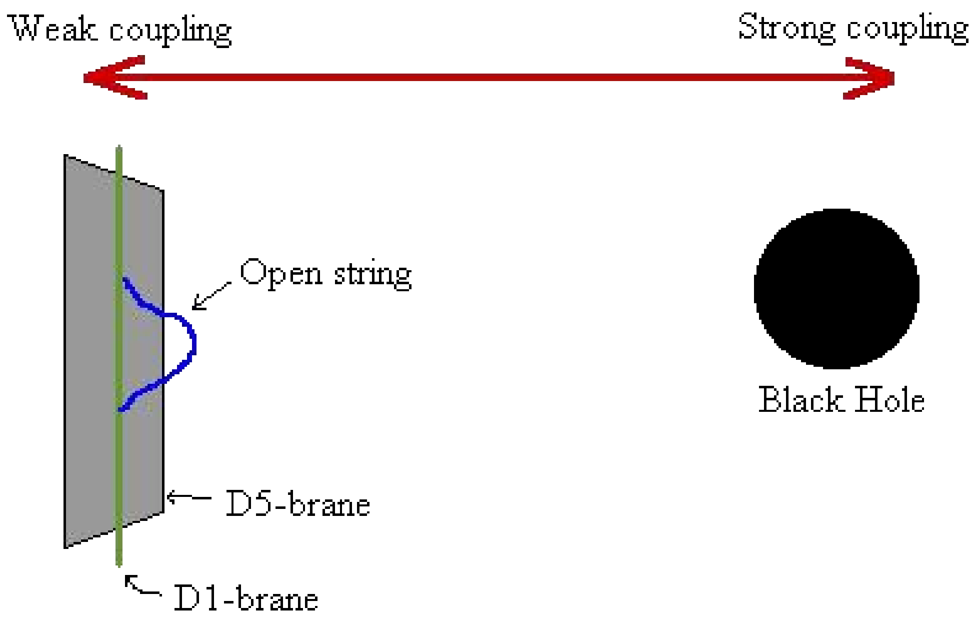
8997 Accesses | 4 Citations | 3528 Altmetric | Metrics

Notion of an 'event horizon', from which nothing can escape, is incompatible with quantum theory, physicist claims.

The View From String

Strominger and Vafa (1996):
Count Black Hole Microstates (branes + strings)
(limited in supersymmetric and extremal BH)

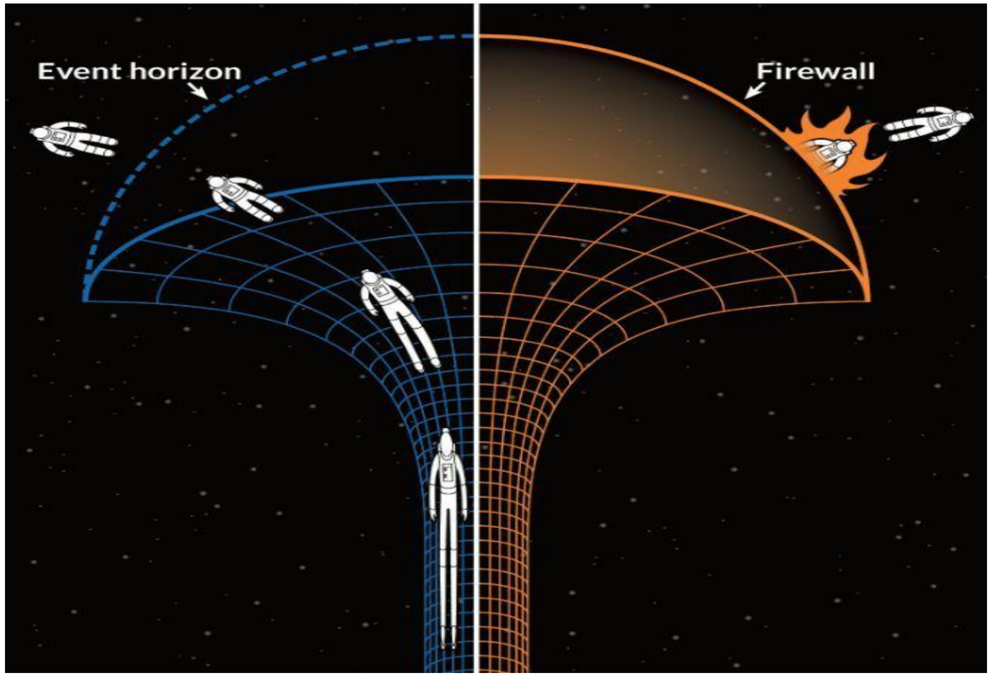


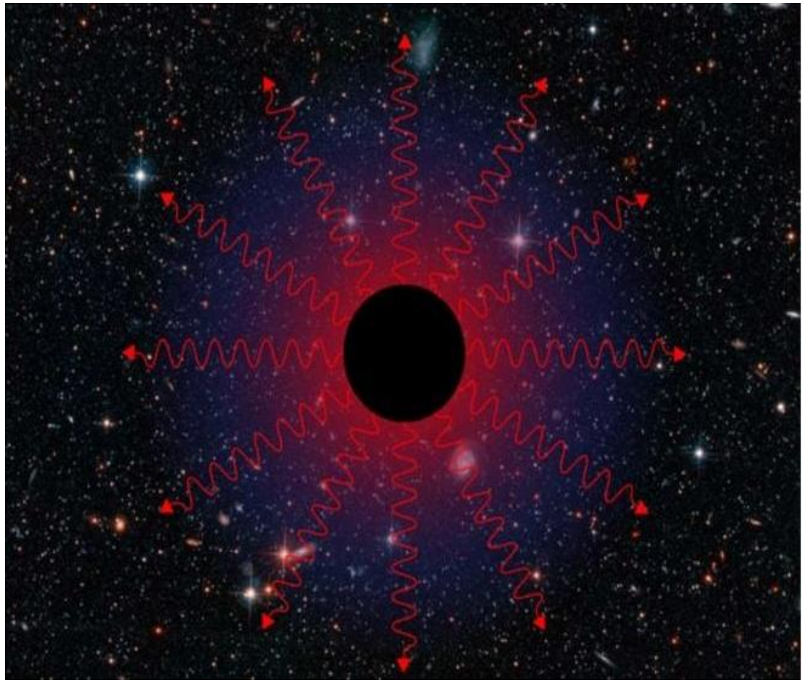


Lunin and Mathur (2002):Fuzzball

The View From quantum gravity

黑洞信息丢失悖论
Hawking 1975





Firewall paradox
Almheiri, Marolf, Polchinski, and Sully 2012

Soft hair
Hawking, Perry and Strominger 2016

Frozen Star

是否存在冻结星解?

当$\boldsymbol{R} -> \boldsymbol{R_g}$，则 $\boldsymbol{g_{tt}|_{R->R_g}} -> \mathbf{0}$

目前存在冻结星的体系

- 规则黑洞 (非线性电磁场) + 物质场

  2305.19057 2312.07224 2312.07400

  2407.11355 2407.17278 2409.14402

- 规则黑洞 (纯引力) + 物质场

  非规则黑洞(纯引力) + 物质场 2406.08813

- Einstein cubic gravity (三次曲率纯引力) 2410.04575

$$\mathcal{P} = 12R_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma}R_{\rho}{}^{\gamma}{}_{\sigma}{}^{\delta}R_{\gamma}{}^{\mu}{}_{\delta}{}^{\nu} + R_{\mu\nu}^{\rho\sigma}R_{\rho\sigma}^{\gamma\delta}R_{\gamma\delta}^{\mu\nu} - 12R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + 8R_{\mu}^{\nu}R_{\nu}^{\rho}R_{\rho}^{\mu} \,.$$

- Einstein Weyl （二次曲率纯引力） 2412.xxxxx

$$I = \int d^4x\sqrt{-g}\left(\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \beta R^2\right)$$

拓扑非平庸空间（虫洞时空）

2305.19819 2309.16379 2311.17557

1. 规则黑洞 (非线性电磁场) + 物质场

经典引力

↓

非规则黑洞
（singularity）

↓

物质 { 落入奇点 / 形成次级毛

奇点 → 事件视界

量子引力

↓

规则黑洞
（No singularity）

↓

物质在黑洞内外如何分布？

没有奇点 → 事件视界

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{4} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \right) \qquad \mathcal{L}^{(2)} = -\nabla_a \psi^* \nabla^a \psi - \mu^2 \psi \psi^*$$

