

2024引力与宇宙学专题研讨会

# Images of Konoplya-Zhidenko rotating black holes with magnetized Accretion Disks

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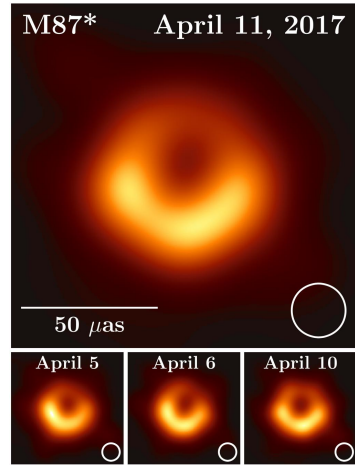
合作者：张泽林、荆继良

2024. 11. 17 中国科技大学

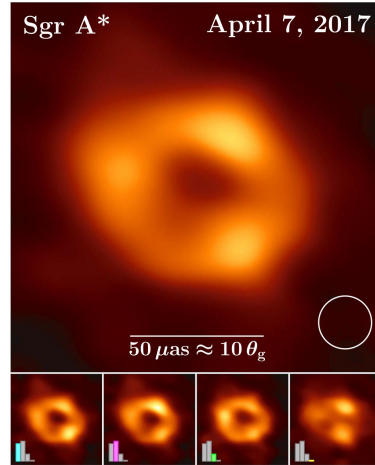
# Outline

- 1. Introduce of black hole images**
- 2. Images of Konoplya-Zhidenko black holes**
  - Geometrically thick magnetized equilibrium tori**
  - Dynamically thick magnetized accretion disk**
- 3. Summary**

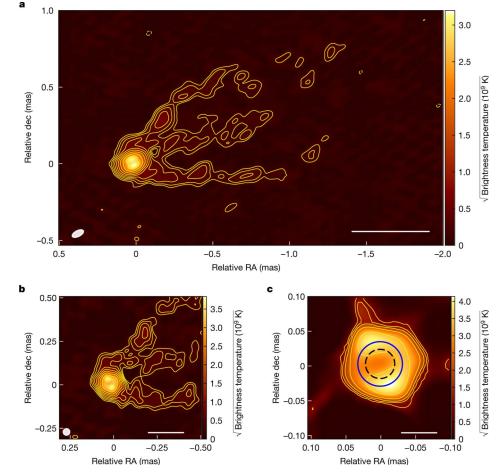
# 1. Observational Images of M87 and SgrA\*



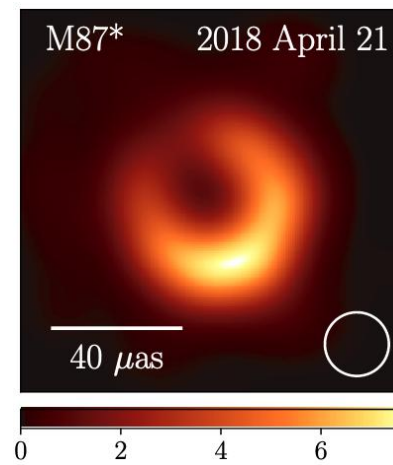
2019 *ApJL* 875 L1  
Event Horizon Telescope Collaboration, et al.



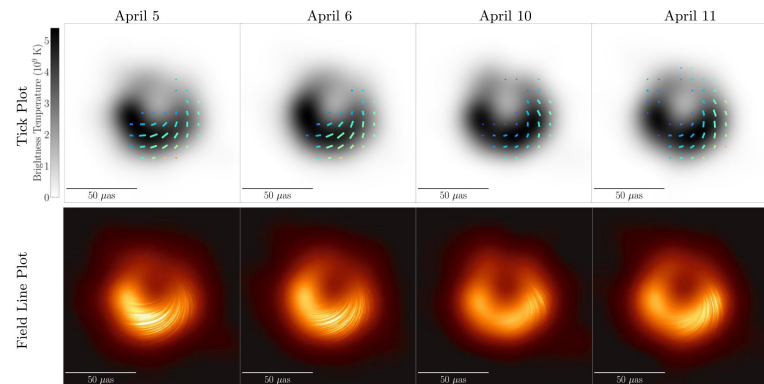
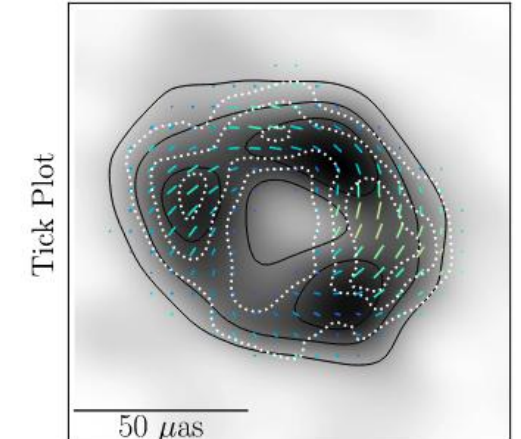
2022 *ApJL* 930 L12



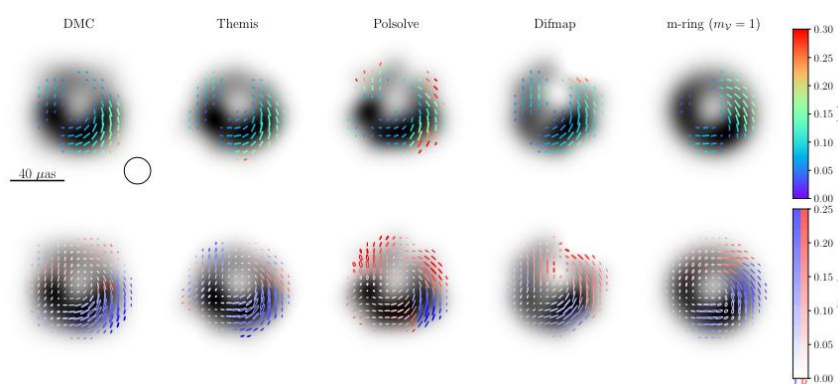
2023 *Nature* 616 686  
Lu, R. S., Asada, K., Krichbaum, T. P., et al.



