中山大學
SUN YAT-SEN UNIVERSITY

中国科学技术大学
University of Science and Technology of China

标量-张量理论构造进展

高显

中山大学

“2024 引力与宇宙学”专题研讨会
彭桓武高能基础理论研究中心（合肥）
2024年11月16日 · 中国科学技术大学

Based on 2004.07752, 2111.08652, 2405.20158, 2410.12680
In collaboration with 胡钰敏、于杨、陈正

Contents

- Evolution of scalar-tensor theories
- Parity violating scalar-tensor theories
- Multiple scalar-tensor theories

Why modifying gravity?

- Phenomenological side:
  - Modified gravity theories provide alternatives to explain the early and late accelerated expansion of our universe;
  - Gravitational waves provide new tools to test gravitation interactions.

- Theoretical side:

  To check the conditions that GR is based on, so that we can understand if and why GR is the correct theory of gravity in nature.

Uniqueness of GR

Lovelock theorem [Lovelock, J.Math.Phys. 12 (1971) 498]

4d + metric theory + diffeomorphism + 2nd derivatives + local Lagrangian

$$\longrightarrow \quad \alpha G_{\mu\nu} + \lambda g_{\mu\nu} = 0$$

Any metric theory of gravity alternative to GR must satisfy (at least):

- extra degrees of freedom (e.g., scalar-tensor theory),
- extra dimensions (e.g., brane world),
- higher derivative terms (e.g., $f(R)$),
- non-Riemannian geometry (e.g., $f(T)$, $f(Q)$),
- giving up locality.

Evolution of scalar-tensor theories

Evolution of scalar-tensor theories with 3 DOFs

The most general theories with 1st order derivatives.

1999 *k*-essence

$$\mathcal{L} = g(\phi) R + F(\phi, \nabla\phi)$$

[Chiba, Okabe, Yamaguchi, astro-ph/9912463]
[Armendariz-Picon, Damour, Mukhanov, hep-th/9904075]

1961 Brans-Dicke theory

$$\mathcal{L} = \phi R - \frac{\omega}{\phi} (\nabla\phi)^2$$

[Brans & Dicke, Phys. Rev. 124 (1961) 925]

Evolution of scalar-tensor theories with 3 DOFs

With 2nd order derivatives? ???? $\mathcal{L} = \mathcal{L}(R, \phi, \nabla\phi, \nabla\nabla\phi)$

The most general theories with 1st order derivatives.

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Evolution of scalar-tensor theories with 3 DOFs

Ostrogradsky instability [Ostrogradsky, Mem. Acad. St. Petersbourg 6 (1850) 385.]

Higher order time derivatives in EoMs (non-degenerate)

⬇

Additional DoFs with a linear instability in Hamiltonian (unbounded)

With 2^{nd} order derivatives? ???? $\mathcal{L} = \mathcal{L}(R, \phi, \nabla\phi, \nabla\nabla\phi)$

The most general theories with 1^{st} order derivatives.

1999 *k*-essence
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Evolution of scalar-tensor theories with 3 DOFs

$$\mathcal{L} = G_2(X,\phi) + G_3(X,\phi)\Box\phi + G_4(X,\phi)\,R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + \cdots$$

The most general theories with 2nd order derivatives.

2011 Horndeski theory
[Horndeski, Int.J.Theor.Phys. 10 (1974) 363]
[Deffayet, XG, Steer & Zahariade, 1103.3260]
[Kobayashi, Yamaguchi, Yokoyama, 1105.5723]

The most general theories with 1st order derivatives.

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[Brans & Dicke, Phys. Rev. 124 (1961) 925]

Evolution of scalar-tensor theories with 3 DOFs

Higher derivatives & higher curvature terms?

???? Beyond Horndeski

The most general theories with 2nd order derivatives.

2011 Horndeski theory

[Horndeski, Int.J.Theor.Phys. 10 (1974) 363]
[Deffayet, XG, Steer & Zahariade, 1103.3260]
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1961 Brans-Dicke theory

[Brans & Dicke, Phys. Rev. 124 (1961) 925]

Evolution of scalar-tensor theories with 3 DOFs

disformal transformation [Bekenstein, gr-qc/9211017]

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = C(\phi, X)\, g_{\mu\nu} + D(\phi, X)\, \nabla_\mu \phi \nabla_\nu \phi$$

2013 — Disformal transformation
[Zumalacárregui & García-Bellido, 1308.4685]

2011 — Horndeski theory
[Horndeski, Int.J.Theor.Phys. 10 (1974) 363]
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Evolution of scalar-tensor theories with 3 DOFs

2021 Higher derivative disformal trans.

[Takahashi, Motohashi and Minamitsuji, 2111.11634]
[Takahashi, Minamitsuji and Motohashi, 2209.02176, 2304.08624]

[Takahashi, 2307.08814]

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = f_0 g_{\mu\nu} + f_1 \nabla_\mu\phi\nabla_\nu\phi + 2f_2\nabla_{(\mu}\phi\nabla_{\nu)}X + 2f_3\nabla_{(\mu}\phi\nabla_{\nu)}Z + \cdots$$

$$X = (\nabla\phi)^2\,, \qquad Z = (\nabla X)^2\,, \cdots$$

2013 Disformal transformation

[Zumalacárregui & García-Bellido, 1308.4685]

2011 Horndeski theory

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2015 — DHOST

[Langlois & Noui, 1510.06930]
[Crisostomi, Koyama & Tasinato, 1602.03119]
[Ben Achour et al, 1602.08398, 1608.08135]

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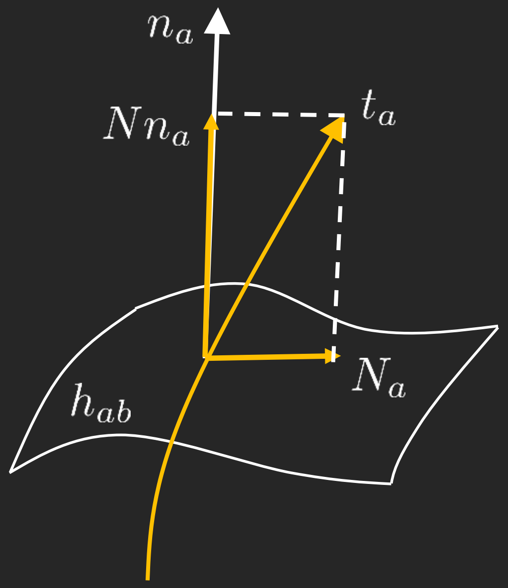
201

Degenerate Higher Order Scalar-Tensor theory



$$K_{ab} = \frac{1}{2}\pounds_{\boldsymbol{n}} h_{ab} \qquad \ddot{\phi} = \pounds_{\boldsymbol{n}}^2 \phi$$

$$\mathcal{L} \supset \mathcal{A}\ddot{\phi}^2 + 2\mathcal{B}^{ab}\ddot{\phi}K_{ab} + \mathcal{C}^{ab,cd}K_{ab}K_{cd}$$

$$\boldsymbol{H} = \begin{pmatrix} \mathcal{A} & \mathcal{B}^{cd} \\ \mathcal{B}^{ab} & \mathcal{C}^{ab,cd} \end{pmatrix}$$ to be degenerate

Evolution of scalar-tensor theories with 3 DOFs

2021 **Higher derivative disformal trans.**
[Takahashi, Motohashi and Minamitsuji, 2111.11634]
[Takahashi, Minamitsuji and Motohashi, 2209.02176, 2304.08624]

2018 **U(nitary)-DHOST**
[De Felice, Langlois, Mukohyama, Noui and Wang, 1803.06241]
[De Felice, Mukohyama, Takahashi, 2110.03194]

2015 **DHOST**
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2014 **GLPV theory**
[Gleyzes, Langlois, Piazza and Vernizzi, 1404.6495]

2013 **Disformal transformation**
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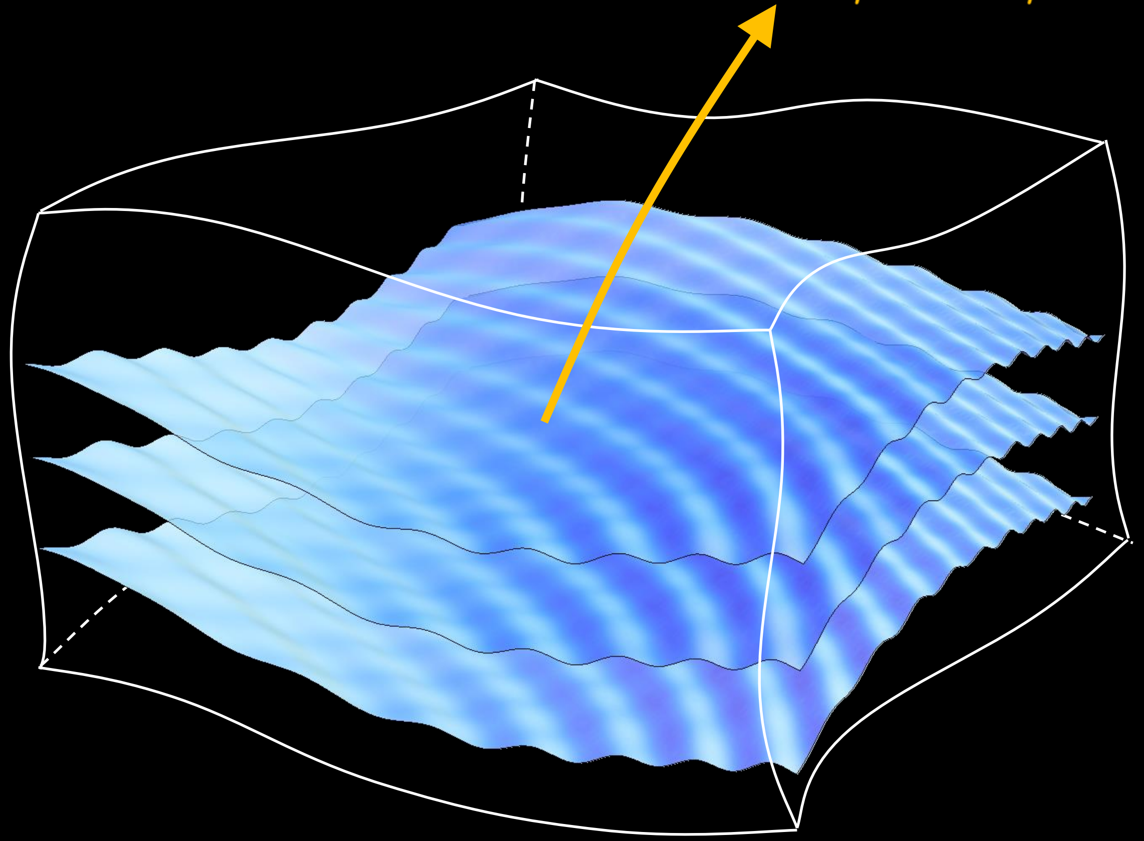
Scalar-tensor theory v.s. Spatially covariant gravity



$\phi(t, \vec{x}) = \text{constant}$

spacelike hypersurfaces

Scalar-tensor theory v.s. Spatially covariant gravity

Gauge fixing: $\phi(t,\vec{x}) \to t$ as long as $\nabla_\mu \phi$ is timelike
(unitary gauge)

Generally covariant
Scalar-tensor theories
$\mathcal{L}(\phi, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu)$

Spatially covariant
gravity theories
$\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$

Gauge recovering: $t \to \phi(t,\vec{x})$
(Stueckelberg trick)

[Motohashi, Suyama, Takahashi, 1608.00071]
[De Felice, Langlois, Mukohyama, Noui & Wang, 1803.06241]
[De Felice, Mukohyama, Takahashi, 2110.03194]

The correspondence

[胡钰敏 & XG, 2004.07752, 2111.08652]