Bardeen (1968)

$$\mathcal{L}^{(1)} = -\frac{3}{2s} \left(\frac{\sqrt{2q^2\mathcal{F}}}{1 + \sqrt{2q^2\mathcal{F}}} \right)^{\frac{5}{2}}$$

Hayward (2005)

$$\mathcal{L}^{(1)} = -\frac{3}{2s} \frac{(2q^2\mathcal{F})^{3/2}}{(1 + (2q^2\mathcal{F})^{3/4})^2},$$

$$\mathcal{F} = \tfrac{1}{4} F_{ab} F^{ab}$$

Ansatzs:

$$ds^2 = -n(r) o^2(r) dt^2 + \frac{dr^2}{n(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right)$$

$$A = q \cos(\theta) d\varphi, \quad \psi = \phi(r) e^{-i\omega t};$$

$\mathcal{L}^{(2)} = 0$ Bardeen （Hayward ） spacetime

$\mathcal{L}^{(1)} = 0$ Boson star

"Geons". Phys Rev 97 (1955)

"gravitational electromagnetic entity"

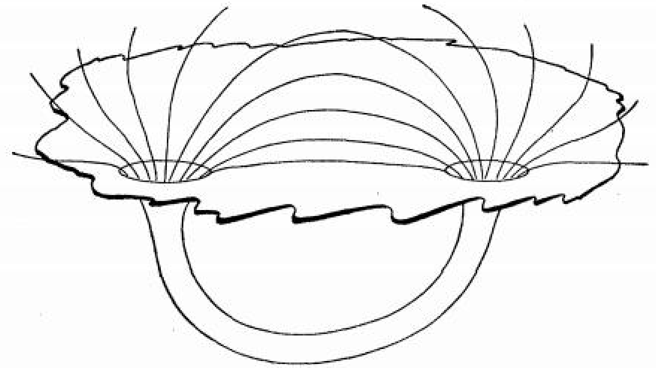
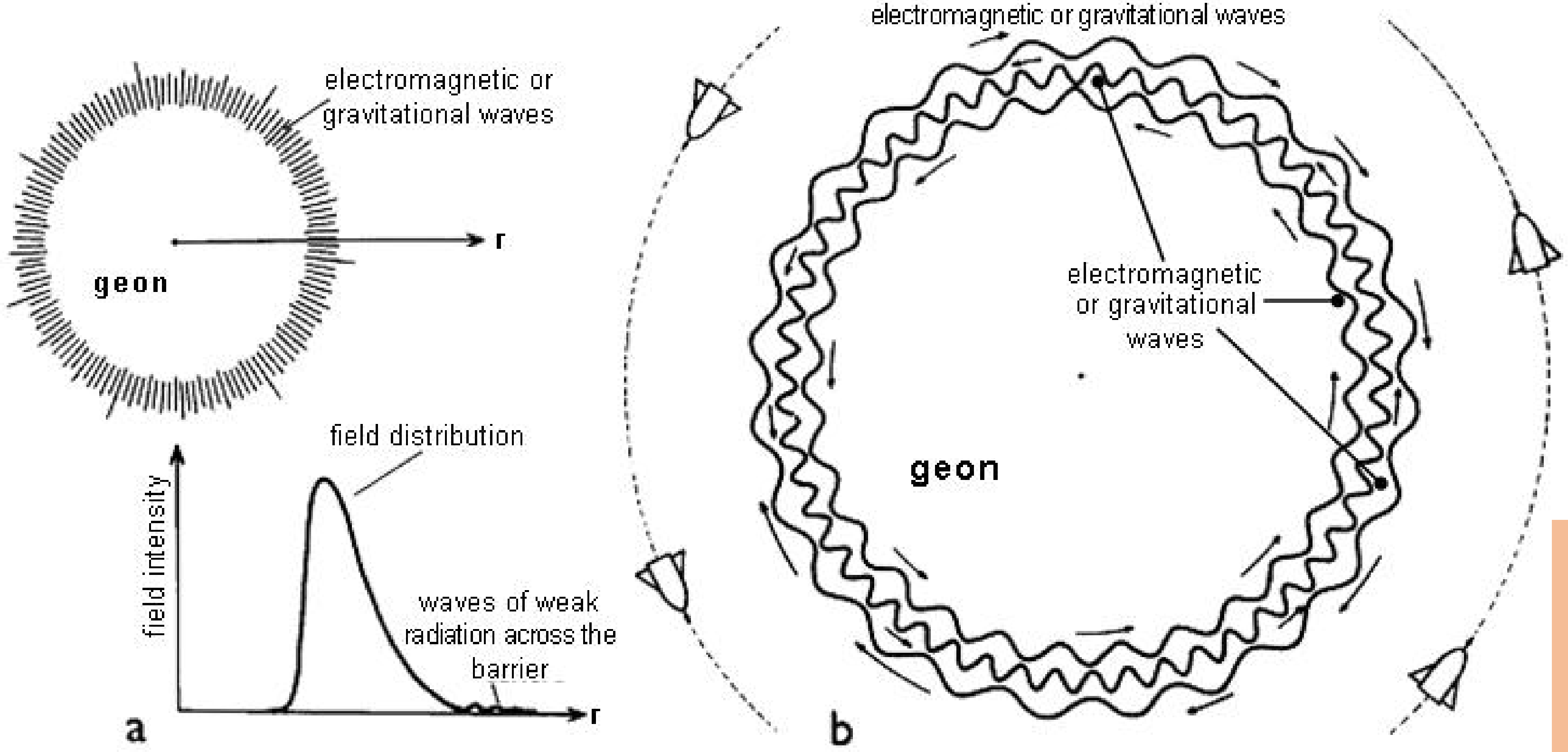
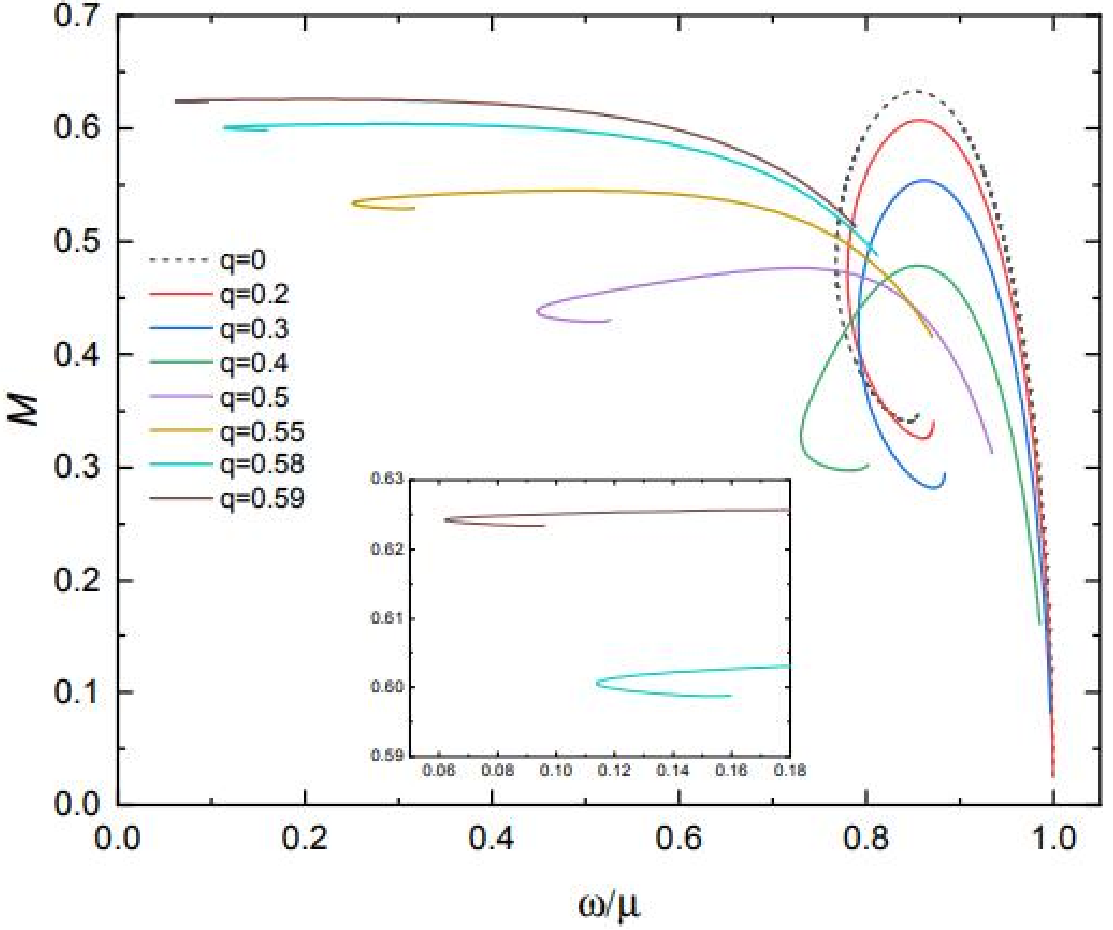