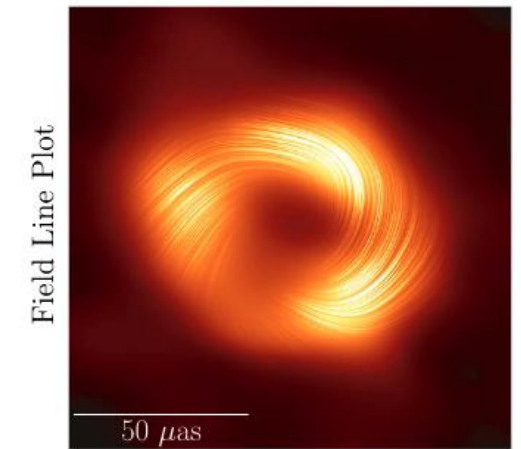
2024 *A&A* 681 A79  
EHT Collaboration, et al.



2021 *ApJL* 910 L12 EHT Collaboration, et al.



2023 *ApJL* 957 L20 EHT Collaboration, et al.



2024 *ApJL* 964 L25 EHT Collaboration, et al.

# 事件视界望远镜项目组 (EHT)



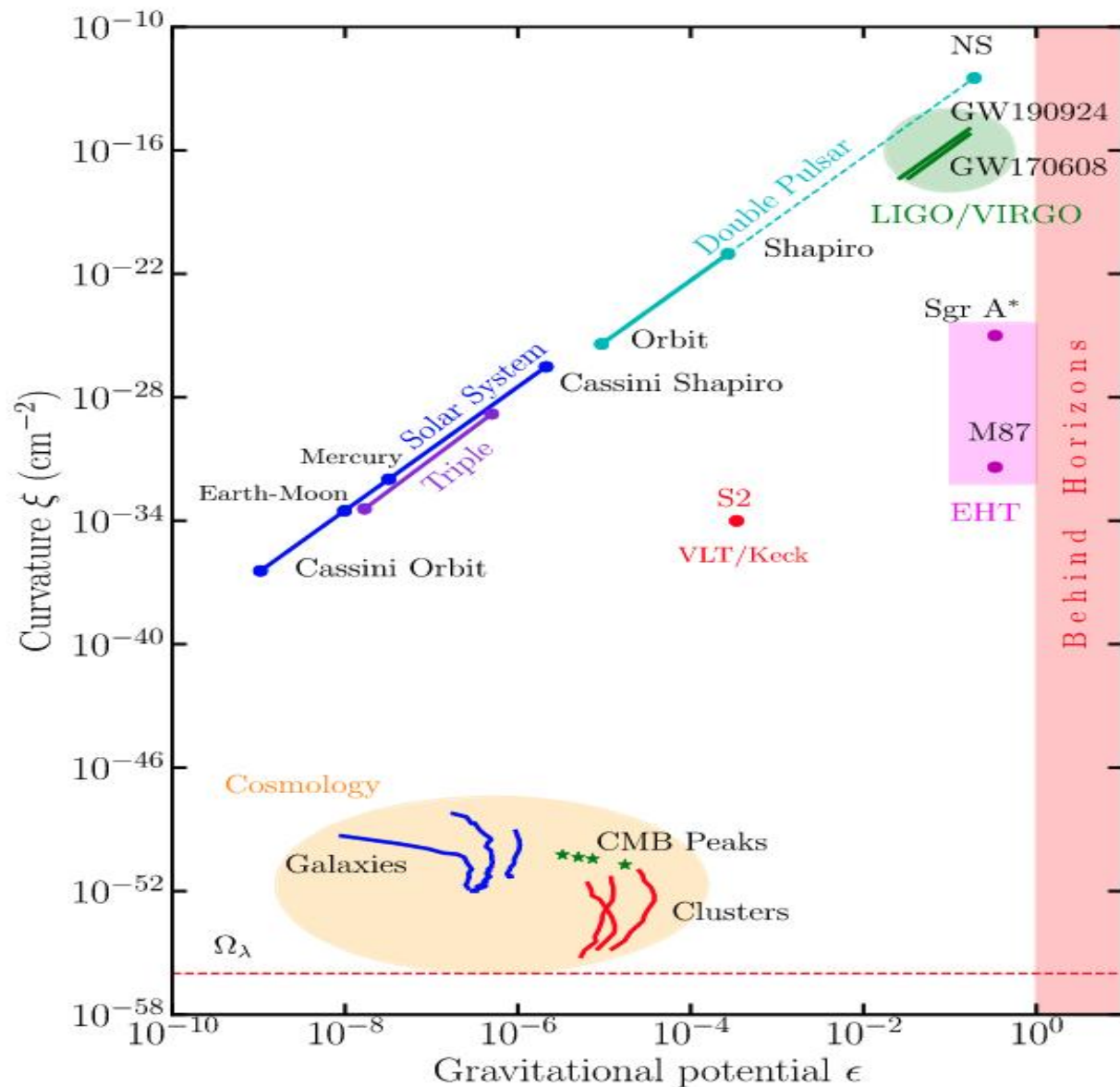
2020年突破奖——基础物理学奖

直接验证了黑洞的存在

有助于检验包括广义相对论在内的引力理论。

有助于理解强引力场中物质的分布和运动及其相关物理。

# Testing the Theory of Gravity



- [Non-rotating Dilaton BH, GRMHD] Mizuno Y., et al., 2018, NatAs, 2, 585.
- [Equivalence Principle] Yan S.F., et al., 2020, PhRvR, 2, 023164.
- [STVG, Polarization] Qin X., et al., 2022, ApJ, 938, 2.
- [Axion, Polarization] Chen Y., et al., 2022, NatAs, 6, 592.
- [Horndeski Gravity] Afrin M., et al., 2022, ApJ, 932, 51.
- [Lorentz symmetry] Khodadi M., et al., 2022, PhRvD, 106, 104050.
- [Modified Gravity] Kuang X.M., et al., 2022, PhRvD, 106, 064012.
- [Loop Quantum Gravity, GRMHD] Jiang H.-X., et al., 2024, JCAP, 101.
- [Naked singularity, GRMHD] Dihingia I.-K., et al., 2024, arXiv:2410.13406.
- [Wormholes, GRMHD] Combi L., et al., 2024, PhRvD, 109, 103034.