- From scalar-tensor theory (4D) to spatially covariant gravity (3D)

$$\nabla_\mu \phi \to \delta^0_\mu$$

$$\nabla_\mu \nabla_\nu \phi \to -\delta^0_\mu \delta^0_\nu \frac{1}{N}\left(\dot{N} - N^i \partial_i N\right) - 2\delta^0_{(\mu} a_{\nu)} - \frac{1}{N} K_{\mu\nu}$$

- From spatially covariant gravity (3D) to scalar-tensor theory (4D)

$$\frac{1}{N} \to \sqrt{2X}$$

$$a_\mu \to -\frac{1}{2}\left[\nabla_\mu \ln X + \frac{1}{2X}\nabla_\mu \phi \left(\nabla^\nu \phi \nabla_\nu \ln X\right)\right]$$

$$K_{\mu\nu} \to -\frac{1}{\sqrt{2X}}\left[\nabla_\mu \nabla_\nu \phi - \nabla_{(\mu} \phi \nabla_{\nu)} \ln X - \frac{1}{4X}\nabla_\mu \phi \nabla_\nu \phi \left(\nabla_\rho \phi \nabla^\rho \ln X\right)\right]$$

The correspondence

- Horndeski theory

$$\mathcal{L}^{\mathrm{H}} = G_2 + G_3 \Box\phi + G_4 \, {}^4R + \frac{\partial G_4}{\partial X}\left[(\Box\phi)^2 - (\nabla_a\nabla_b\phi)^2\right] + G_5 \, {}^4G^{ab}\nabla_a\nabla_b\phi - \frac{1}{6}\frac{\partial G_5}{\partial X}\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_a\nabla_b\phi)^2 + 2\,(\nabla_a\nabla_b\phi)^3\right]$$

- Horndeski theory in unitary gauge (with ADM coordinates)

$$\mathcal{L}^{\mathrm{H,unitary}} \simeq N\sqrt{h}\Big(a_0\,K + a_1\,{}^3R\,K + a_2\,{}^3R_{ij}K^{ij} + b_1\,K^2 + b_2 K_{ij}K^{ij} + c_1\,K^3 + c_2\,K K_{ij}K^{ij} + c_3\,K^i_j K^j_k K^k_j + d_0 + d_1\,{}^3R\Big)$$

[Gleyzes, Langlois, Vernizzi, 1411.3712]
[Gleyzes, Langlois, Piazza, Vernizzi, 1404.6495]

Spatially covariant gravity

[XG, et al: 1406.0822 , 1409.6708, 1806.02811, 1902.07702, 1910.13995, 2004.07752, 2006.15633, 2111.08652, 2405.20158, 2410.12680]

Spatially covariant gravity

$$S = \int \mathrm{d}t\mathrm{d}^3x \, N\sqrt{h}\mathcal{L}(t, N, h_{ij}, R_{ij}, K_{ij}, \nabla_i)$$

Gauge-fixed version (viewed in a special coordinate system) of a generally covariant scalar-tensor theory (with a single scalar field).

2 tensor + 1 scalar DoFs with higher derivative EoMs, as long as the Lagrangian is generally nonlinear in the lapse function N.

U-DHOST v.s. Spatially covariant gravity

correspondence

U-DHOST

2018 U-DHOST
[De Felice, Langlois, Mukohyama, Noui & Wang, 1803.06241]

2014 GLPV theory
[Gleyzes, Langlois, Piazza and Vernizzi, 1404.6495]

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[Deffayet, XG, Steer & Zahariade, 1103.3260]
[Kobayashi, Yamaguchi, Yokoyama, 1105.5723]

Spatially covariant gravity

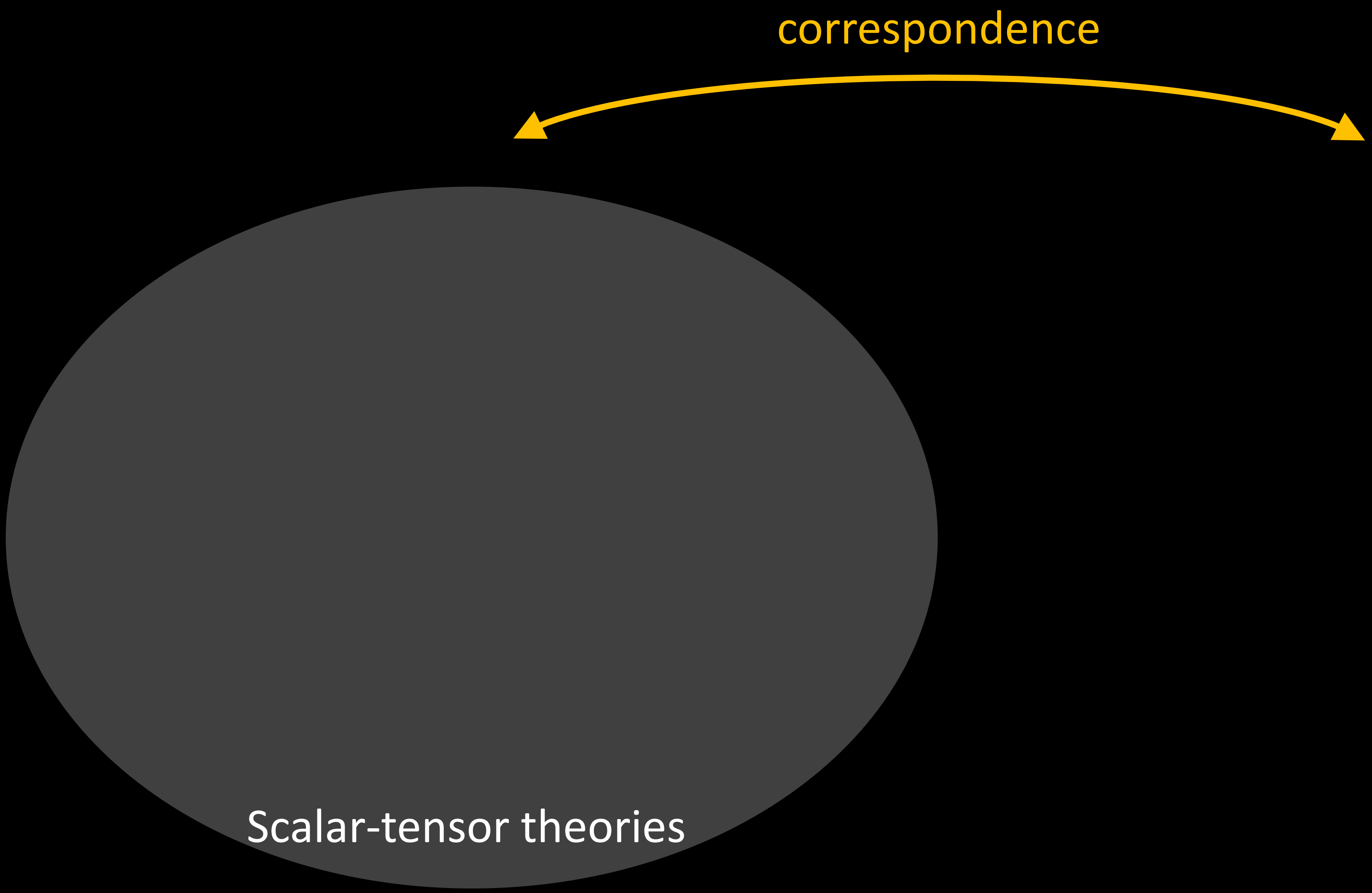
2014 Spatially covariant gravity
[XG, 1406.0822, 1409.6708]

2009 Hořava gravity
[Horava, 0901.3775]

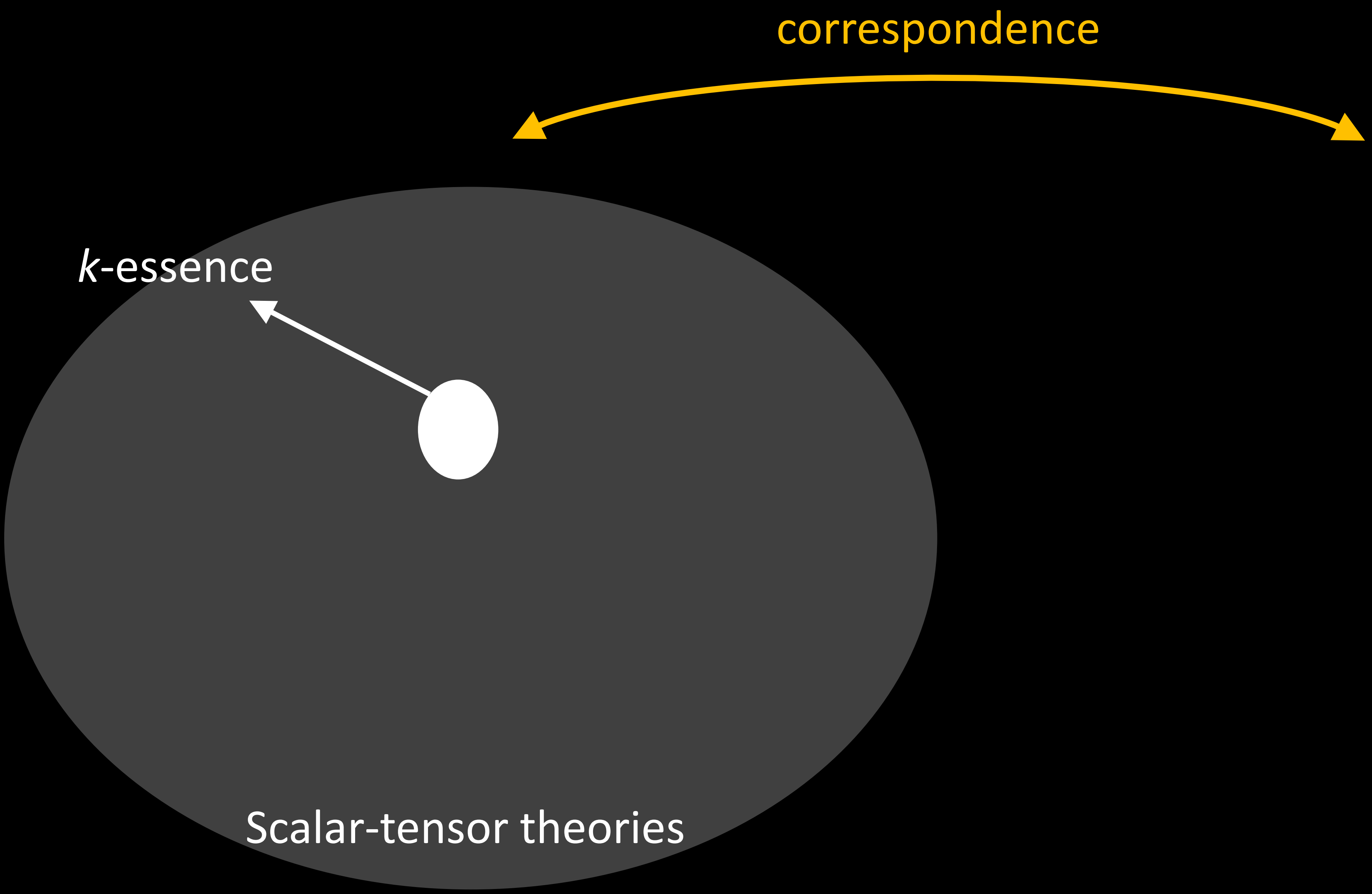
2007 EFT of inflation
[Cheung, Creminelli, Fitzpatrick, Kaplan & Senatore, 0709.0293]