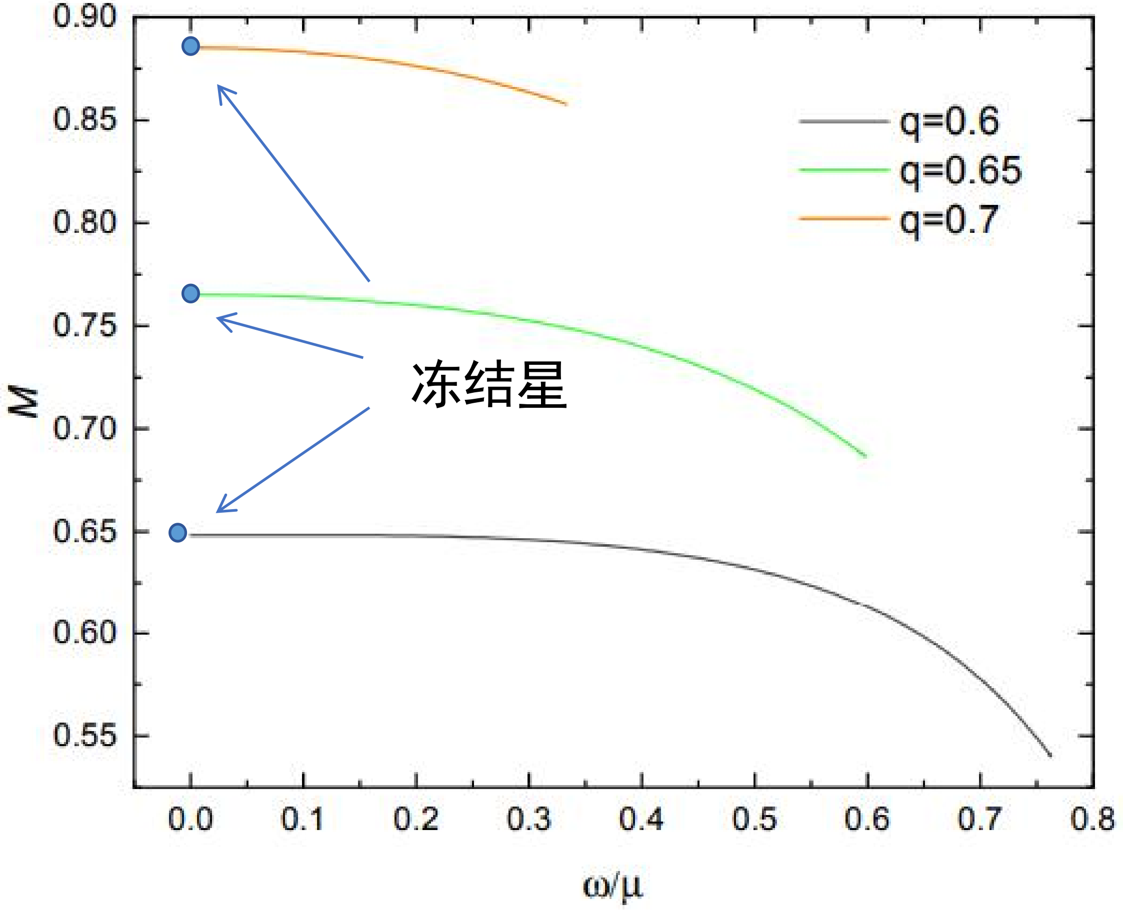
FIG. 7. Schematic representation of lines of force in a doubly-connected space. In the upper continuum the lines of force behave much as if the tunnel mouths were the seats of equal and opposite charges.