2022 *ApJL* **930** L17 EHT Collaboration, et al.

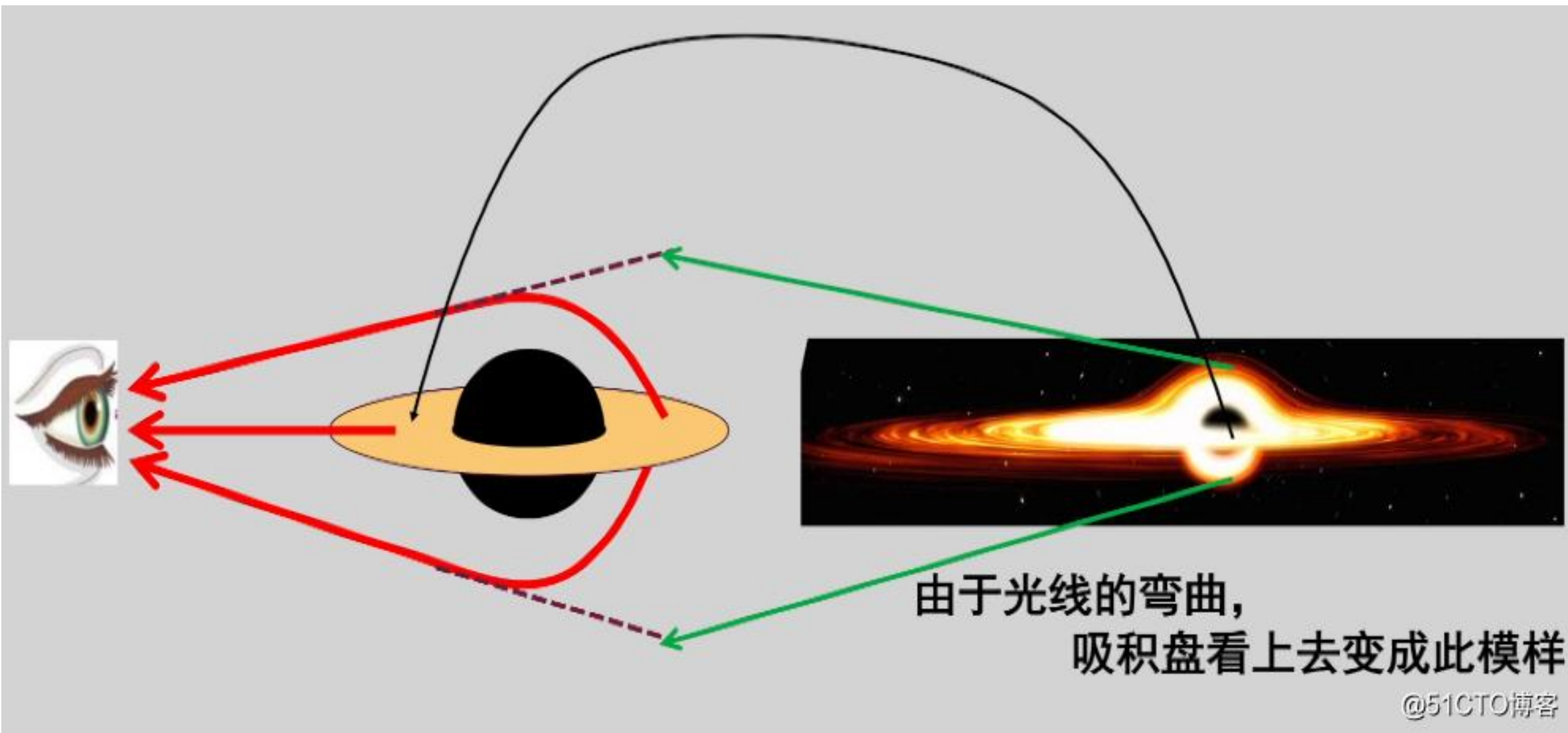