2004 Ghost condensation
[Arkani-Hamed, Cheng, Luty & Mukohyama, hep-th/0312099]

U-DHOST v.s. Spatially covariant gravity



U-DHOST v.s. Spatially covariant gravity



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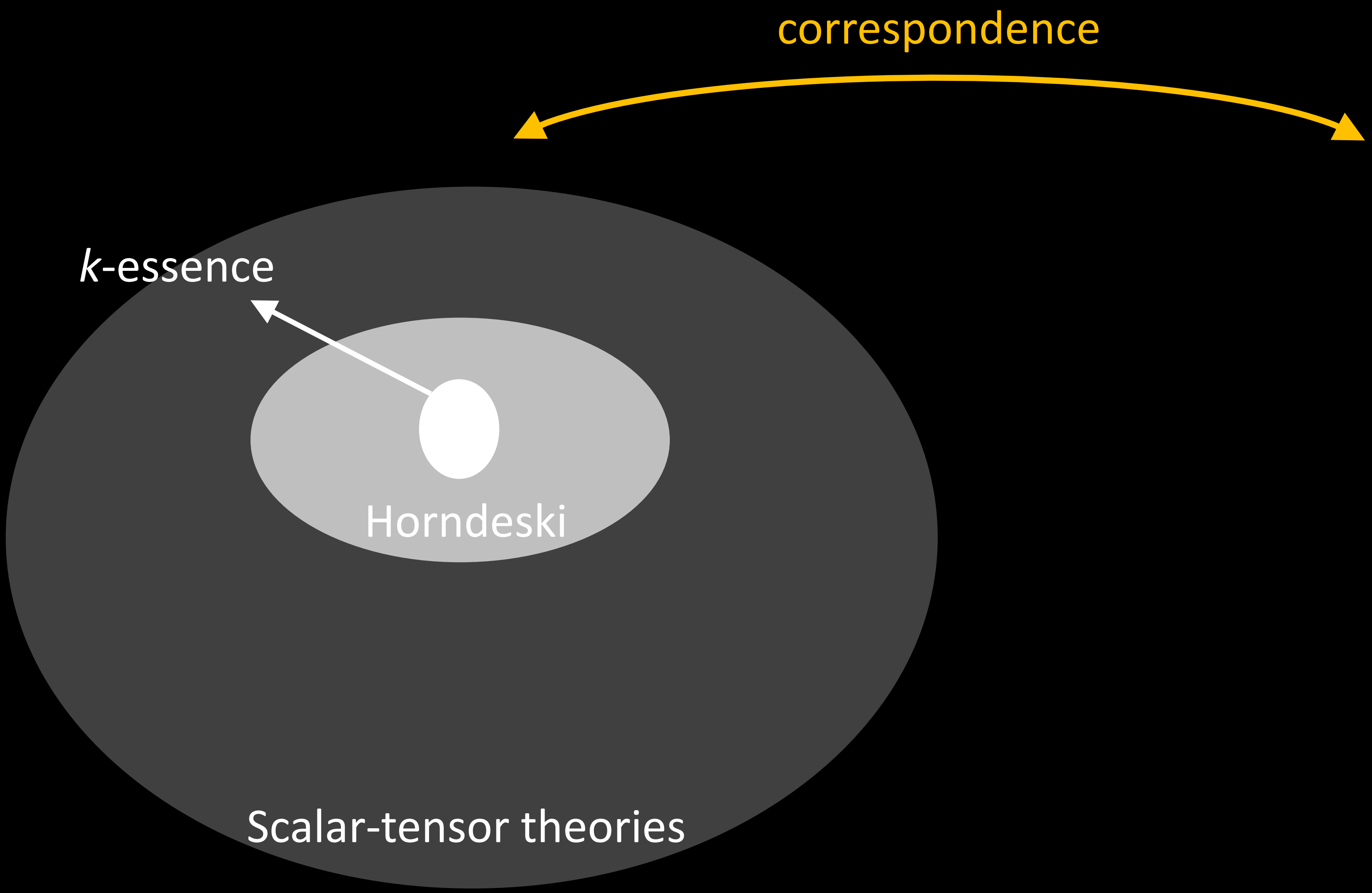
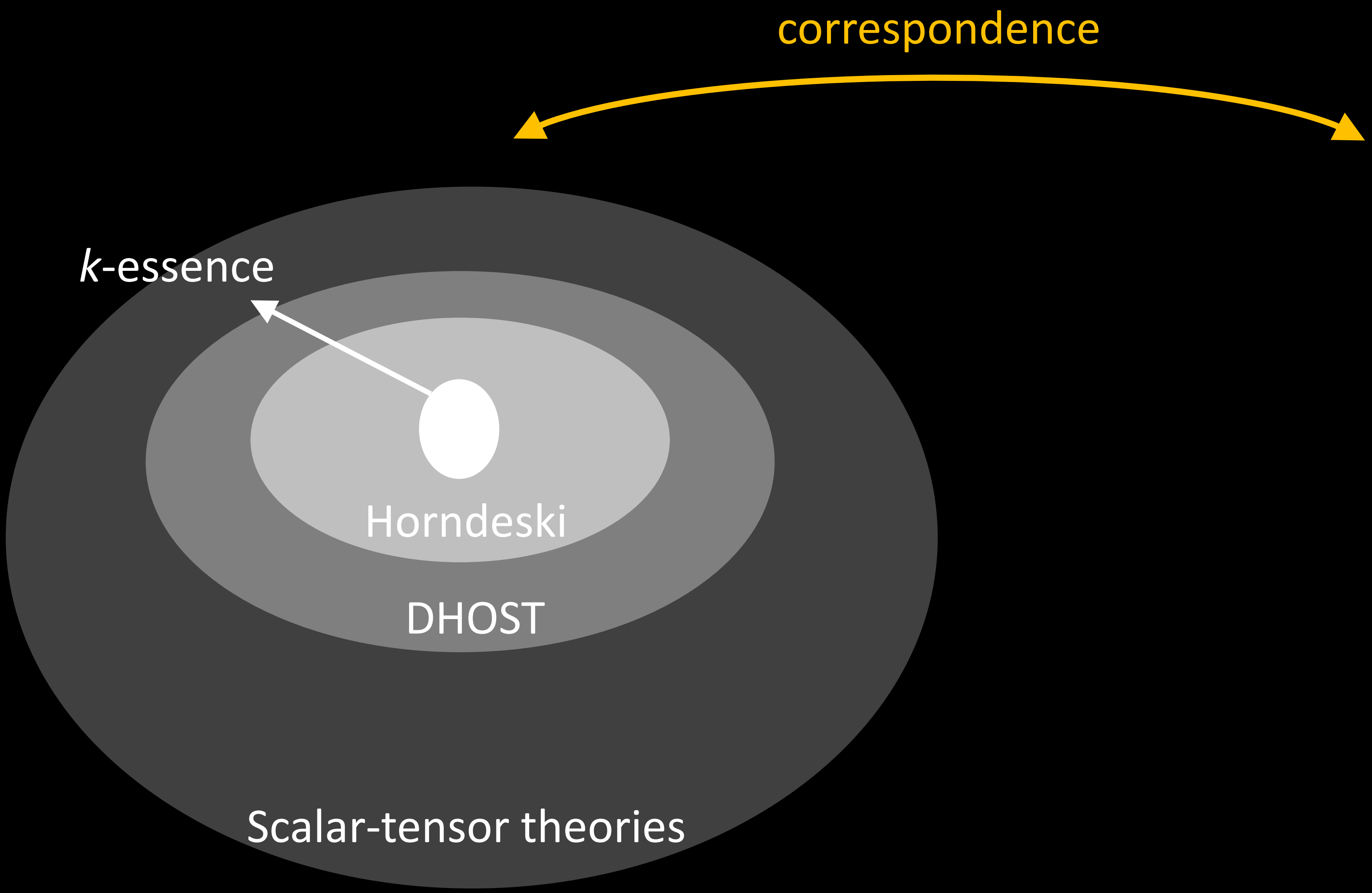
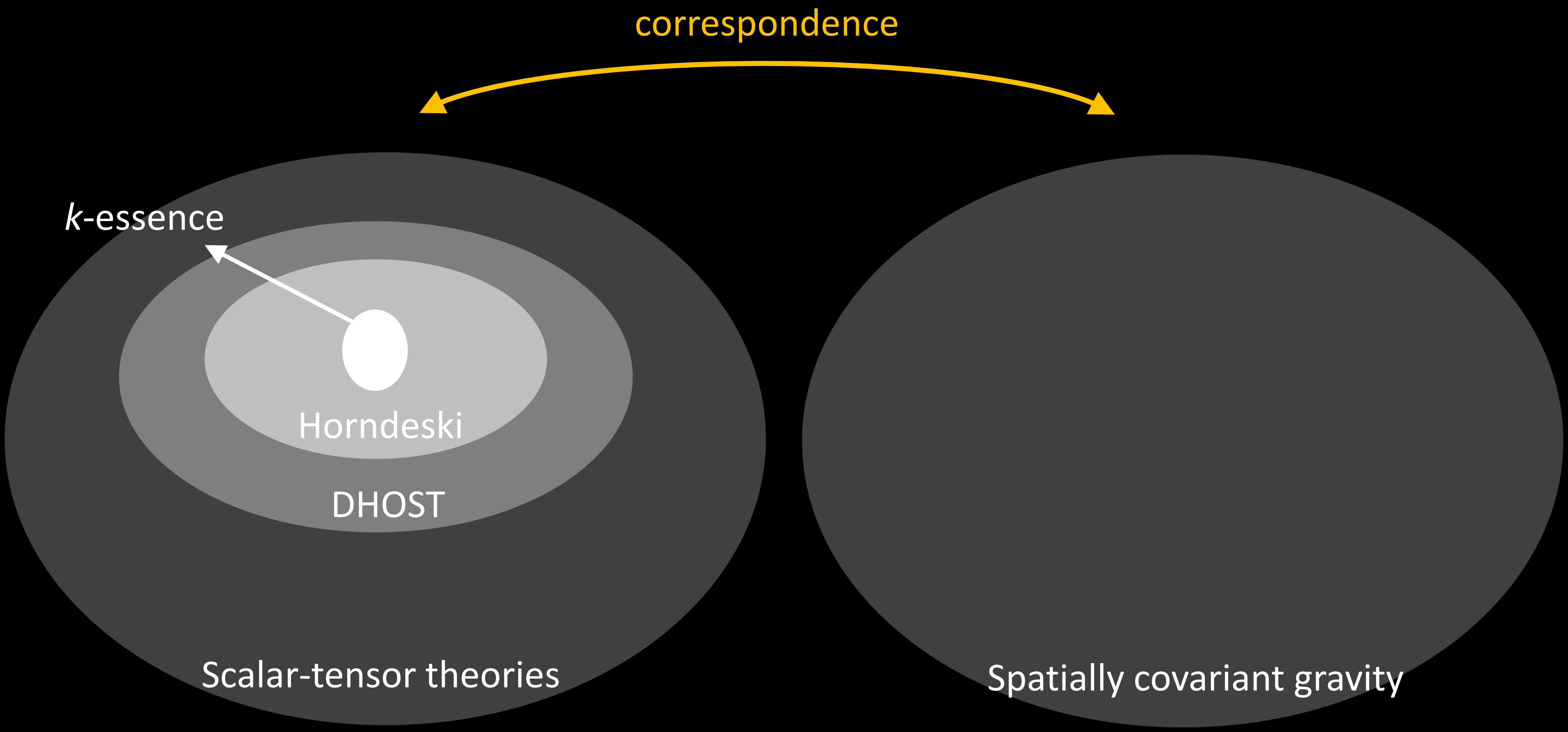