D. Kaup (1968)
R. Ruffini，S. Bonazzola (1969)
Einstein+scalar field
"Boson star"

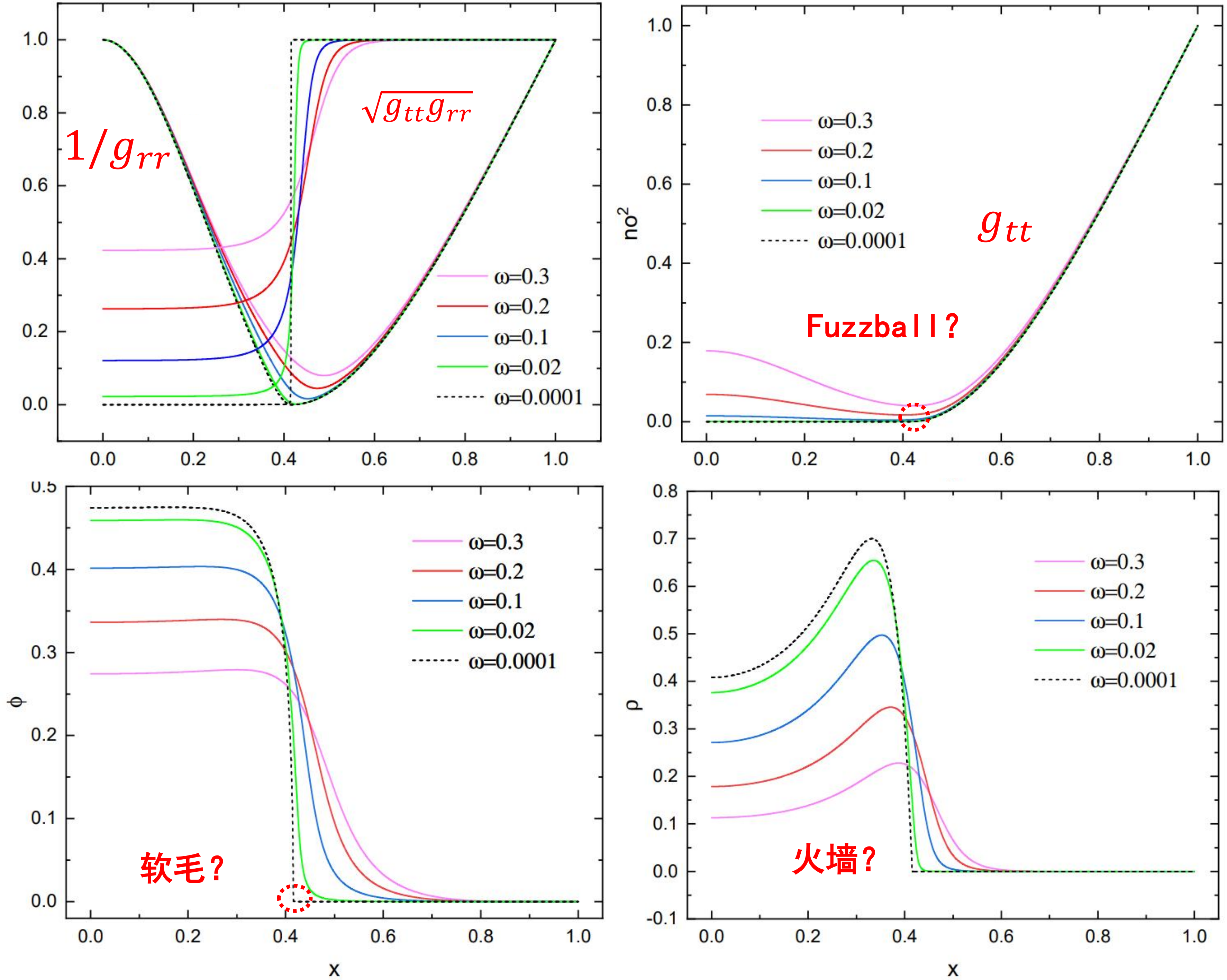
Bardeen-boson Star *2305.19057*





出现Bardeen-玻色冻结星（ω ⟶ 0）时q取值范围：

0.598 < q <0.7208（极端Bardeen BH）



Bardeen-boson Star
2305.19057

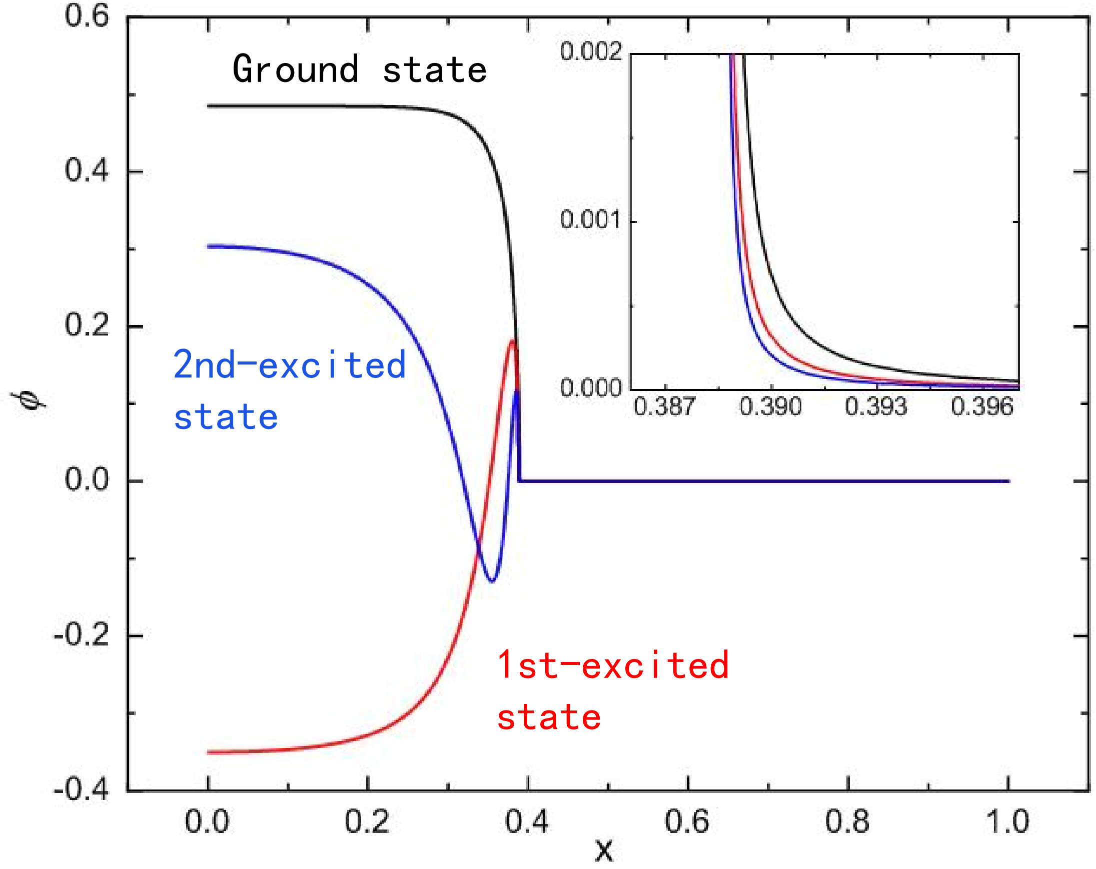
$$r = Tan(\frac{\pi}{2}x)$$

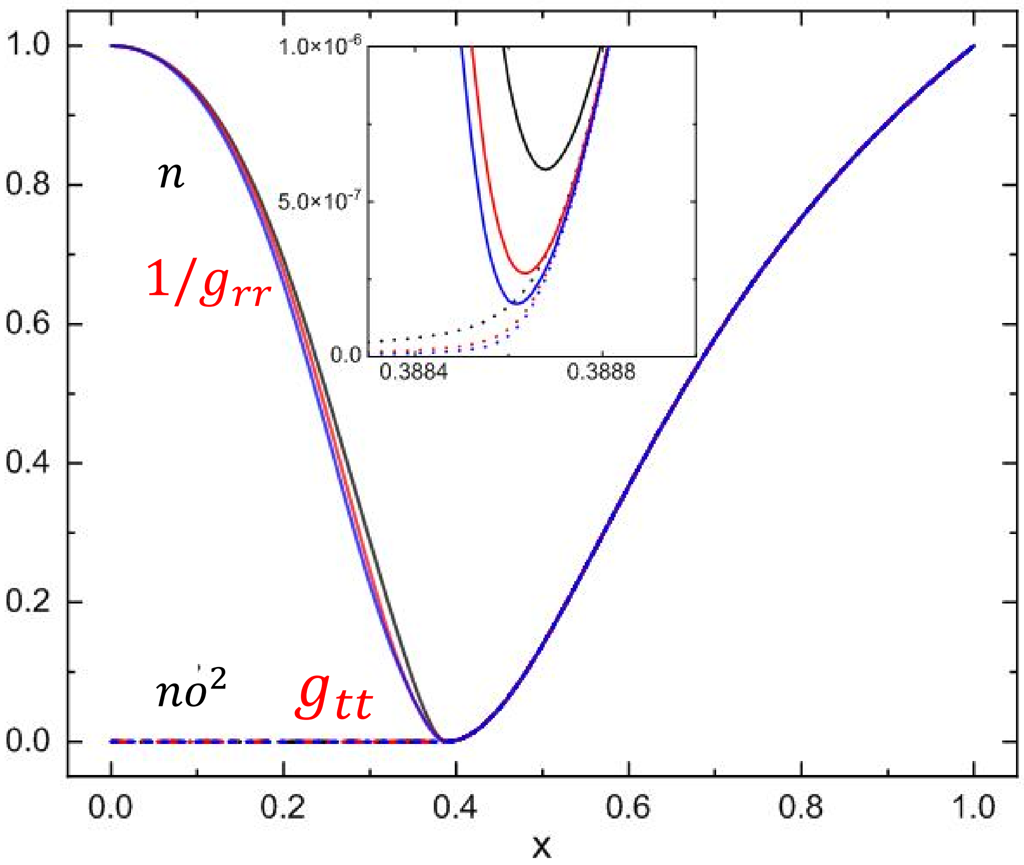
$s = 0.2$, $q = 0.65$

Hayward-boson star *2312. 07224*

$$r = \frac{x}{1-x}$$

$s = 0.2$, q=0.56 ω =0.00001





M=0.49612 !!!

2. 纯引力 + 物质场

Quasi-Topological gravity

- Second-order linearized equations
- Admit single-function ($g_{tt}g_{rr} = -1$) non-hairy generalizations of the Schwarzschild black hole.

$$I_{\rm QT} = \frac{1}{16\pi G}\int \mathrm{d}^D x\sqrt{|g|}\left[R + \sum_{n=2}^{n_{\max}} \alpha_n \mathcal{Z}_n\right]$$

$$\mathrm{d}s^2 = -N(r)^2 f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2_{D-2}\,,$$

$$\frac{dN}{dr} = 0\,, \quad \frac{d}{dr}\left[r^{D-1}h(\psi)\right] = 0\,,$$

$$h(\psi) \equiv \psi + \sum_{n=2}^{n_{\max}} \alpha_n \psi^n\,, \quad \psi \equiv \frac{1-f(r)}{r^2}\,. \qquad h(\psi) = \frac{m}{r^{D-1}}\,,$$