**黑洞图像取决于：**

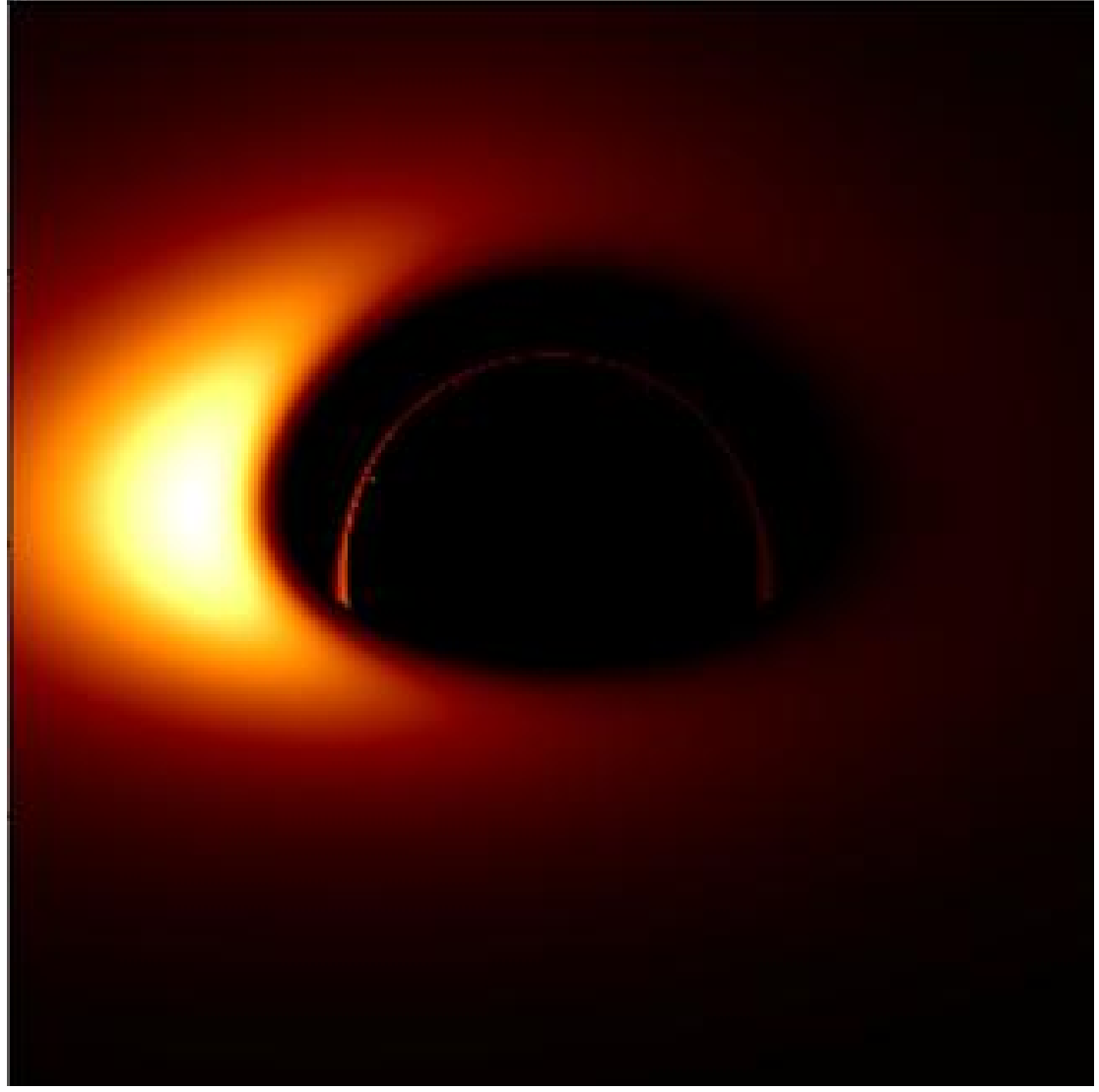
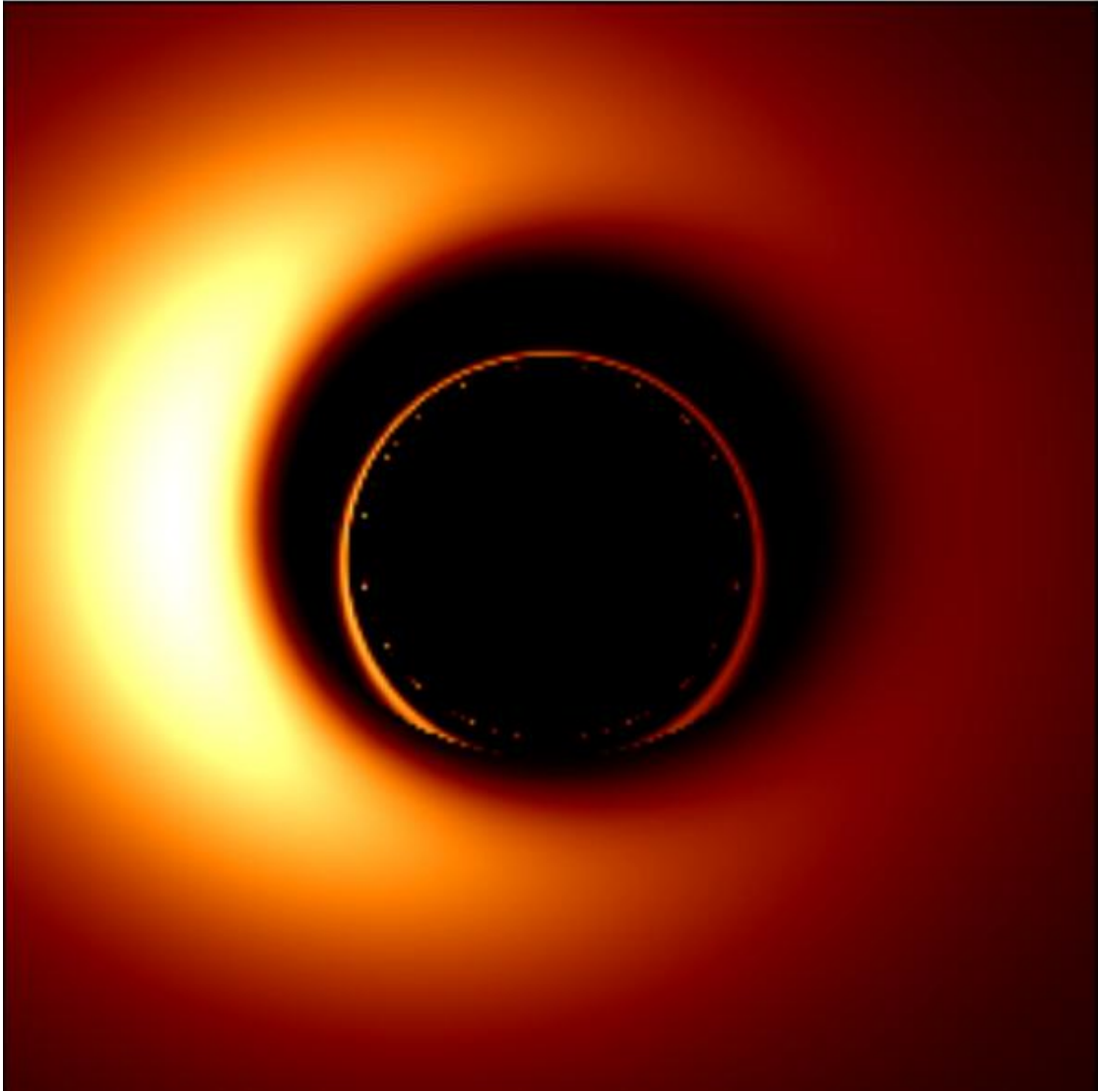
**黑洞本身的时空特征**