U-DHOST v.s. Spatially covariant gravity



U-DHOST v.s. Spatially covariant gravity



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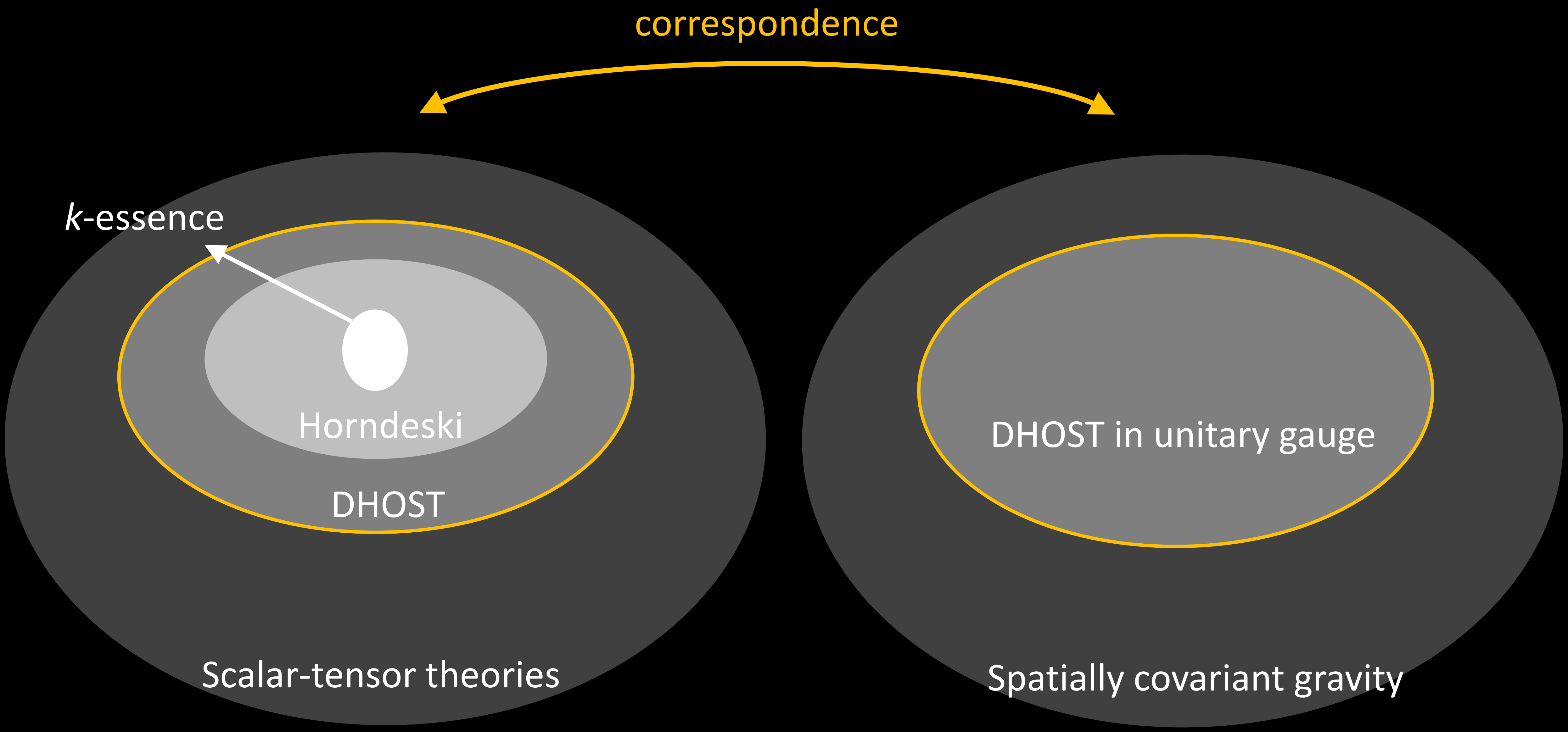
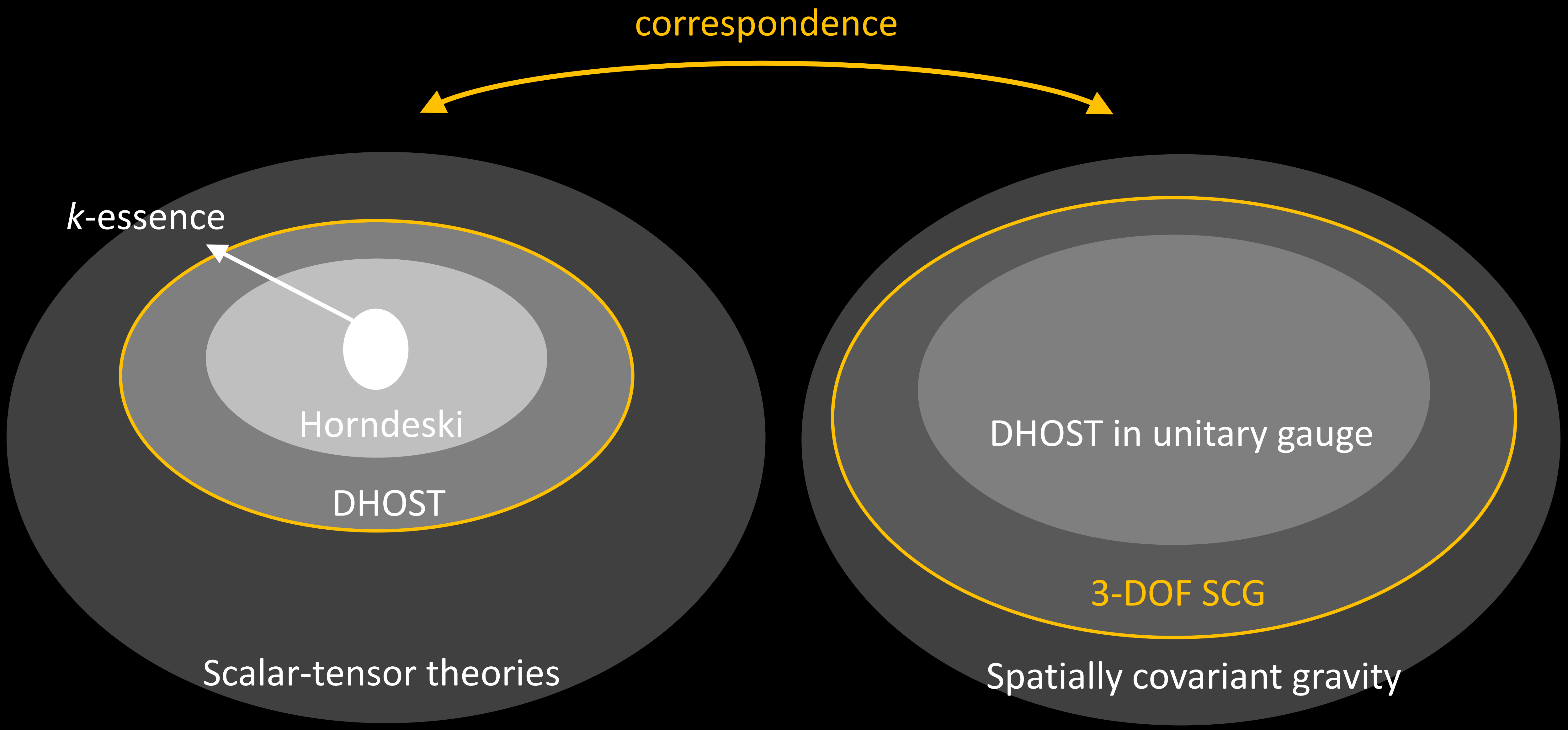
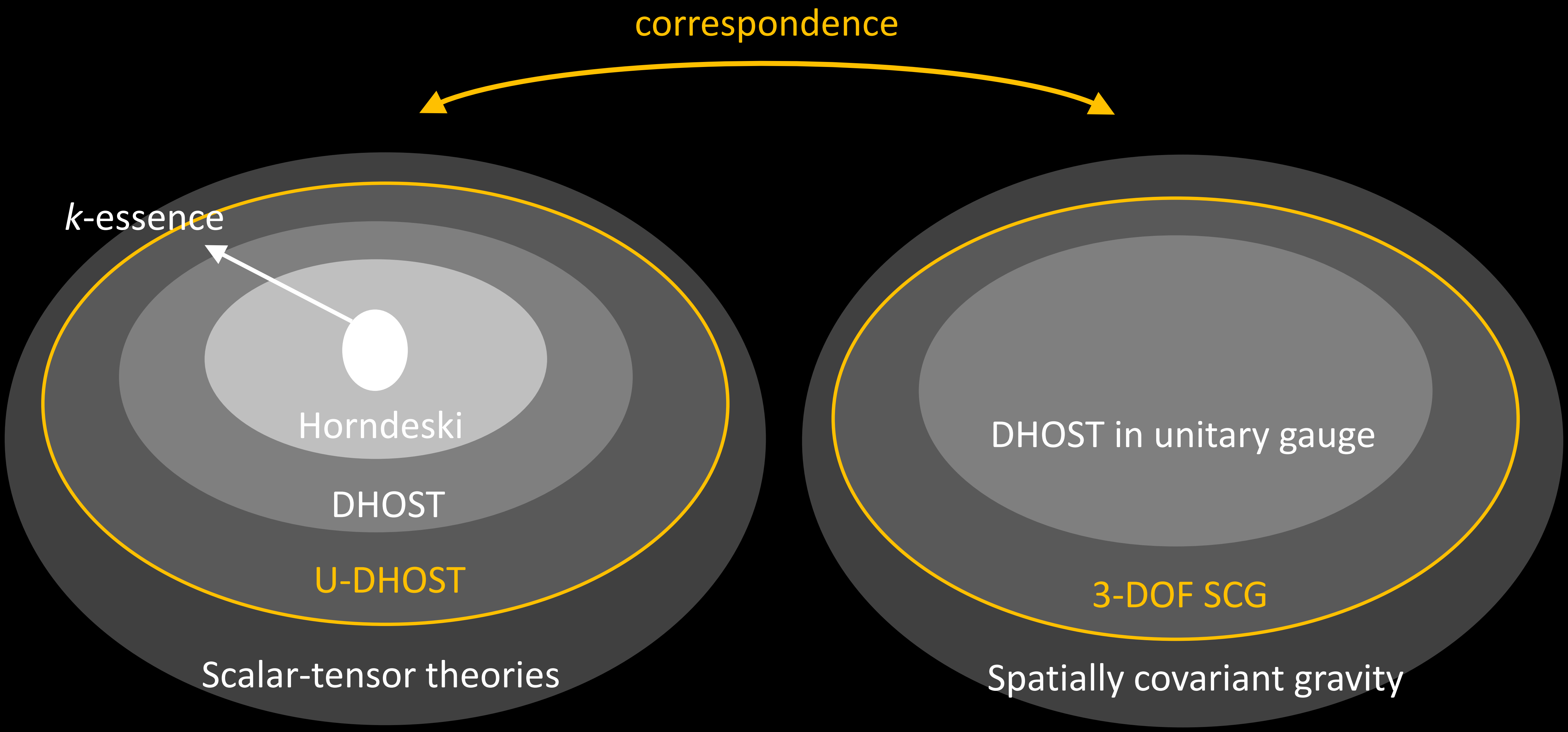


U-DHOST v.s. Spatially covariant gravity



U-DHOST v.s. Spatially covariant gravity



Developments of SCG

2014 Spatially covariant gravity
[XG, 1406.0822, 1409.6708]

2-DOF (TTDOF) theories

2019 TTDOF theory
[XG, 姚志邦, 1910.13995]
[胡钰敏, XG, 2104.07615]
[Lin, Gong, Lu, Zhang, 2011.05739]

2020 TTDOF with auxiliary constraints
[姚志邦, Oliosi, XG, Mukohyama, 2011.00805]

2023 Multiple auxiliary constraints
[姚志邦, Oliosi, XG, Mukohyama, 2302.02090]

Developments of SCG

3-DOF theories

- 2024 Multiple fields
  [于杨, 陈正, 胡钰敏, XG, 2410.12680]
- 2024 Parity violation GST
  [胡钰敏, XG, 2405.20158]
- 2019 Parity violation SCG
  [XG, 洪巽阳, 1906.07131]
- 2018 Dynamical lapse function
  [XG, 姚志邦, 1806.02811]

2-DOF (TTDOF) theories

- 2023 Multiple auxiliary constraints
  [姚志邦, Oliosi, XG, Mukohyama, 2302.02090]
- 2020 TTDOF with auxiliary constraints
  [姚志邦, Oliosi, XG, Mukohyama, 2011.00805]
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  [胡钰敏, XG, 2104.07615]
  [Lin, Gong, Lu, Zhang, 2011.05739]