$$f = 1 - m/r^{D-3} \qquad \alpha_n = 0$$

where $$\mathcal{Z}_n = -\frac{1}{(D-2n)}\tilde{\mathcal{Z}}_n$$

$$\tilde{\mathcal{Z}}_1 = -R\,, \tag{A5}$$

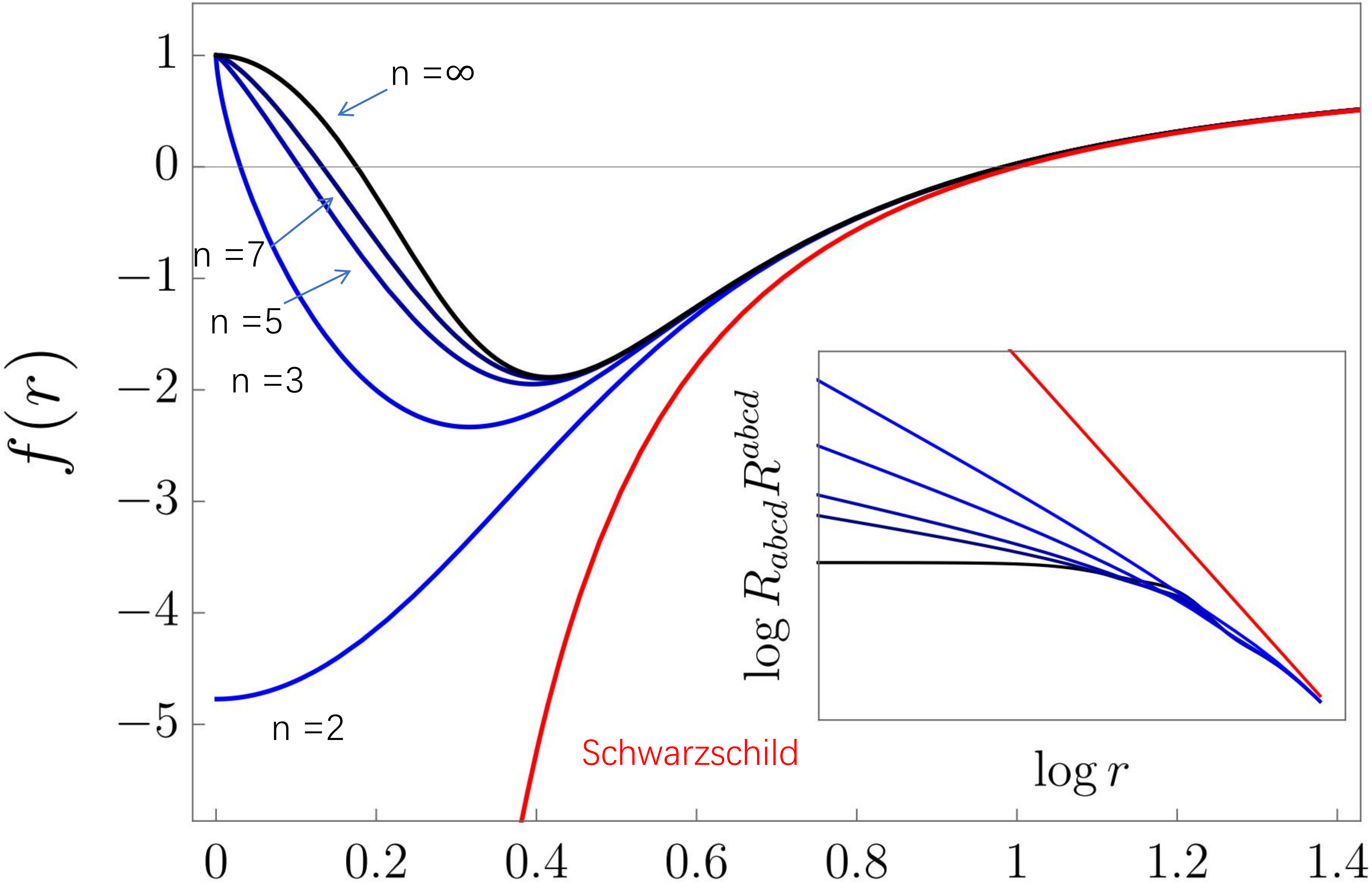
$$\tilde{\mathcal{Z}}_2 = -\frac{1}{(D-2)(D-3)}\left[R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}\right] \tag{A6}$$

$$\begin{aligned}\tilde{\mathcal{Z}}_3 = -\frac{8(2D-3)}{(D-2)(D-3)(D-4)(3D^2-15D+16)}\Bigg[&(D-4)R_a{}^b{}_c{}^d R_b{}^e{}_d{}^f R_e{}^a{}_f{}^c + \frac{3(3D-8)}{8(2D-3)}R_{abcd}R^{abcd}R \\ &- \frac{3(3D-4)}{2(2D-3)}R_a{}^c R_c{}^a R - \frac{3(D-2)}{(2D-3)}R_{acbd}R^{acb}{}_e R^{de} + \frac{3D}{(2D-3)}R_{acbd}R^{ab}R^{cd} \\ &+ \frac{6(D-2)}{(2D-3)}R_a{}^c R_c{}^b R_b{}^a + \frac{3D}{8(2D-3)}R^3\Bigg]\,,\end{aligned} \tag{A7}$$

$$\begin{aligned}\tilde{\mathcal{Z}}_4 = &-\frac{384(D-8)R^a_b R^c_a R^d_c R^b_d}{(D-2)^5(D^3-8D^2+48D-96)} - \frac{1152R_{ab}R^{ab}R_{cd}R^{cd}}{(D-2)^5(D^3-8D^2+48D-96)} \\ &- \frac{64(D^3-10D^2+40D+24)RR^c_a R^b_c R^a_b}{(D-1)(D-2)^5(D^3-8D^2+48D-96)} + \frac{24(D^4-6D^3+20D^2+104D-64)R^2R_{ab}R^{ab}}{(D-1)^2(D-2)^5(D^3-8D^2+48D-96)}\end{aligned}$$

α_n	$h(\psi)$	$f(r)$	Differentiablility at $r=0$
α^{n-1}	$\dfrac{\psi}{1-\alpha\psi}$	$1-\dfrac{mr^2}{r^{D-1}+\alpha m}$	$\mathcal{C}^\infty$ if D odd, $\mathcal{C}^{D+1}$ if D even
$\dfrac{\alpha^{n-1}}{n}$	$-\dfrac{\log(1-\alpha\psi)}{\alpha}$	$1-\dfrac{r^2}{\alpha}\left(1-e^{-\alpha m/r^{D-1}}\right)$	$\mathcal{C}^\infty$
$n\alpha^{n-1}$	$\dfrac{\psi}{(1-\alpha\psi)^2}$	$1-\dfrac{2mr^2}{r^{D-1}+2\alpha m+\sqrt{r^{2(D-1)}+4\alpha m r^{D-1}}}$	$\mathcal{C}^\infty$ if $D=1 \bmod 4$, else $\mathcal{C}^{\lfloor (D+3)/2\rfloor}$
$\dfrac{(1-(-1)^n)}{2}\alpha^{n-1}$	$\dfrac{\psi}{1-\alpha^2\psi^2}$	$1-\dfrac{2mr^2}{r^{D-1}+\sqrt{r^{2(D-1)}+4\alpha^2 m^2}}$	$\mathcal{C}^\infty$ if D odd, $\mathcal{C}^{D+1}$ if D even
$\dfrac{(1-(-1)^n)\Gamma\left(\frac{n}{2}\right)}{2\sqrt{\pi}\Gamma\left(\frac{n+1}{2}\right)}\alpha^{n-1}$	$\dfrac{\psi}{\sqrt{1-\alpha^2\psi^2}}$	$1-\dfrac{mr^2}{\sqrt{r^{2(D-1)}+\alpha^2 m^2}}$	$\mathcal{C}^\infty$

5d Hayward Black Hole.



Higher-derivative Gravity Theory

higher-curvature terms*

Action: $S=\int d^5x\frac{\sqrt{|g|}}{16\pi G}\left[R+\sum_{n=2}^{n_{max}}\alpha_n\mathcal{Z}_n\right]-g^{\mu\nu}\bar{\Phi}_{,\mu}\Phi_{,\nu}-\mu^2\bar{\Phi}\Phi$

ansatz: $\mathrm{d}s^2=-\sigma(r)^2N(r)\mathrm{d}t^2+\frac{\mathrm{d}r^2}{N(r)}+r^2\mathrm{d}\Omega^2_{D-2},\qquad \Phi=\phi(r)e^{-iwt}$