**吸积盘中的流体动力学过程以及辐射机制**

**辐射在黑洞时空中的传输过程**

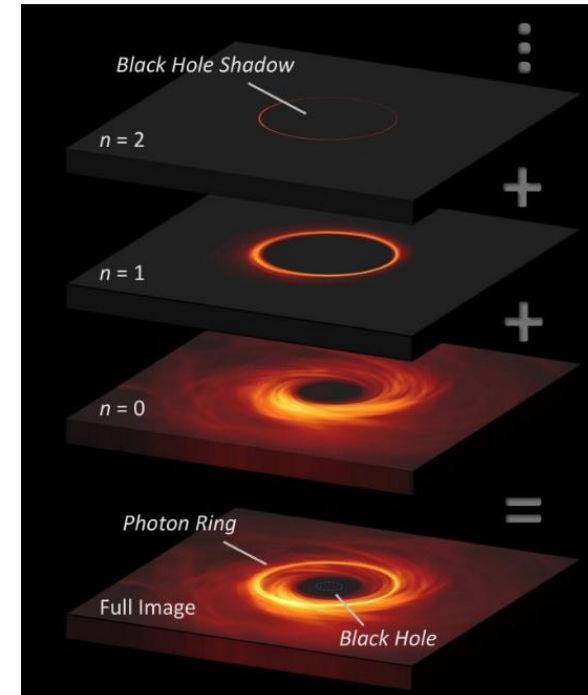
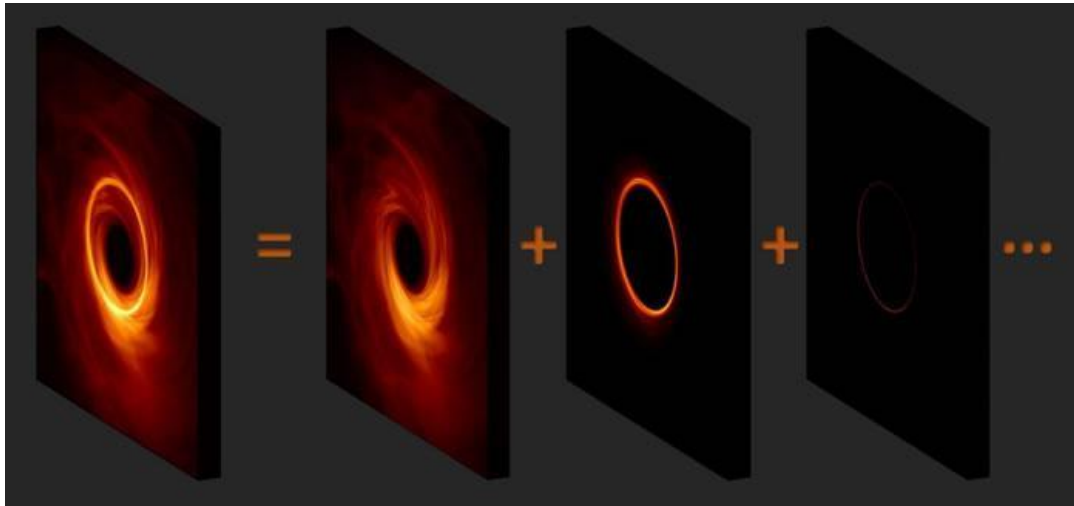
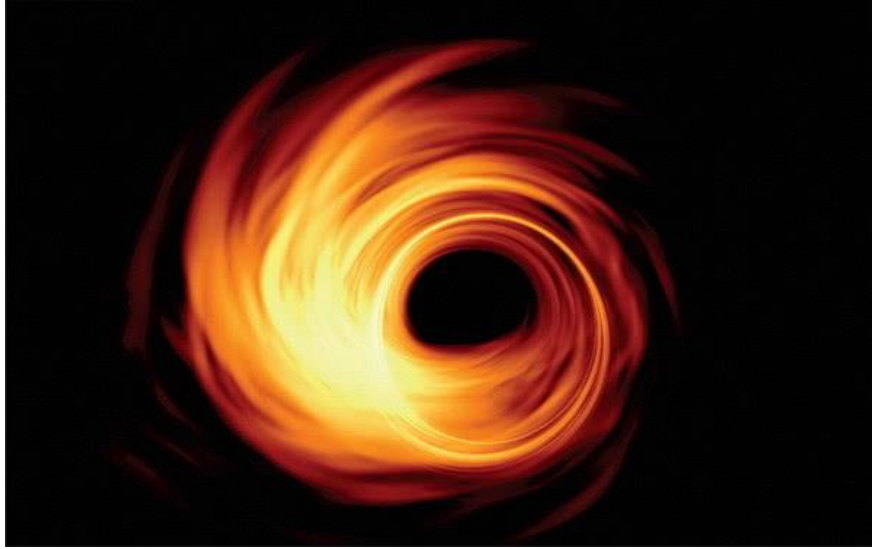
# 薄盘