2014 Spatially covariant gravity
[XG, 1406.0822, 1409.6708]

Parity violating scalar-tensor theory

Gravity theories with parity violation

- Axion fields [Phys. Rev. D 41, 1231 (1990)], [0808.0673], [0802.4210]

$$\mathcal{L}_\phi = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{g}{4}\phi\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

- Chern-Simons modified gravity [Annals Math. 99, 48 (1974)], [gr-qc/0308071], [0907.2562]

$$\mathcal{L}_{\rm CS} = f(\phi)\,\varepsilon^{\mu\nu\rho\sigma}R_{\rho\sigma\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}$$

- Parity violating scalar-tensor theory [1710.04531], [2405.20158]
- Parity violating Horava-Lifshitz gravity/spatially covariant gravity [1111.5345], [1208.5490], [2112.06446], [1906.07131]
- Nieh-Yan (NY) gravity [J. Math. Phys. 23 (1982) 373] [2007.08038], [2012.14423], [2105.06870], …

$$\mathcal{L}_{\rm NY} = \frac{c}{4}\phi\varepsilon^{\mu\nu\alpha\beta}T_{\rho\mu\nu}T^{\rho}{}_{\alpha\beta}$$

- Parity violating non-metricity theory [1908.04313], [2108.01337], [2212.14362], …
- Parity violating metric-affine gravity [2112.09154], [1506.02882], [1810.12276], …

Chiral scalar-tensor theory

"Chiral scalar-tensor theory" with 3 DoFs [Crisostomi et al, 1710.04531]

$$\mathcal{L}_{\rm PV1} = \sum_{n=1}^{4} a_n(\phi, X)\mathcal{A}_n$$

$$\begin{aligned}
\mathcal{A}_1 &= \varepsilon^{abcd} R_{cdef} R_{ab}{}^{e}{}_{g} \phi^f \phi^g, \\
\mathcal{A}_2 &= \varepsilon^{abcd} R_{cdef} R^{f}{}_{b} \phi^e \phi_a, \\
\mathcal{A}_3 &= \varepsilon^{abcd} R_{cdef} R_{ag}{}^{ef} \phi_b \phi^g, \\
\mathcal{A}_4 &= \varepsilon^{abcd} R_{cdef} R^{ef}{}_{ab} \phi_g \phi^g.
\end{aligned}$$

The combination is ghost-free (in unitary gauge) when coefficients satisfy the condition:

$$4a_1 + a_2 + 2a_3 + 8a_4 = 0$$

Chiral scalar-tensor theory

"Chiral scalar-tensor theory" with 3 DoFs

[Crisostomi et al, 1710.04531]

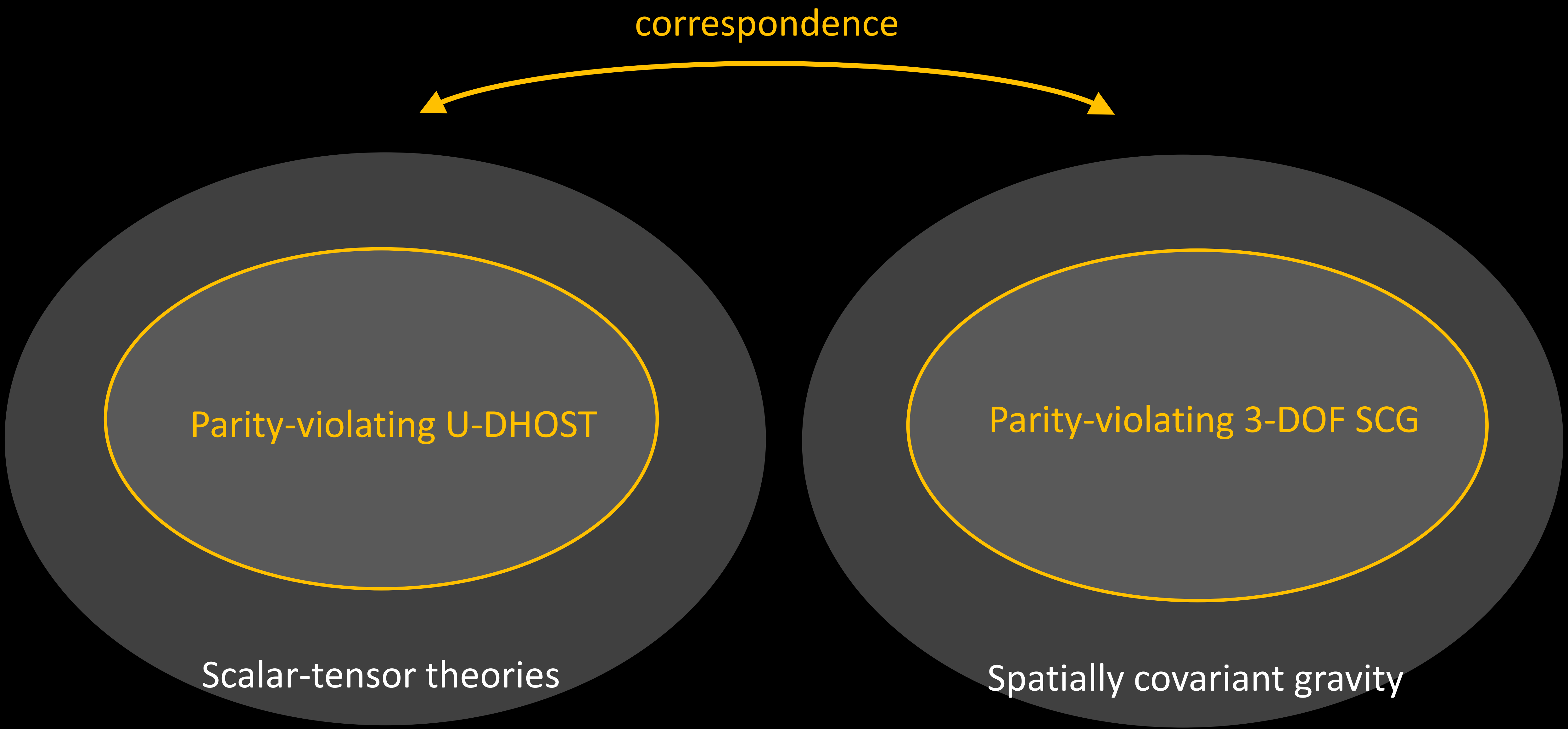
$$\mathcal{L}_{\rm PV3} = \sum_{n=1}^{6} c_n(\phi, X)\mathcal{C}_n$$

$$\begin{aligned}
\mathcal{C}_1 &= \varepsilon^{abcd} R_{cdef}\phi^e_a\phi^f_b\phi_g\phi^g, \\
\mathcal{C}_2 &= \varepsilon^{abcd} R_{cdef}\phi^e_a\phi^g_b\phi^f\phi_g, \\
\mathcal{C}_3 &= \varepsilon^{abcd} R_{cefg}\phi^f_a\phi^g_b\phi^e\phi_d, \\
\mathcal{C}_4 &= \varepsilon^{abcd} R_{cdef}\phi^e_a\phi^f_g\phi_b\phi^g, \\
\mathcal{C}_5 &= \varepsilon^{abcd} R_{de}\phi^e_a\phi^f_b\phi_c\phi_f, \\
\mathcal{C}_6 &= \varepsilon^{abcd} R_{cdef}\phi^e\phi_a\phi^f_b\Box\phi.
\end{aligned}$$

The combination is ghost-free (in unitary gauge) when coefficients satisfy the conditions:

$$4c_1 + 2c_2 + 2c_3 - c_5 = 0, \qquad 2c_1 + c_2 + c_4 = 0$$

U-DHOST v.s. Spatially covariant gravity



Playing with monomials

- A general generally covariant scalar-tensor (GST) monomial

$$\cdots \underbrace{\cdots {}^4R \cdots}_{c_0} \underbrace{\cdots \nabla\, {}^4R \cdots}_{c_1} \underbrace{\cdots \nabla\nabla\, {}^4R \cdots}_{c_2} \cdots \underbrace{\cdots \nabla\nabla\phi \cdots}_{d_2} \underbrace{\cdots \nabla\nabla\nabla\phi \cdots}_{d_3} \underbrace{\cdots \nabla\nabla\nabla\nabla\phi \cdots}_{d_4} \cdots$$

with indices contracted by g^{ab} and/or ε^{abcd}.

- A general spatially covariant gravity (SCG) monomial

$$\underbrace{\cdots {}^3R \cdots}_{r_0} \underbrace{\cdots \Delta\, {}^3R \cdots}_{r_1} \cdots \underbrace{\cdots K \cdots}_{k_0} \underbrace{\cdots \Delta K \cdots}_{k_1} \cdots \underbrace{\cdots a \cdots}_{a_0} \underbrace{\cdots \Delta a \cdots}_{a_1} \cdots$$

with indices contracted by h^{ij} and/or ε^{ijk}.
(Δ stands for both spatial/temporal derivatives)

Parity-violating SCG monomials up to $d = 4$

[胡钰敏 & XG, 2405.20158]

d	Category	Form	Irreducible	Reducible	Number
3	$(0;1,1)$	$[K\nabla K]$	$\varepsilon_{ijk}K^i_l\nabla^j K^{kl}$	–	1
4	$(0;0,2)$	$[\pounds K\nabla K]$	–	$\varepsilon_{ijk}\pounds_{\boldsymbol{u}}K^{li}\nabla^j K^k_l$	0
		$[\nabla a\nabla K]$	–	$\varepsilon_{ijk}\nabla^l a^i\nabla^j K^k_l$	
	$(0;2,1)$	$[aK\nabla a]$	$\varepsilon_{ijk}a^i\nabla^l a^j K^k_l$	–	6
		$[aa\nabla K]$	–	$\varepsilon_{ijk}a^l a^i\nabla^j K^k_l$	
		$[KK\nabla K]$	$\varepsilon_{ijk}K^i_l K^j_m\nabla^m K^{kl}$, $\varepsilon_{ijk}K^i_l K^{ml}\nabla^j K^k_m$, $\varepsilon_{ijk}KK^i_l\nabla^j K^{kl}$	–	
		$[aK\pounds K]$	$\varepsilon_{ijk}a^i K^{jl}\pounds_{\boldsymbol{u}}K^k_l$	–	
		$[FK\nabla K]$	$\varepsilon_{ijk}FK^i_l\nabla^j K^{kl}$	–	
	$(1;2,0)$	$[RKa]$	$\varepsilon_{ijk}a^i K^{jl\,3}R^k_l$	–	1
	$(1;0,1)$	$[R\nabla K]$	$\varepsilon_{ijk}{}^3R^{li}\nabla^j K^k_l$	–	1

Parity-violating SCG monomials up to $d = 4$

[胡钰敏 & XG, 2405.20158]

d	Category	Form	Irreducible	Reducible	Number
3	$(0;1,1)$	$[K\nabla K]$	$\varepsilon_{ijk}K^i_l\nabla^j K^{kl}$	–	1
4	$(0;0,2)$	$[\pounds K\nabla K]$	–	$\varepsilon_{ijk}\pounds_{\boldsymbol{u}}K^{li}\nabla^j K^k_l$	0
		$[\nabla a\nabla K]$	–	$\varepsilon_{ijk}\nabla^l a^i\nabla^j K^k_l$	
	$(0;2,1)$	$[aK\nabla a]$	$\varepsilon_{ijk}a^i\nabla^l a^j K^k_l$	–	6
		$[aa\nabla K]$	–	$\varepsilon_{ijk}a^l a^i\nabla^j K^k_l$	
		$[KK\nabla K]$	$\varepsilon_{ijk}K^i_l K^j_m\nabla^m K^{kl}$, $\varepsilon_{ijk}K^i_l K^{ml}\nabla^j K^k_m$, $\varepsilon_{ijk}KK^i_l\nabla^j K^{kl}$	–	
		$[aK\pounds K]$	$\varepsilon_{ijk}a^i K^{jl}\pounds_{\boldsymbol{u}}K^k_l$	–	
		$[FK\nabla K]$	$\varepsilon_{ijk}FK^i_l\nabla^j K^{kl}$	–	
	$(1;2,0)$	$[RKa]$	$\varepsilon_{ijk}a^i K^{jl\,3}R^k_l$	–	1
	$(1;0,1)$	$[R\nabla K]$	$\varepsilon_{ijk}{}^3R^{li}\nabla^j K^k_l$	–	1