equation of motion

$$3[r^4h(\psi)]'=16\pi Gr^3(\mu^2\phi^2+\frac{w^2\phi^2}{N\sigma^2}+N\phi'^2),$$

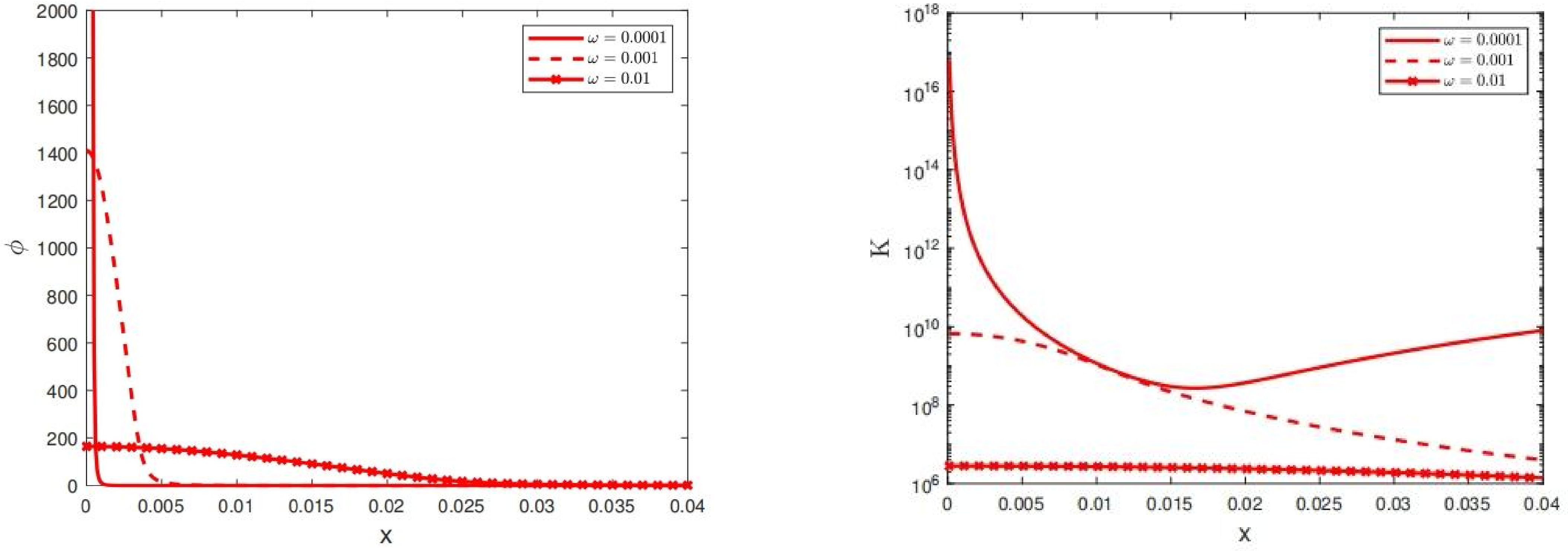
$$3r^2\sigma'\frac{dh(\psi)}{d\psi}=16\pi Gr^3\frac{w^2\phi^2+N^2\sigma^2\phi'^2}{N^2\sigma},$$

$$\phi''+\left(\frac{3}{r}+\frac{N'}{N}+\frac{\sigma'}{\sigma}\right)\phi'+\left(\frac{\omega^2}{N\sigma^2}-\mu^2\right)\frac{\phi}{N}=0,$$

where $h(\psi)\equiv\psi+\sum_{n=2}^{n_{\max}}\alpha^{n-1}\psi^n,\quad \psi\equiv\frac{1-N(r)}{r^2}.$