# 黑洞几何厚盘图像一般特征



# 检验非爱因斯坦引力理论的思路

1. 找出非爱因斯坦引力理论的黑洞解及其可观测的特征信息
2. 在Kerr黑洞时空直接加入一些偏离参数：可观测的特征信息

## 2. Konoplya-Zhidenko BH Metric

Konoplya R., Zhidenko A., PhLB, 756, 350 (2016)

$$ds^2 = - \left( 1 - \frac{2Mr^2 + \eta}{r\rho^2} \right)^2 dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 +$$

$$\sin^2\theta \left[ r^2 + a^2 + \frac{(2Mr^2 + \eta)a^2 \sin^2\theta}{r\rho^2} \right] d\phi^2$$

$$- 2 \left( \frac{2Mr^2 + \eta}{r\rho^2} \right) a \sin^2\theta dt d\phi$$

$$\Delta = r^2 - 2Mr + a^2 - \frac{\eta}{r} \quad \rho^2 = r^2 + a^2 \cos^2\theta$$

---

Static deformation from Kerr BH  $M \rightarrow M + \frac{\eta}{2r^2}$

$$\eta_1 = \frac{2}{27} (\sqrt{4M^2 - 3a^2} - 2M)^2 (\sqrt{4M^2 - 3a^2} + M)$$

$$\eta_2 = -\frac{2}{27} (\sqrt{4M^2 - 3a^2} + 2M)^2 (\sqrt{4M^2 - 3a^2} - M)$$

Horizons	Parameter range
0	$\eta < \eta_2 < 0$ or $ a  > \frac{2\sqrt{3}}{3}$ & $\eta < 0$
1	$\eta > \eta_1$ or $0 < \eta < \eta_2$ or $ a  > \frac{2\sqrt{3}}{3}$ & $\eta > 0$
2	$\eta_2 \leq \eta \leq 0$ or $\eta = \eta_1$
3	$0 < \eta < \eta_1$ & $\eta_2 < 0$ or $\eta_2 < \eta < \eta_1$ & $\eta_2 > 0$

# 几何厚盘成像要解决的问题:

## (1) 盘的几何形状、物质分布和动态演化及亮度分布

### 3+1 GRMHD equations

Valencia Formulation: No Christoffel symbols but only metric derivatives on the rhs  
Conserved Form

$$\partial_t(\sqrt{\gamma}\mathbf{U}) + \partial_i(\sqrt{\gamma}\mathbf{F}^i) = \sqrt{\gamma}\mathbf{S}$$

Metric :  $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$   
 $\alpha$ : lapse function,  $\beta^i$ : shift vector

Primitive variables

Conserved variables

Flux

$$\mathbf{P} = \begin{bmatrix} \rho \\ \Gamma v^j \\ p \\ B^j \\ \phi \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} D \\ S_j \\ \tau \\ B^j \\ \phi \end{bmatrix}$$

$$\mathbf{F}^i = \begin{bmatrix} \mathcal{V}^i D \\ \alpha W_j^i - \beta^i S_j \\ \alpha(S^i - v^i D) - \beta^i \tau \\ \mathcal{V}^i B^j - B^j \mathcal{V}^i - B^i \beta^j \\ \alpha B^i - \phi \beta^i \end{bmatrix}$$

transport velocity

$$\mathcal{V}^i = \alpha v^i - \beta^i$$

Magnetic field 4-vector

$$b^0 = \Gamma(B^i v_i)/\alpha, \quad b^i = (B^i + \alpha b^0 u^i)/\Gamma$$

Source term

$$\mathbf{S} = \begin{bmatrix} 0 \\ \frac{1}{2}\alpha W^{ik}\partial_j\gamma_{ik} + S_i\partial_j\beta^i - U\partial_j\alpha \\ \frac{1}{2}W^{ik}\beta^j\partial_j\gamma_{ik} + W_i^j\partial_j\beta^i - S^j\partial_j\alpha \\ -B^i\partial_i\beta^j - \alpha\gamma^{ij}\partial_i\phi \\ -\alpha\kappa\phi - \phi\partial_i\beta^i - \frac{1}{2}\phi\gamma^{ij}\beta^k\partial_k\gamma_{ij} + B^i\partial_i\alpha \end{bmatrix}$$

Energy-momentum tensor

$$T^{\mu\nu} = (\rho h + b^2)u^\mu u^\nu + (p + \frac{b^2}{2})g^{\mu\nu} - b^\mu b^\nu$$

$$W^{ij} := h_\mu^i h_\nu^j T^{\mu\nu}$$

For AMR grid, we solve "Augmented Faraday's law"

$$\nabla_\nu(F^{*\mu\nu} - \phi g^{\mu\nu}) = -\kappa n^\mu \phi$$

广义相对论磁流体动力学



## (2) 盘上的辐射及其传输

Observer  $(t_0, r_0, \theta_0, \phi_0)$

Geodesic equation

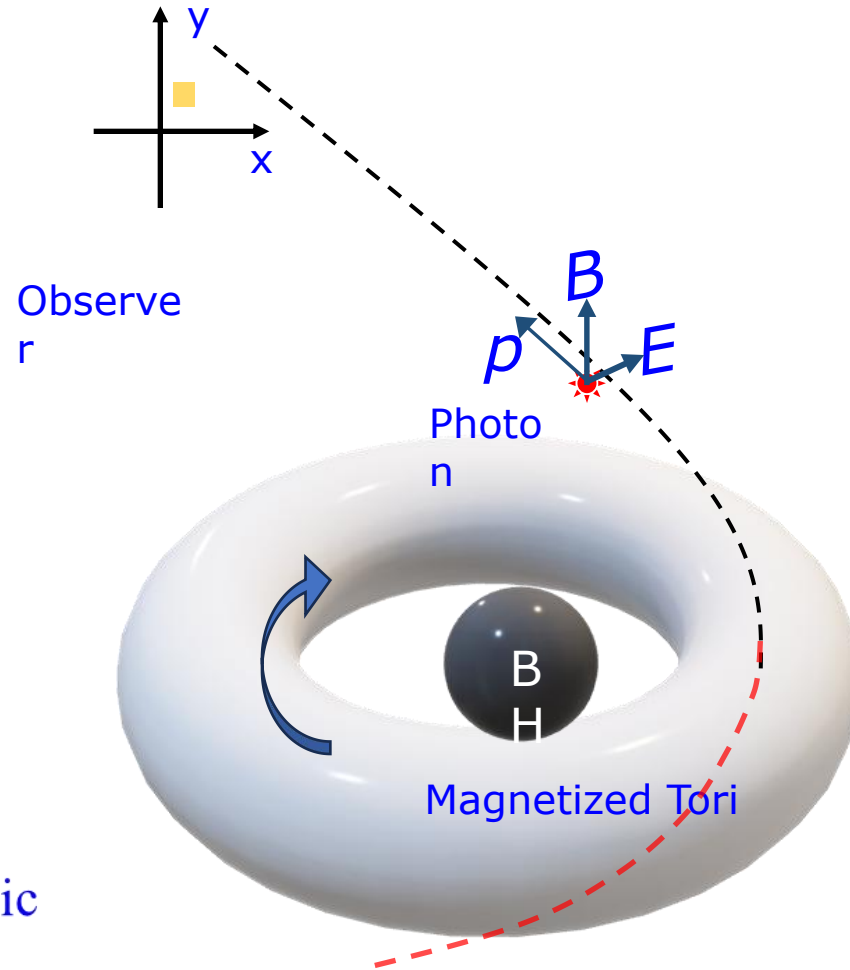
$$\ddot{x}^\mu + \Gamma_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta = 0$$