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		$[\nabla a\nabla K]$	–	$\varepsilon_{ijk}\nabla^l a^i\nabla^j K^k_l$	
	$(0;2,1)$	$[aK\nabla a]$	$\varepsilon_{ijk}a^i\nabla^l a^j K^k_l$	–	6
		$[aa\nabla K]$	–	$\varepsilon_{ijk}a^l a^i\nabla^j K^k_l$	
		$[KK\nabla K]$	$\varepsilon_{ijk}K^i_l K^j_m\nabla^m K^{kl}$, $\varepsilon_{ijk}K^i_l K^{ml}\nabla^j K^k_m$, $\varepsilon_{ijk}KK^i_l\nabla^j K^{kl}$	–	
		$[aK\pounds K]$	$\varepsilon_{ijk}a^i K^{jl}\pounds_{\boldsymbol{u}}K^k_l$	–	
		$[FK\nabla K]$	$\varepsilon_{ijk}FK^i_l\nabla^j K^{kl}$	–	
	$(1;2,0)$	$[RKa]$	$\varepsilon_{ijk}a^i K^{jl}\,{}^3R^k_l$	–	1
	$(1;0,1)$	$[R\nabla K]$	$\varepsilon_{ijk}\,{}^3R^{li}\nabla^j K^k_l$	–	1

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[胡钰敏 & XG, 2405.20158]

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4	$(0;0,2)$	$[\pounds K\nabla K]$	–	$\varepsilon_{ijk}\pounds_{\boldsymbol{u}}K^{li}\nabla^j K^k_l$	0
		$[\nabla a\nabla K]$	–	$\varepsilon_{ijk}\nabla^l a^i\nabla^j K^k_l$	
	$(0;2,1)$	$[aK\nabla a]$	$\varepsilon_{ijk}a^i\nabla^l a^j K^k_l$	–	6
		$[aa\nabla K]$	–	$\varepsilon_{ijk}a^l a^i\nabla^j K^k_l$	
		$[KK\nabla K]$	$\varepsilon_{ijk}K^i_l K^j_m\nabla^m K^{kl}$, $\varepsilon_{ijk}K^i_l K^{ml}\nabla^j K^k_m$, $\varepsilon_{ijk}KK^i_l\nabla^j K^{kl}$	–	
		$[aK\pounds K]$	$\varepsilon_{ijk}a^i K^{jl}\pounds_{\boldsymbol{u}}K^k_l$	–	
		$[FK\nabla K]$	$\varepsilon_{ijk}FK^i_l\nabla^j K^{kl}$	–	
	$(1;2,0)$	$[RKa]$	$\varepsilon_{ijk}a^i K^{jl\,3}R^k_l$	–	1
	$(1;0,1)$	$[R\nabla K]$	$\varepsilon_{ijk}{}^3R^{li}\nabla^j K^k_l$	–	1

Parity-violating scalar-tensor theory up to $d = 4$

[胡钰敏 & XG, 2405.20158]

$d = 3$

$$\mathcal{L}_1 \equiv \frac{1}{\sigma^3}\varepsilon_{abcd}R_{ef}{}^{cd}\phi^a\phi^e\phi^{bf},$$

$d = 4$

$$\mathcal{L}_2 \equiv \frac{1}{\sigma^4}\varepsilon_{abcd}R_{ef}{}^{cd}\phi^a\phi^e\phi^b_m\phi^{fm},$$

$$\mathcal{L}_3 \equiv \frac{1}{\sigma^6}\varepsilon_{abcd}R_{ef}{}^{cd}\phi^m\phi^n\phi^e\phi^a\phi^f_m\phi^b_n,$$

$$\mathcal{L}_4 \equiv \frac{1}{\sigma^4}\varepsilon_{abcd}R^{amen}\phi^b\phi^f\phi^c_e\phi^d_f\left(g_{mn}+\frac{1}{2X}\phi_n\phi_m\right),$$

$$\mathcal{L}_5 \equiv \frac{1}{\sigma^4}\varepsilon_{abcd}R_{ef}{}^{cm}\phi^a\phi^e\phi^{bf}\phi^{dn}\left(g_{mn}+\frac{1}{2X}\phi_m\phi_n\right),$$

$$\mathcal{L}_6 \equiv \frac{1}{\sigma^4}\varepsilon_{abcd}R_{ef}{}^{cd}\phi^a\phi^e\phi^{bf}\phi^{mn}\left(g_{mn}+\frac{1}{2X}\phi_m\phi_n\right),$$

$$\mathcal{L}_7 \equiv \frac{1}{\sigma^2}\varepsilon_{abcd}R_{ef}{}^{cd}R^{amen}\phi^b\phi^f\left(g_{mn}+\frac{1}{2X}\phi_m\phi_n\right).$$

Multiple scalar-tensor theories

Evolution of multi-field scalar-tensor theory

$$\omega^a_{p+1} = \mathrm{d}A^a_p$$

$$S = \int \mathrm{d}^D x \, \varepsilon^{\mu\nu\cdots} \varepsilon^{\alpha\beta\cdots} \omega^a_{\mu\nu\cdots} \omega^b_{\alpha\beta\cdots} \left(\partial_\rho \omega^c_{\gamma\delta\cdots} \cdots\right) \left(\partial_\epsilon \omega^d_{\sigma\tau\cdots} \cdots\right)$$

2010 — Mixed combinations of *p*-form fields

[Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

$$\mathcal{L}_{\pi,\xi} = \sum_{0\leqslant m+n\leqslant 4} (\alpha_{m,n}\pi + \beta_{m,n}\xi)\mathcal{E}_{m,n}$$

$$\mathcal{E}_{m,n} = (m+n)!\delta^{\mu_1}_{[\nu_1}\dots\delta^{\mu_m}_{\nu_m}\delta^{\rho_1}_{\sigma_1}\dots\delta^{\rho_n}_{\sigma_n]}(\partial_{\mu_1}\partial^{\nu_1}\pi)\dots(\partial_{\mu_m}\partial^{\nu_m}\pi)(\partial_{\rho_1}\partial^{\sigma_1}\xi)\dots(\partial_{\rho_n}\partial^{\sigma_n}\xi)$$

2010 Bi-galileons

[Padilla, Saffin, Zhou, 1007.5424, 1008.3312]

2010 Mixed combinations of *p*-form fields

[Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

$$\mathcal{L}_{4,\text{single}} = G_4(X,\phi)R + \frac{\partial G_4(X,\phi)}{\partial X}\left(\Box\phi\Box\phi - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right) \qquad X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

$$\mathcal{L}_{4,\text{multiple}} = G_4\left(X^{IJ},\phi^I\right)R + \frac{\partial G_4\left(X^{IJ},\phi^I\right)}{\partial X^{IJ}}\left(\Box\phi^I\Box\phi^J - \nabla_\mu\nabla_\nu\phi^I\nabla^\mu\nabla^\nu\phi^J\right)$$

$$I,J = 1,2,\cdots \qquad X^{IJ} = -\frac{1}{2}\nabla_\mu\phi^I\nabla^\mu\phi^J$$

2012 — **Multi-galileons**
[Padilla & Sivanesan, 1210.4026]
[Sivanesan, 1307.8081]

2010 — **Bi-galileons**
[Padilla, Saffin, Zhou, 1007.5424, 1008.3312]

2010 — **Mixed combinations of *p*-form fields**
[Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

multi-field DBI-type Galileon

[Renaux-Petel, 1105.6366]
[Renaux-Petel, Mizuno, Koyama, 1108.0305]

$$S = \int \mathrm{d}^4 x \left(\frac{M_\mathrm{P}^2}{2} \sqrt{-g} R\left[g\right] + \frac{M^2}{2} \sqrt{-\gamma} R\left[\gamma\right] + \sqrt{-g} \mathcal{L}_\mathrm{brane} \right)$$

$$\gamma_{\mu\nu} = h^{-1/2} \left(g_{\mu\nu} + h G_{IJ} \nabla_\mu \phi^I \nabla_\nu \phi^J \right)$$

$$\mathcal{L}_\mathrm{brane} = -\frac{1}{f\left(\phi^I\right)} \left(\sqrt{\det\left(\delta^\mu_\nu + f G_{IJ} \nabla^\mu \phi^I \nabla_\nu \phi^J \right)} - 1 \right) - V\left(\phi^I\right)$$

2012 — Multi-galileons
[Padilla & Sivanesan, 1210.4026]
[Sivanesan, 1307.8081]

2010 — Bi-galileons
[Padilla, Saffin, Zhou, 1007.5424, 1008.3312]

2010 — Mixed combinations of *p*-form fields
[Deffayet, Deser and Esposito-Farese, 1007.5278]

2011 — Probe brane method
[de Rham & Tolley, 1003.5917]
[Trodden & Hinterbichler, 1104.2088]
[Van Acoleyen & Van Doorsselaere, 1102.0487]

Evolution of multi-field scalar-tensor theory

$$
\begin{aligned}
0 = \mathcal{G}^a_b = {} & A\delta^a_b + \left[-2\mathcal{F}_{,I} - 4\mathcal{W}_{,I} + 2\left(D_{JKI} + 8J_{J[K,I]}\right)X^{JK} - 8E_{JKLMI}X^{JK}X^{LM}\right]\delta^{ac}_{bd}\phi^{I|d}_{|c} \\
& + \Big(-2\mathcal{F}_{,I,J} - 4\mathcal{W}_{,I,J} + A_{,IJ} + 2D_{IKJ,L}X^{KL} - 16E_{KIMNJ,L}X^{KL}X^{MN} - 16J_{K[I,L],J}X^{KL}\Big)\phi^{(I|a}\phi^{J)}_{|b} \\
& + D_{IJK}\delta^{ace}_{bdf}\phi^I_{|c}\phi^{J|d}\phi^{K|f}_{|e} + E_{IJKLM}\delta^{aceg}_{bdfh}\phi^I_{|c}\phi^{J|d}\phi^K_{|e}\phi^{L|f}\phi^{M|h}_{|g} + \left(\frac{1}{2}\mathcal{F} + \mathcal{W}\right)\delta^{ace}_{bdf}R^{df}_{ce} + \mathcal{F}_{,IJ}\delta^{ace}_{bdf}\phi^{I|d}_{|c}\phi^{J|f}_{|e} \\
& + J_{IJ}\delta^{aceg}_{bdfh}\phi^I_{|c}\phi^{J|d}R^{fh}_{eg} + 2J_{IJ,KL}\delta^{aceg}_{bdfh}\phi^I_{|c}\phi^{J|d}\phi^{K|f}_{|e}\phi^{L|h}_{|g} + K_I\delta^{aceg}_{bdfh}\phi^{I|d}_{|c}R^{fh}_{eg} + \frac{2}{3}K_{I,JK}\delta^{aceg}_{bdfh}\phi^{I|d}_{|c}\phi^{J|f}_{|e}\phi^{K|h}_{|g}
\end{aligned}
$$

2015 — The most general 2nd-order EOM for bi-scalar–tensor theory
[Ohashi, Tanahashi, Kobayashi & Yamaguchi, 1505.06029]

2012 — Multi-galileons
[Padilla & Sivanesan, 1210.4026]
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2011 — Probe brane method
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[Trodden & Hinterbichler, 1104.2088]
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[Padilla, Saffin, Zhou, 1007.5424, 1008.3312]

2010 — Mixed combinations of *p*-form fields
[Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

2016 New terms for the multi-Galileon
[Allys, 1612.01972]
[Akama & Kobayashi, 1701.02926]

2015 The most general 2^{nd}-order EOM for bi-scalar–tensor theory
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[Padilla, Saffin, Zhou, 1007.5424, 1008.3312]

2010 Mixed combinations of *p*-form fields
[Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

- **2024** — Conjecture of 2nd-order Bi-Scalar-Tensor Field Equations
  [Horndeski, 2405.08303]
- **2016** — New terms for the multi-Galileon
  [Allys, 1612.01972]
  [Akama & Kobayashi, 1701.02926]
- **2015** — The most general 2nd-order EOM for bi-scalar–tensor theory
  [Ohashi, Tanahashi, Kobayashi & Yamaguchi, 1505.06029]
- **2012** — Multi-galileons
  [Padilla & Sivanesan, 1210.4026]
  [Sivanesan, 1307.8081]
- **2011** — Probe brane method
  [de Rham & Tolley, 1003.5917]
  [Trodden & Hinterbichler, 1104.2088]
  [Van Acoleyen & Van Doorsselaere, 1102.0487]
- **2010** — Bi-galileons
  [Padilla, Saffin, Zhou, 1007.5424, 1008.3312]
- **2010** — Mixed combinations of *p*-form fields
  [Deffayet, Deser and Esposito-Farese, 1007.5278]

Evolution of multi-field scalar-tensor theory

The most general ghost-free multi-scalar-tensor theory is still missing.