(non-Frozen) Boson Star Solution

n=2



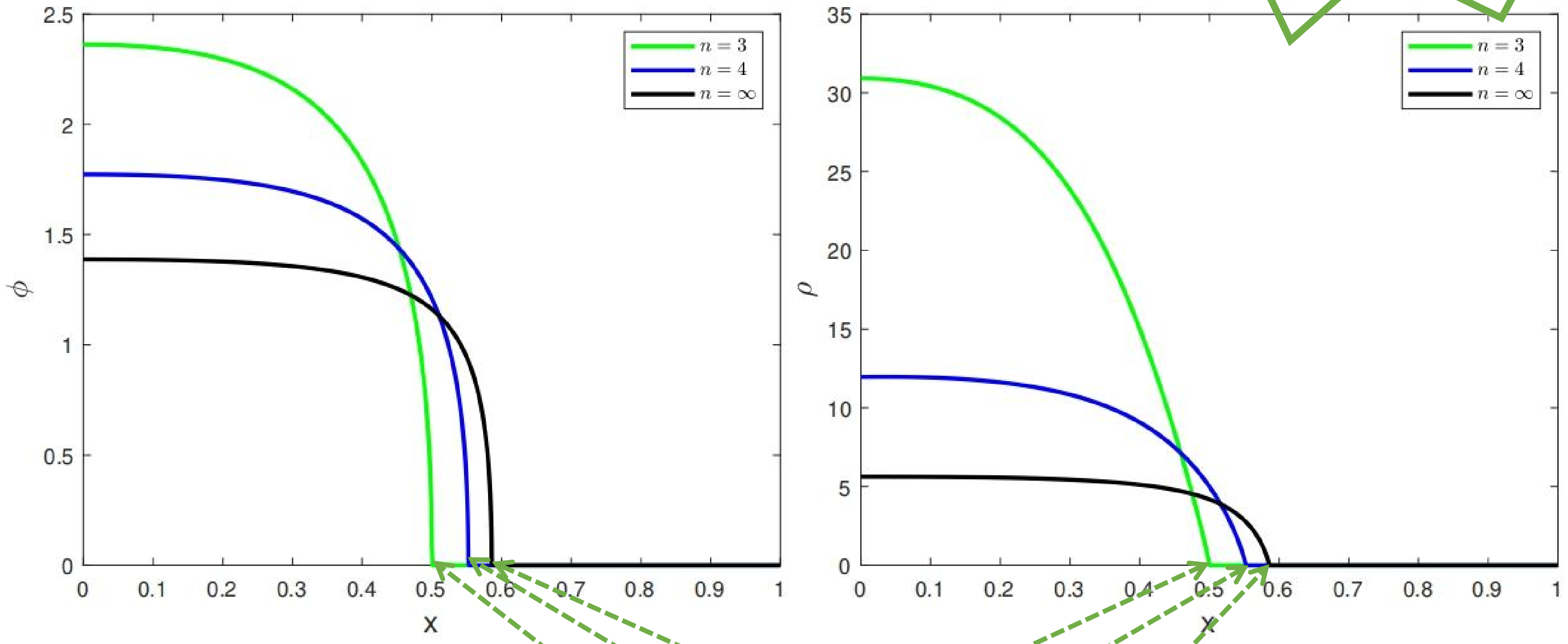
diverge at center as $\omega \to 0$

*The results presented below all have α=1

Frozen Boson Star Solution

n=3,4,∞

no divergence

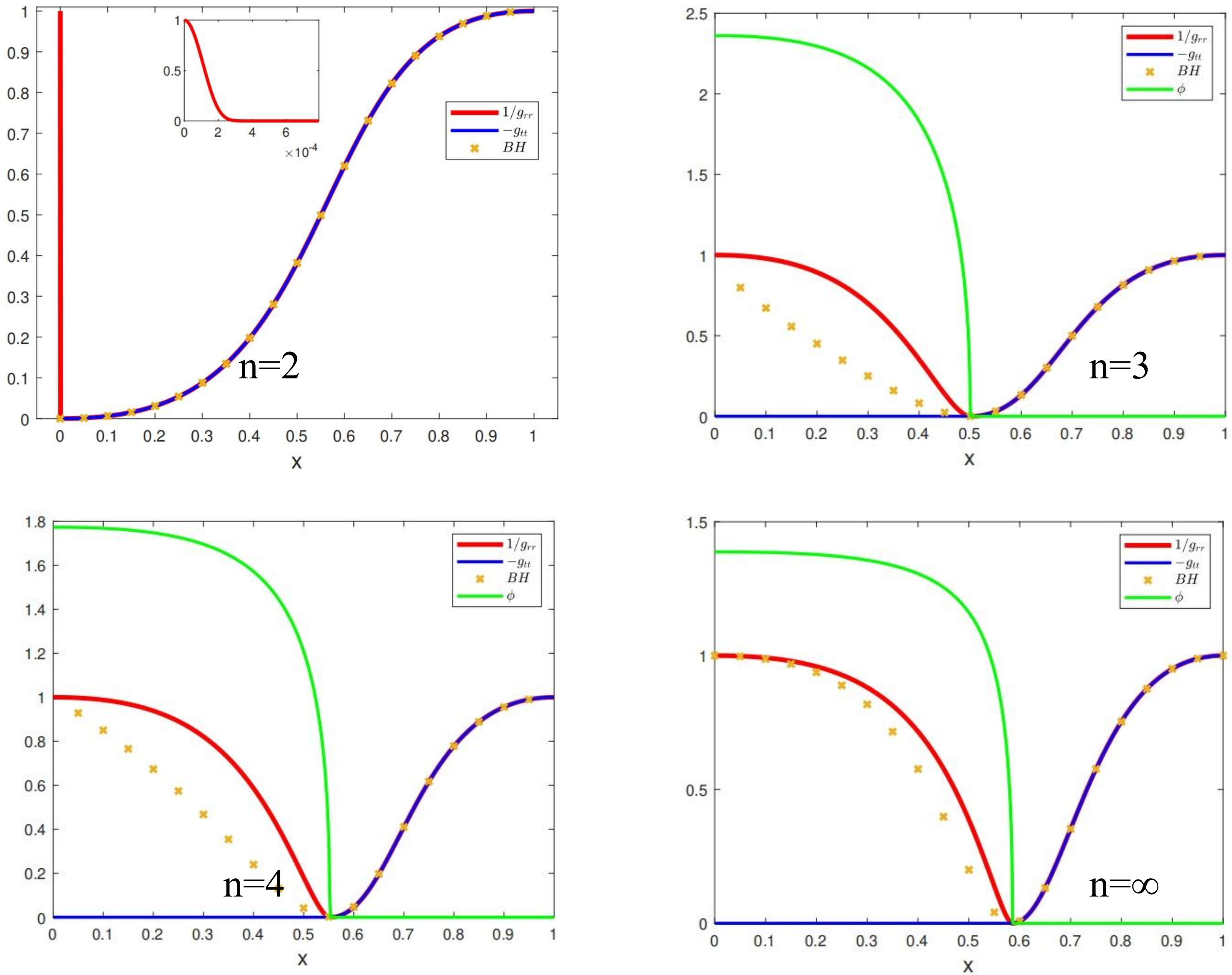


critical horizon

Frozen Boson Star Solution

$n = 2,\ BH: g_{tt} = 1/g_{rr}$

$$= 1 - \frac{-r^2 + \sqrt{\frac{32\alpha GM}{3\pi} + r^4}}{2\alpha}$$



$n = \infty,\ BH:\ g_{tt} = 1/g_{rr}$

$$= 1 - \frac{8GMr^2}{3\pi r^4 + 8GM\alpha}$$

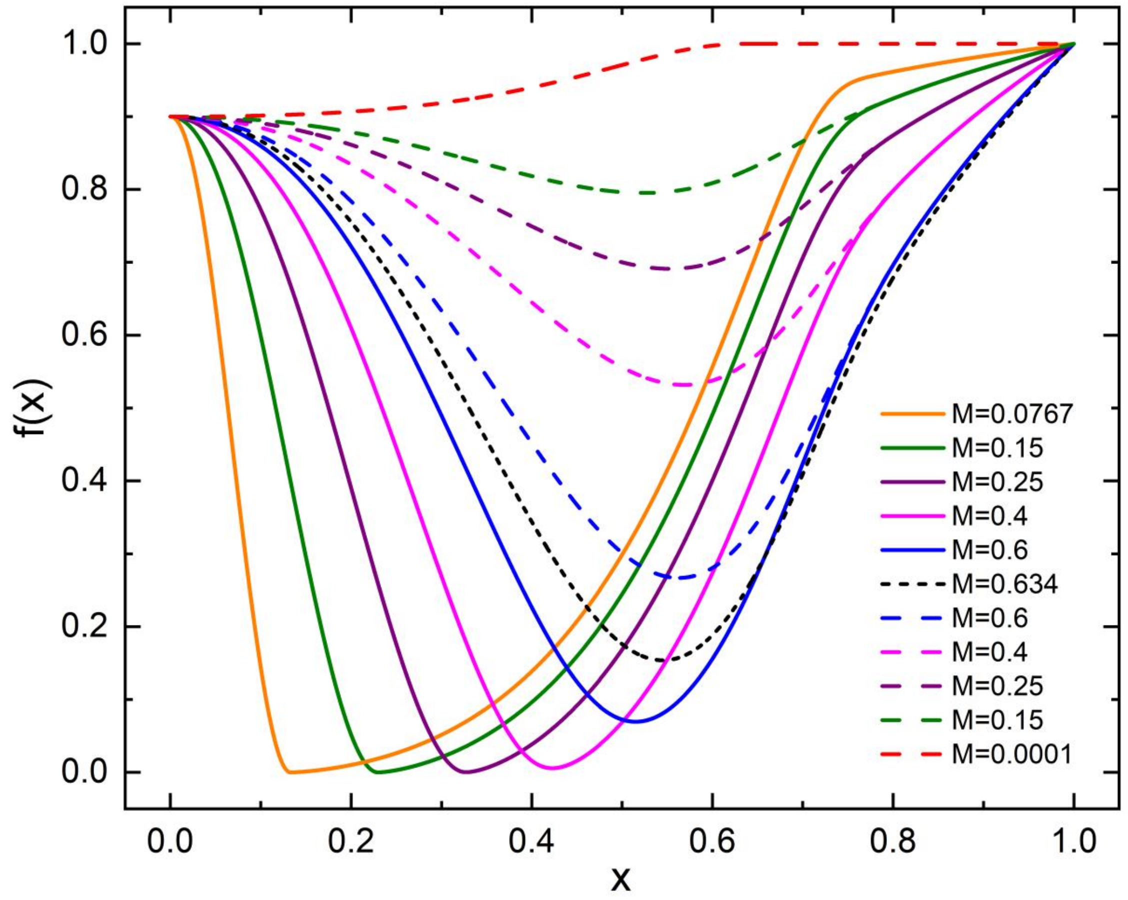
3. Einsteinian cubic gravity 2410.04575

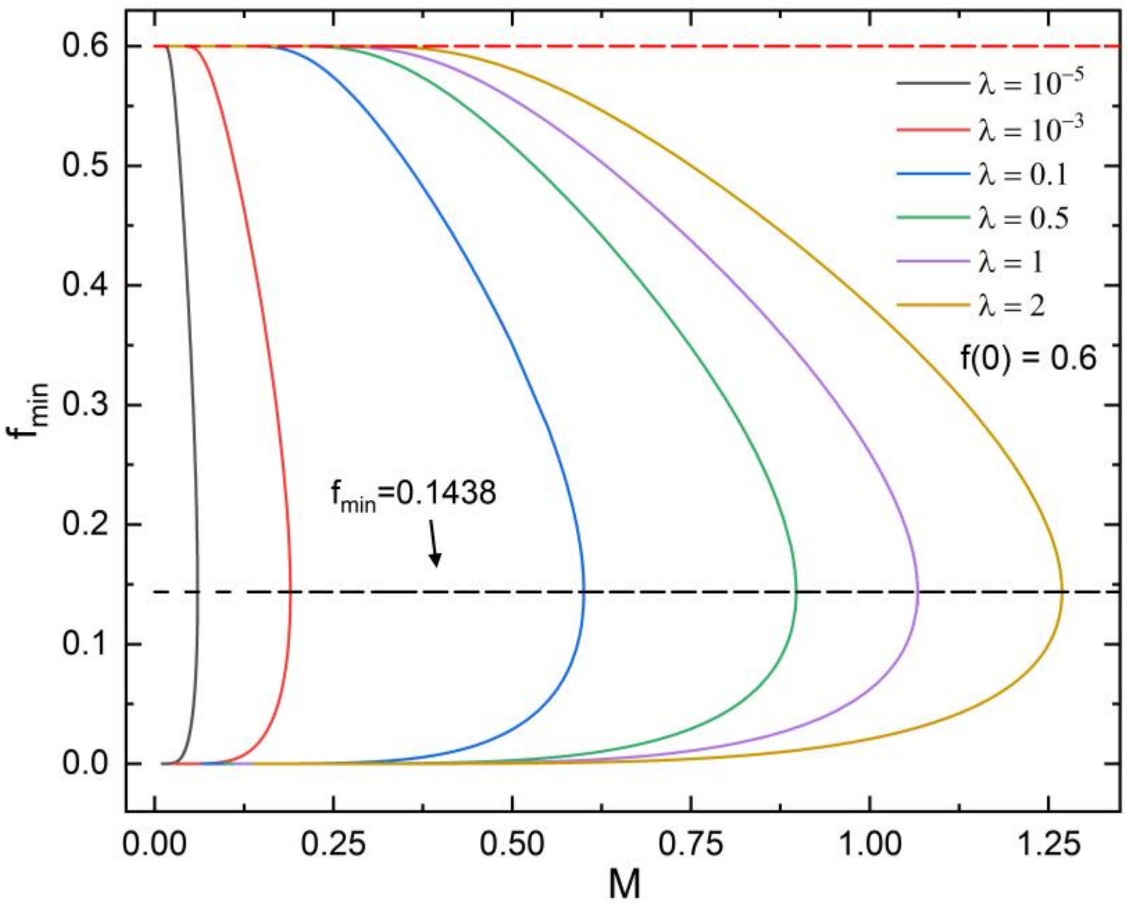
$$S = \frac{1}{16\pi G}\int \sqrt{-g}\, d^4x \left(R - G^2 \lambda \mathcal{P}\right)$$