Radiative transfer equation

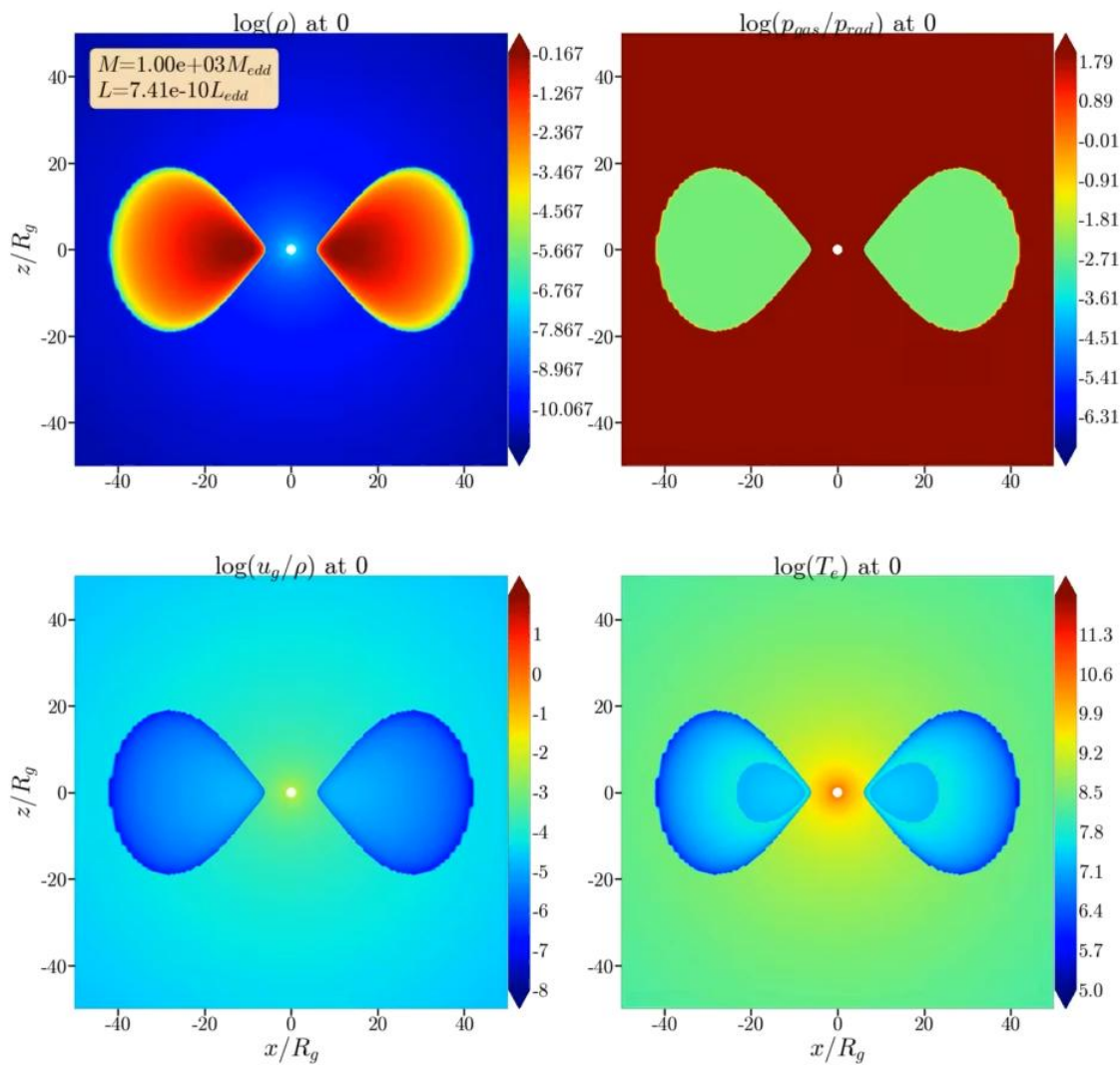
$$\frac{d\mathbb{I}_\nu}{d\lambda} = L \frac{u_a(p_{obs})k^a(p_{obs})}{\nu_0} (\mathbb{J}_\nu - \mathbb{I}_\nu \mathbb{M}_\nu)$$