2024 Conjecture of 2^{nd}-order Bi-Scalar-Tensor Field Equations
[Horndeski, 2405.08303]

2016 New terms for the multi-Galileon
[Allys, 1612.01972]
[Akama & Kobayashi, 1701.02926]

2015 The most general 2^{nd}-order EOM for bi-scalar–tensor theory
[Ohashi, Tanahashi, Kobayashi & Yamaguchi, 1505.06029]

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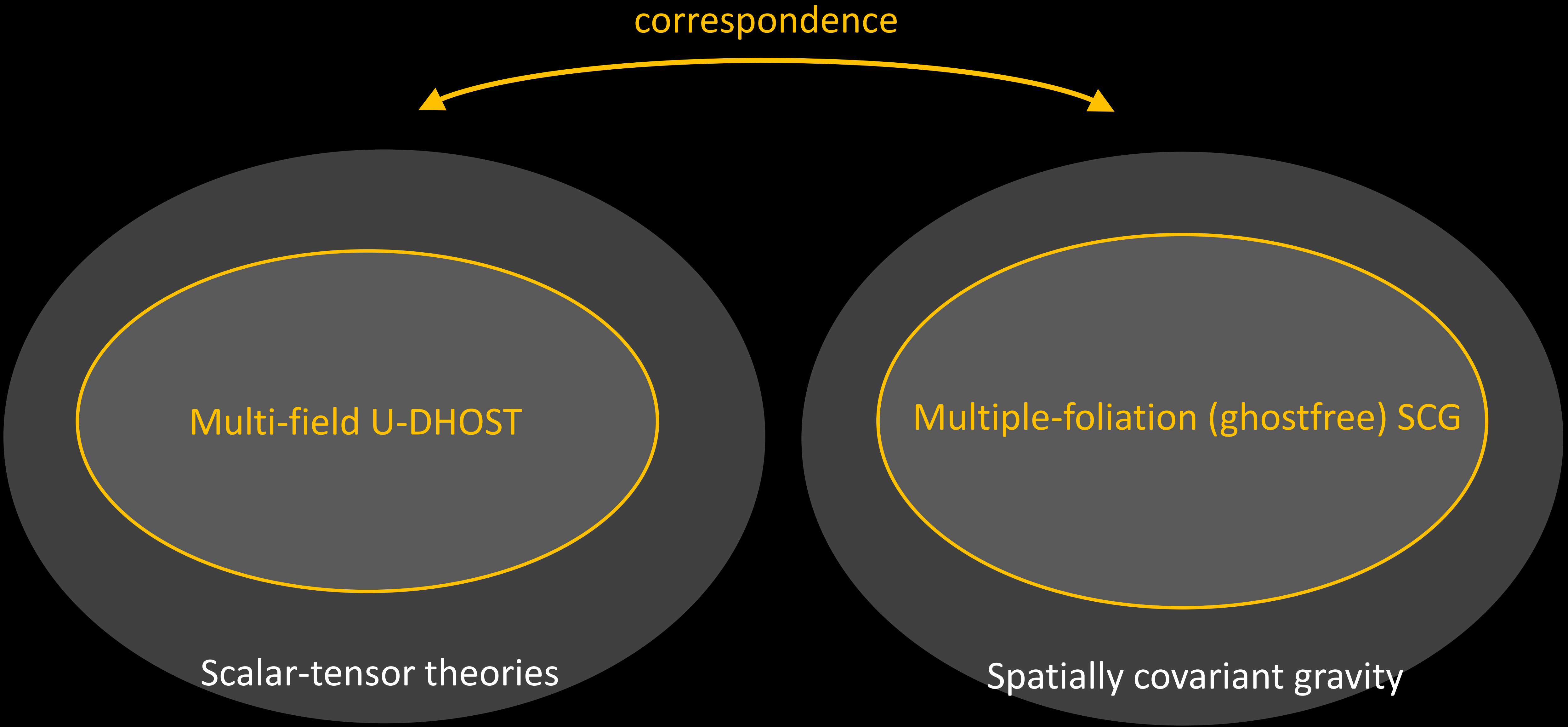
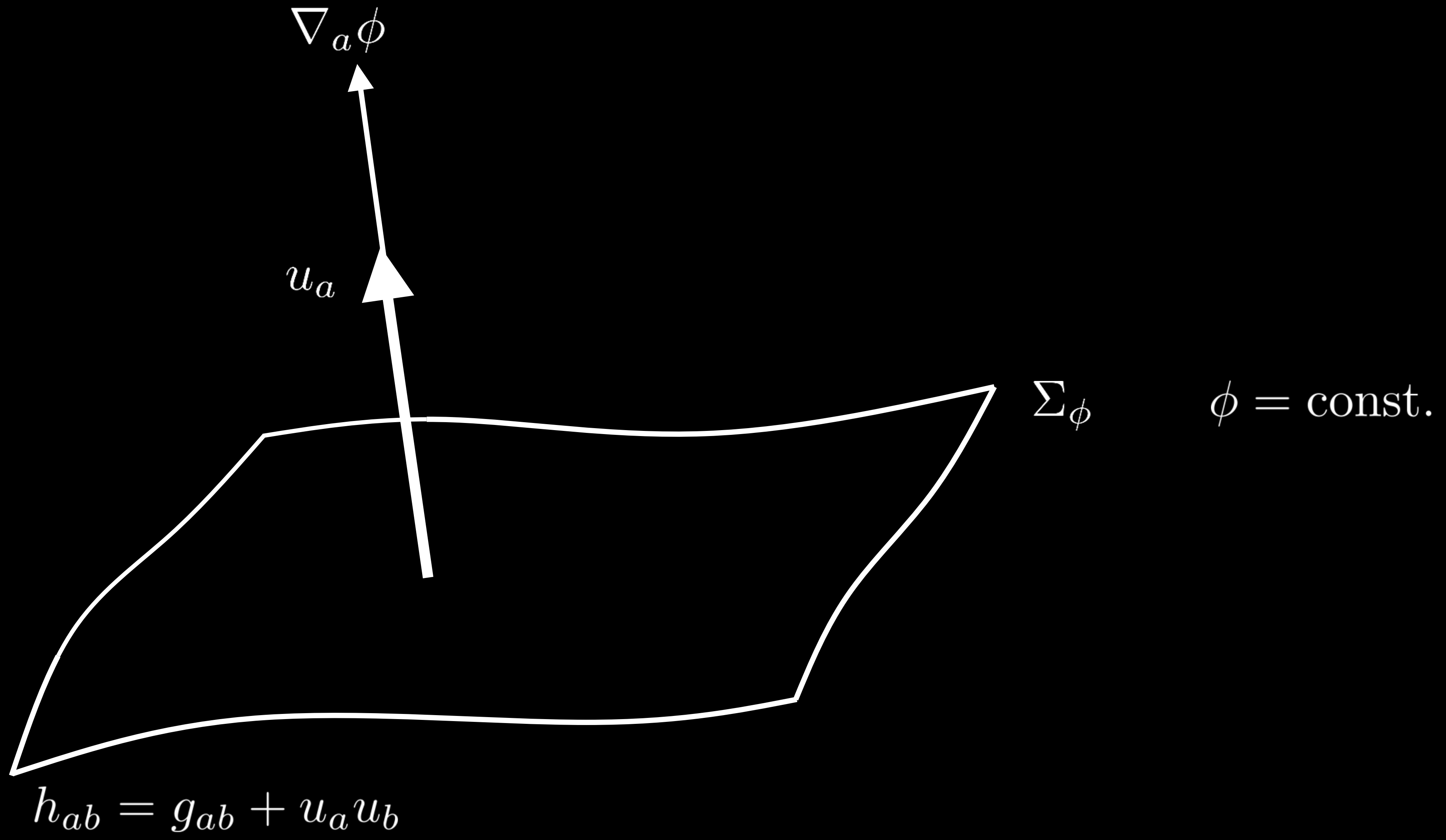
2010 Mixed combinations of *p*-form fields
[Deffayet, Deser and Esposito-Farese, 1007.5278]

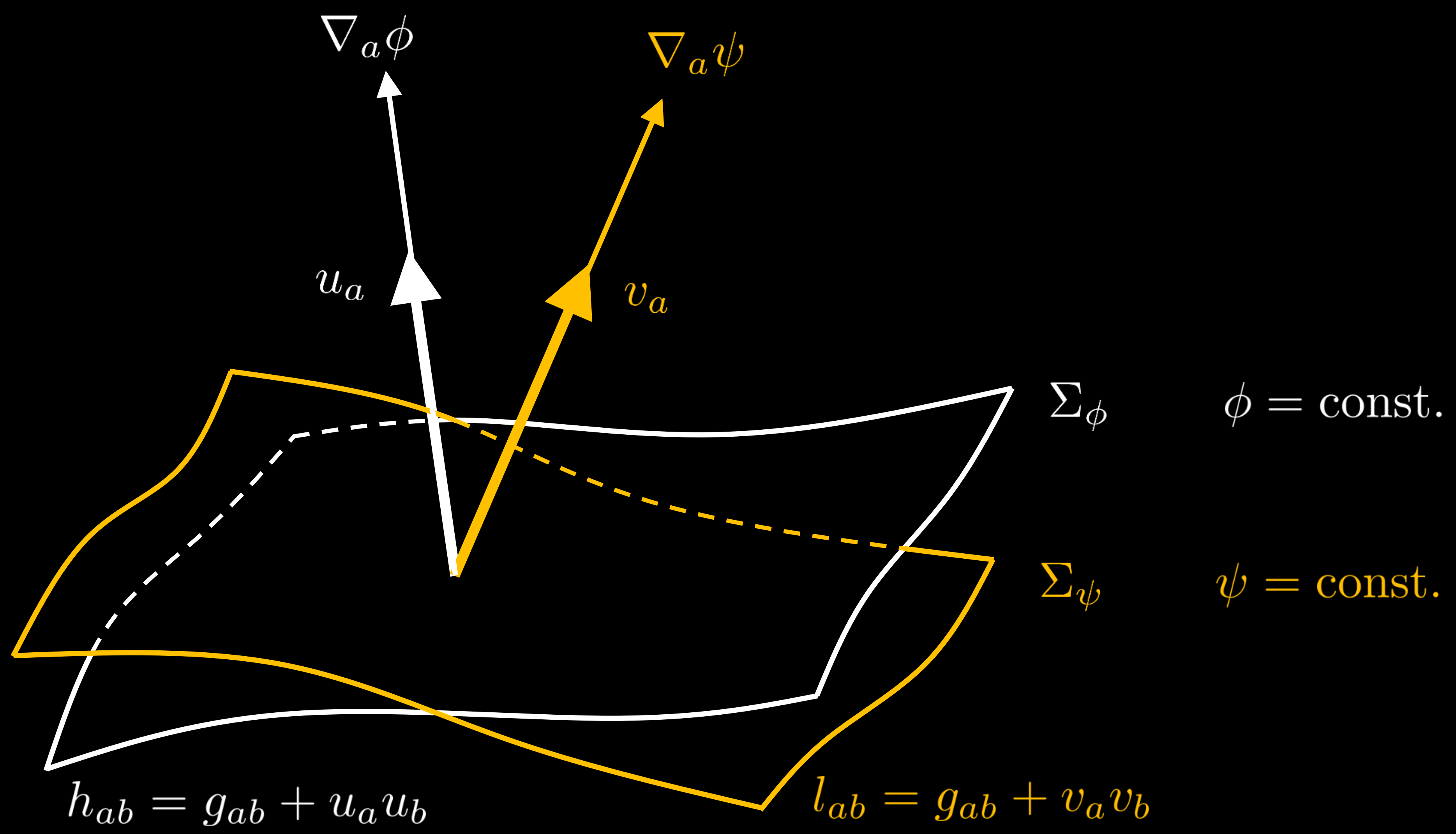
U-DHOST v.s. Spatially covariant gravity