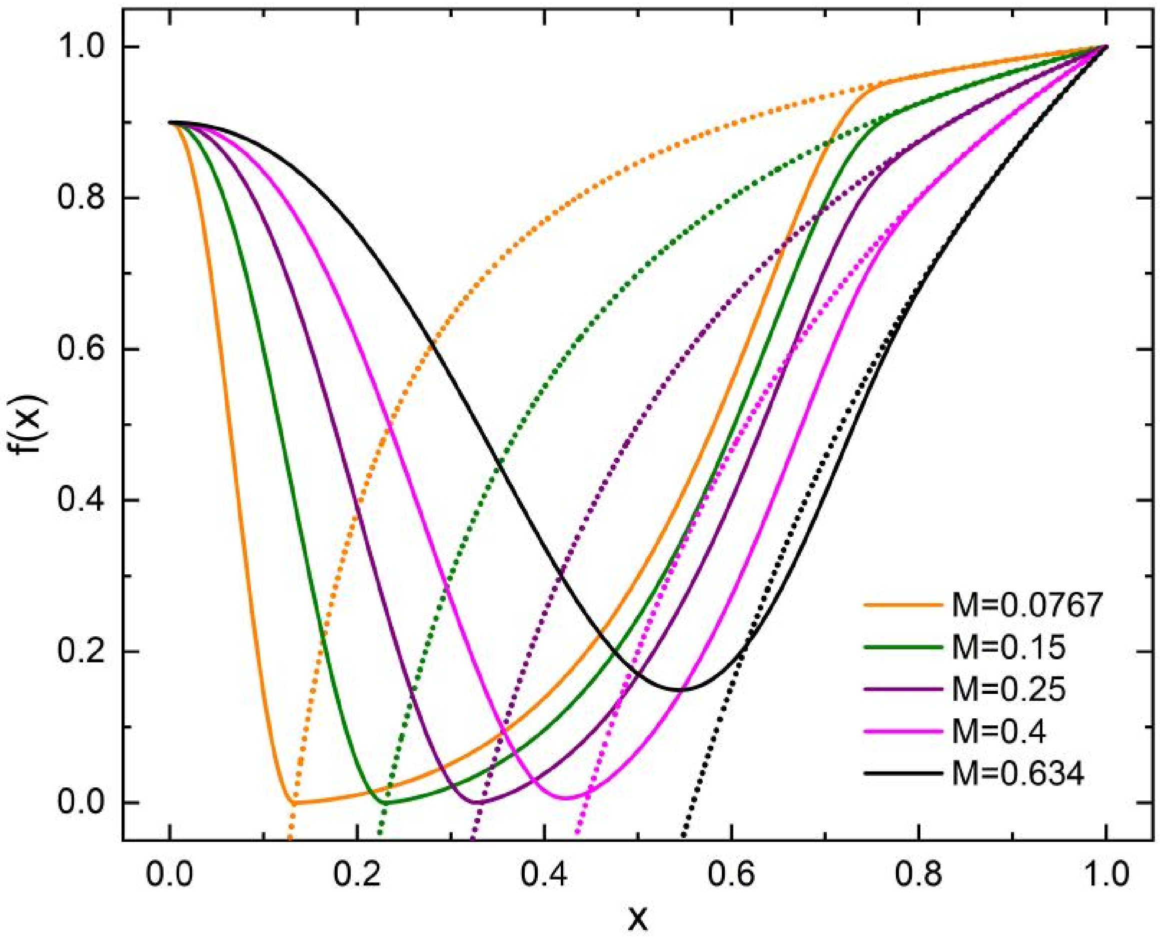
$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

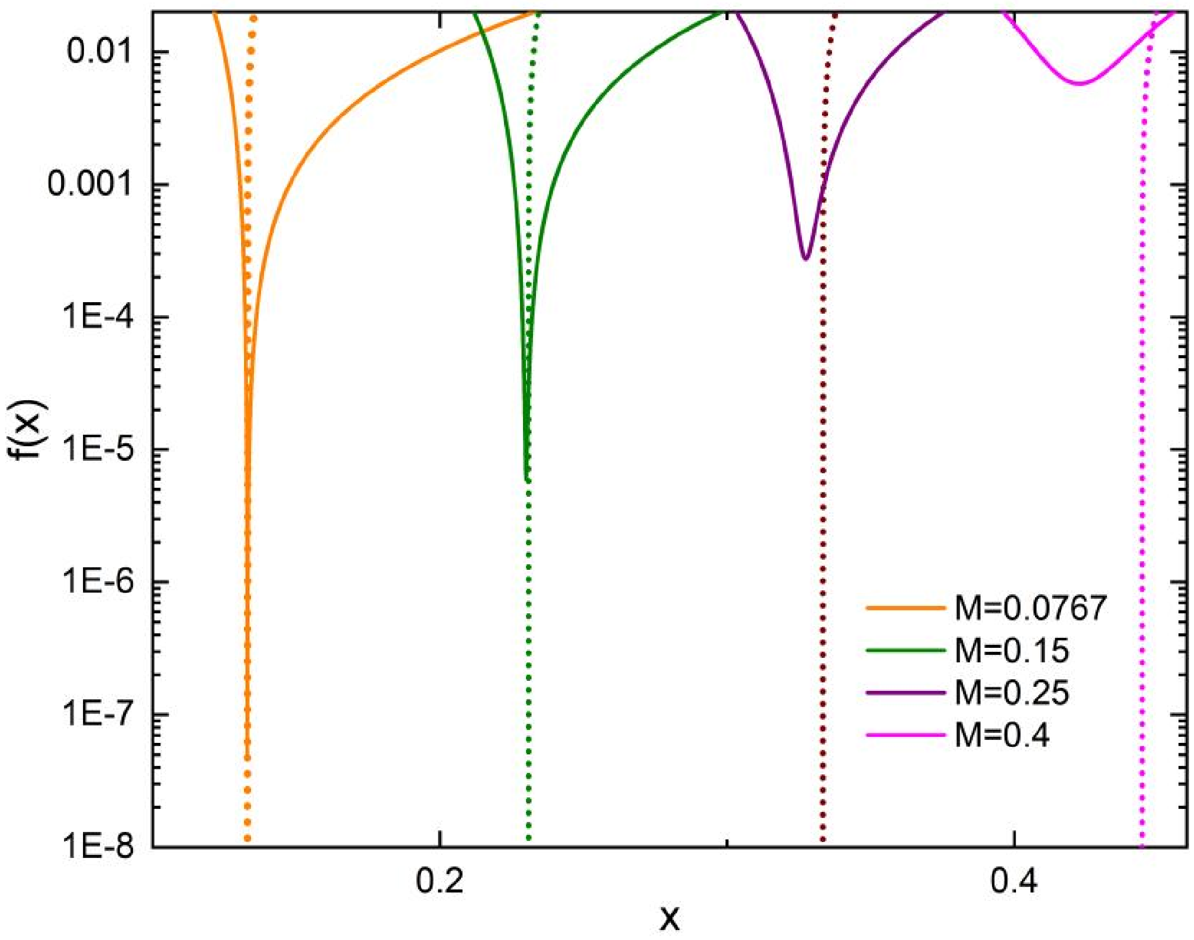
$$\mathcal{P} = 12 R_a{}^c{}_b{}^d R_c{}^e{}_d{}^f R_e{}^a{}_f{}^b + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab} - 12 R_{abcg} R^{ac} R^{bd} + 8 R_a^b R_b^c R_c^a .$$

$$x = \frac{r}{1+r}$$









$$R_{abcd}R^{abcd} = \frac{4(f(0)-1)^2}{r^4} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

裸奇点解： 奇点定理的失效

总结

- 物质场将破坏 Event horizon

猜测：Nature abhors event horizon

- 冻结星　　　　极端黑洞　　　　外部

- 修改引力下会出现冻结星

奇点->事件视界->黑洞

猜测：

无奇点->无事件视界->冻结星

Thank you