Multiple foliations of hypersurfaces



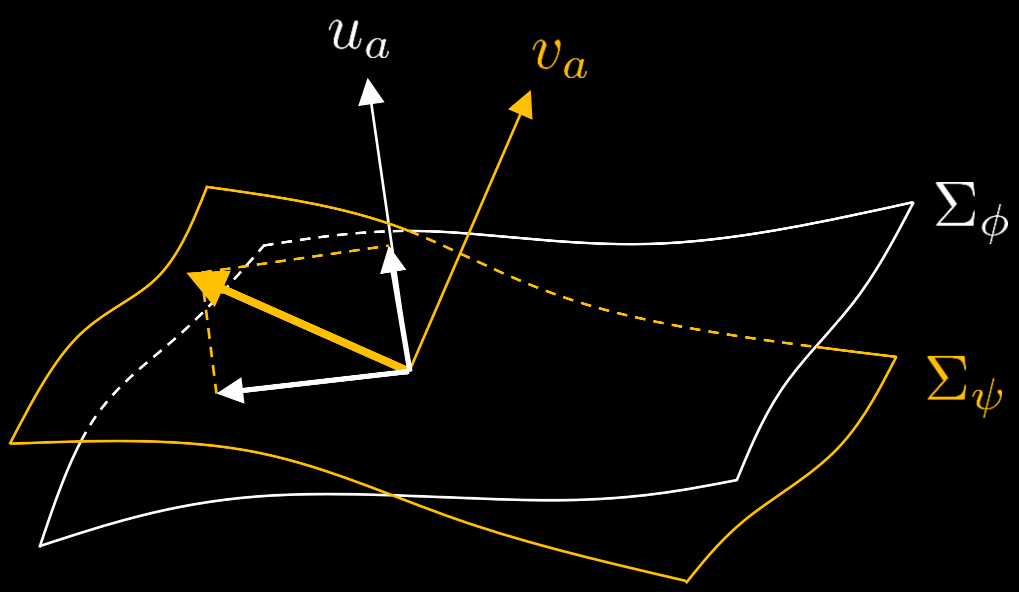
Multiple foliations of hypersurfaces



Building blocks

Geometric quantities	w.r.t. Σ_ϕ	w.r.t. Σ_ψ
Normal vectors	$u_a := -N\nabla_a\phi$	$v_a := -M\nabla_a\psi$
Induced metrics	$h_{ab} := g_{ab} + u_a u_b$	$l_{ab} := g_{ab} + v_a v_b$
Normalization factors	$N := 1/\sqrt{2X}$	$M := 1/\sqrt{2Y}$
Accelerations	$a_a := \mathrm{D}_a \ln N$	$b_a := \tilde{\mathrm{D}}_a \ln M$
Extrinsic curvatures	$K_{ab} := \frac{1}{2}\pounds_{\boldsymbol{u}} h_{ab}$	$L_{ab} := \frac{1}{2}\pounds_{\boldsymbol{v}} l_{ab}$
Ricci tensors	${}^3R_{ab} \equiv {}^3R_{ab}(h)$	${}^3\tilde{R}_{ab} \equiv {}^3\tilde{R}_{ab}(l)$

Healthiness of the construction



Spatial tensors w.r.t. one foliation (e.g., Σ_ϕ) will not be spatial w.r.t the other foliation (e.g., Σ_ψ).

Spatial derivative D_a contains temporal parts w.r.t. Σ_ψ.

Higher time derivatives (extra modes)? ➡ Superficial instantaneous/shadowy modes

As long as operators are safe in their own unitary gauge, they are always safe.

[De Felice, Langlois, Mukohyama, Noui, Wang, 1803.06241]
[De Felice, Mukohyama, Takahashi, 2110.03194]

The action

The most general action describing two foliations of hypersurfaces:

$$S = \int \mathrm{d}^4x \sqrt{g} \mathcal{L} \left(\underbrace{\phi, N, u_a, h_{ab}, {}^3R_{ab}, \mathrm{D}_a, \pounds_{\boldsymbol{u}}}_{\Sigma_\phi} ; \underbrace{\psi, M, v_a, l_{ab}, {}^3\tilde{R}_{ab}, \tilde{\mathrm{D}}_a, \pounds_{\boldsymbol{v}}}_{\Sigma_\psi} \right)$$

An analogue of 3-DOF SCG:

$$S = \int \mathrm{d}^4x \sqrt{g} \mathcal{L} \left(\phi, N, u_a, h_{ab}, K_{ab}, {}^3R_{ab}, \mathrm{D}_a ; \psi, M, v_a, l_{ab}, L_{ab}, {}^3\tilde{R}_{ab}, \tilde{\mathrm{D}}_a \right)$$

Temporal derivatives enter only through extrinsic curvature.

Monomials

d=1

B.b.	Monomials
K	$[K]\,,[Kvv]$
L	$[L]\,,[Luu]$
a	$[av]$
b	$[bu]$

d=2

B.b.	Unfactorizable monomials	Factorizable monomials
KK	$[KK]$, $[KvKv]$	$[K]^2, [Kvv]^2, [K][Kvv]$
LL	$[LL]$, $[LuLu]$	$[L]^2, [Luu]^2, [L][Luu]$
KL	$[KL]$, $[KvLu]$	$[K][L], [Kvv][L], [K][Luu], [Kvv][Luu]$
Ka	$[Kva]$	$[K][av], [Kvv][av]$
Kb	$[Kvb]$	$[K][bu], [Kvv][bu]$
La	$[Lua]$	$[L][av], [Luu][av]$
Lb	$[Lub]$	$[L][bu], [Luu][bu]$
aa	$[aa]$	$[av]^2$
ab	$[ab]$	$[av][bu]$
bb	$[bb]$	$[bu]^2$
${}^3R$	$[{}^3R]\,,\,[{}^3Rvv]$	-
${}^3\tilde{R}$	$[{}^3\tilde{R}]\,,\,[{}^3\tilde{R}uu]$	-
$\mathrm{D}K$	$[v\mathrm{D}K]$, $[\mathrm{D}Kv]$, $[v\mathrm{D}Kvv]$	-
$\tilde{\mathrm{D}}L$	$[u\tilde{\mathrm{D}}L]\,,\,[\tilde{\mathrm{D}}Lu]\,,\,[u\tilde{\mathrm{D}}Luu]$	-
$\mathrm{D}a$	$[\mathrm{D}a]$, $[v\mathrm{D}av]$	-
$\tilde{\mathrm{D}}b$	$[\tilde{\mathrm{D}}b]\,,\,[u\tilde{\mathrm{D}}bu]$	-

Correspondence

Examples of correspondence:

$$K^{ab}K_{ab} \to \frac{1}{2X}\phi_{ab}\phi^{ab} + \frac{1}{2X^2}\phi^a\phi^b\phi_a{}^c\phi_{bc} + \frac{1}{8X^3}(\phi^a\phi^b\phi_{ab})^2 \qquad X = -\frac{1}{2}\nabla_a\phi\nabla^a\phi$$

$$L^{ab}L_{ab} \to \frac{1}{2Y}\psi_{ab}\psi^{ab} + \frac{1}{2Y^2}\psi^a\psi^b\psi_a{}^c\psi_{bc} + \frac{1}{8Y^3}(\psi^a\psi^b\psi_{ab})^2 \qquad Y = -\frac{1}{2}\nabla_a\psi\nabla^a\psi$$

Mixing derivative terms:

$$a_a v^a \to -\frac{1}{2X\sqrt{2Y}}\left(\psi^a - \frac{Z}{X}\phi^a\right)\phi^b\phi_{ab} \qquad Z = -\frac{1}{2}\nabla_a\phi\nabla^a\psi$$

$$K_{ab}v^a v^b \to -\frac{1}{\sqrt{2X2Y}}\phi_{ab}\left(\psi^a - \frac{Z}{X}\phi^a\right)\left(\psi^b - \frac{Z}{X}\phi^a\right)$$

A concrete model

A simple model of $d = 2$

$$\mathcal{L}_2 = c_1 K_{ab}K^{ab} + c_2 K^2 + c_3 K_{ab}L^{ab} + c_4 KL + c_5 L_{ab}L^{ab} + c_6 L^2 + c_7 a_a a^a + c_8 a_a b^a + c_9 b_a b^a + c_{10}\,{}^3R + c_{11}\,{}^3\tilde{R},$$

This model is already far beyond previous results of multi-scalar-tensor theories.

Scalar perturbations:

$$g_{\mu\nu} = \begin{pmatrix} -e^{2A} + \delta^{ij}\partial_i B \partial_j B & a\partial_i B \\ a\partial_j B & a^2 e^{2\zeta}\delta_{ij} \end{pmatrix}$$

$$\{A, B, \zeta, \not{E}, \delta\not{\phi}, \delta\psi\}$$

2 dynamical DoFs

$$L^{2s} \simeq \mathcal{C}_{\dot\zeta\dot\zeta}\dot\zeta^2 + \mathcal{C}_{\dot\zeta\delta\dot\psi}\dot\zeta\delta\dot\psi + \mathcal{C}_{\dot\zeta\partial^2 B}\dot\zeta\partial^2 B + \mathcal{C}_{\delta\dot\psi\delta\dot\psi}\delta\dot\psi^2 + \mathcal{C}_{\partial\delta\dot\psi\partial\delta\dot\psi}\partial_i\delta\dot\psi\partial^i\delta\dot\psi + \mathcal{C}_{\delta\dot\psi A}\delta\dot\psi A + \mathcal{C}_{\partial\delta\dot\psi\partial A}\partial^i\delta\dot\psi\partial_i A + \mathcal{C}_{\delta\dot\psi\partial^2 B}\delta\dot\psi\partial^2 B + \mathcal{C}_{\zeta\delta\dot\psi}\zeta\delta\dot\psi + \mathcal{C}_{\partial^2\zeta\delta\dot\psi}\partial^2\zeta\delta\dot\psi + \text{terms without time derivatives}$$

Summary

1. The spatially covariant gravity theory is an efficient method to generate scalar-tensor theories ghost-free in unitary gauge.

2. We construct the most general (unitary-gauge) ghost-free parity-violating scalar-tensor theory up to $d = 4$ by employing the correspondence between scalar-tensor theory and spatially covariant gravity.

3. We propose a novel method to construct (unitary-gauge) ghost-free multiple scalar-tensor theories by generalizing the single-field spatially covariant gravity.

Thank you for your attention!