Radiation type

Synchrotron emission from hot relativistic electrons



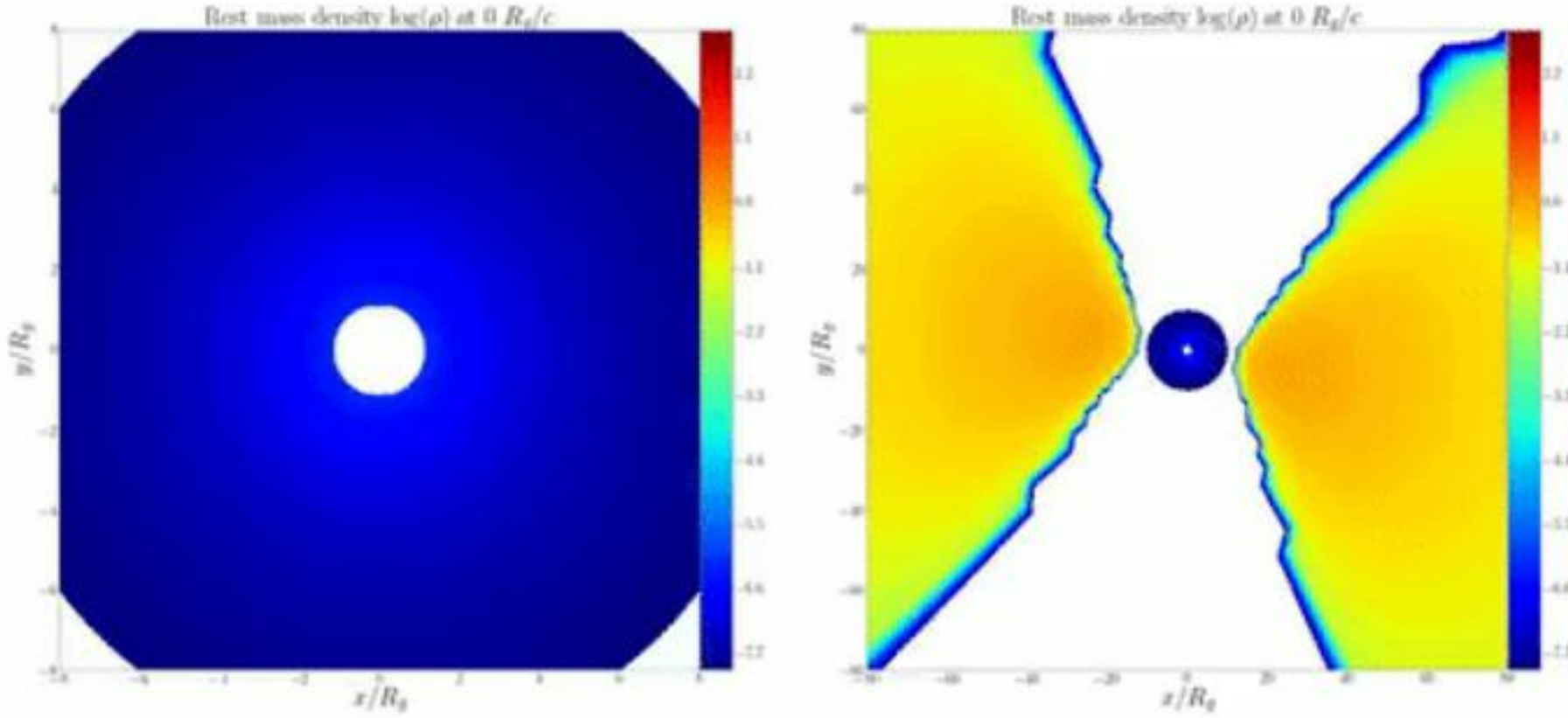
### (3) 要借助高精度的数值模拟



CPU144

2.6万元

Liska et al. 2023,  
arXiv:2309.15926.



缺点：复杂、算力要求高、烧钱、不便做一般性讨论

# a) Geometrically thick magnetized equilibrium tori

## • Basic Equations

➤  $\nabla_\alpha T^{\alpha\beta} = 0, \quad \nabla_\alpha {}^*F^{\alpha\beta} = 0, \quad \nabla_\alpha \rho u^\alpha = 0$

➤ Ideal relativistic MHD:

$$T^{\alpha\beta} = (h + b^2)u^\alpha u^\beta + \left(p + \frac{1}{2}b^2\right)g^{\alpha\beta} - b^\alpha b^\beta$$

➤ enthalpy  $h$ , pressure  $p$ , 4-velocity  $u^\alpha$ , magnetic field 4-vector  $b^\alpha$

➤  $\Omega = u^\phi / u^t, \quad l = -u_\phi / u_t$

## • Assumptions

➤ Flow is both stationary and axisymmetric

$$g_{\mu\nu,t} = g_{\mu\nu,\phi} = 0, \quad f_{,t} = f_{,\phi} = 0$$

➤ Velocity, magnetic field is purely azimuthal

$$u^r = u^\theta = 0, \quad b^r = b^\theta = 0$$

## • Equipotential Surface

➤ For a barotropic equation of state

$$h = h(p), \quad \tilde{h} = \tilde{h}(\tilde{p}_m), \quad \Omega = \Omega(l)$$

$$W - W_{in} + \int_0^p \frac{dp}{h} + \int_0^{\tilde{p}_m} \frac{d\tilde{p}_m}{\tilde{h}} = 0$$

where  $W = \ln|u_t| + \int_l^{l_\infty} \frac{\Omega dl}{1-\Omega}$  (potential)

➤ Simplest case  $l = l_0$

➤ Equation of state  $p = Kh^\kappa, \quad \tilde{p}_m = K_m \tilde{h}^\eta$

$$W - W_{in} + \frac{\kappa}{\kappa - 1} \frac{p}{h} + \frac{\eta}{\eta - 1} \frac{\tilde{p}_m}{\tilde{h}} = 0$$



## 该模型的优点:

盘的形状完全由黑洞度规和流体的比角动量决定。

## 盘上物质的分布

$$p = Kh^\kappa, \quad \tilde{p}_m = K_m \tilde{h}^\eta$$

$$W - W_{in} + \frac{\kappa}{\kappa - 1} \frac{p}{h} + \frac{\eta}{\eta - 1} \frac{p_m}{h} = 0$$

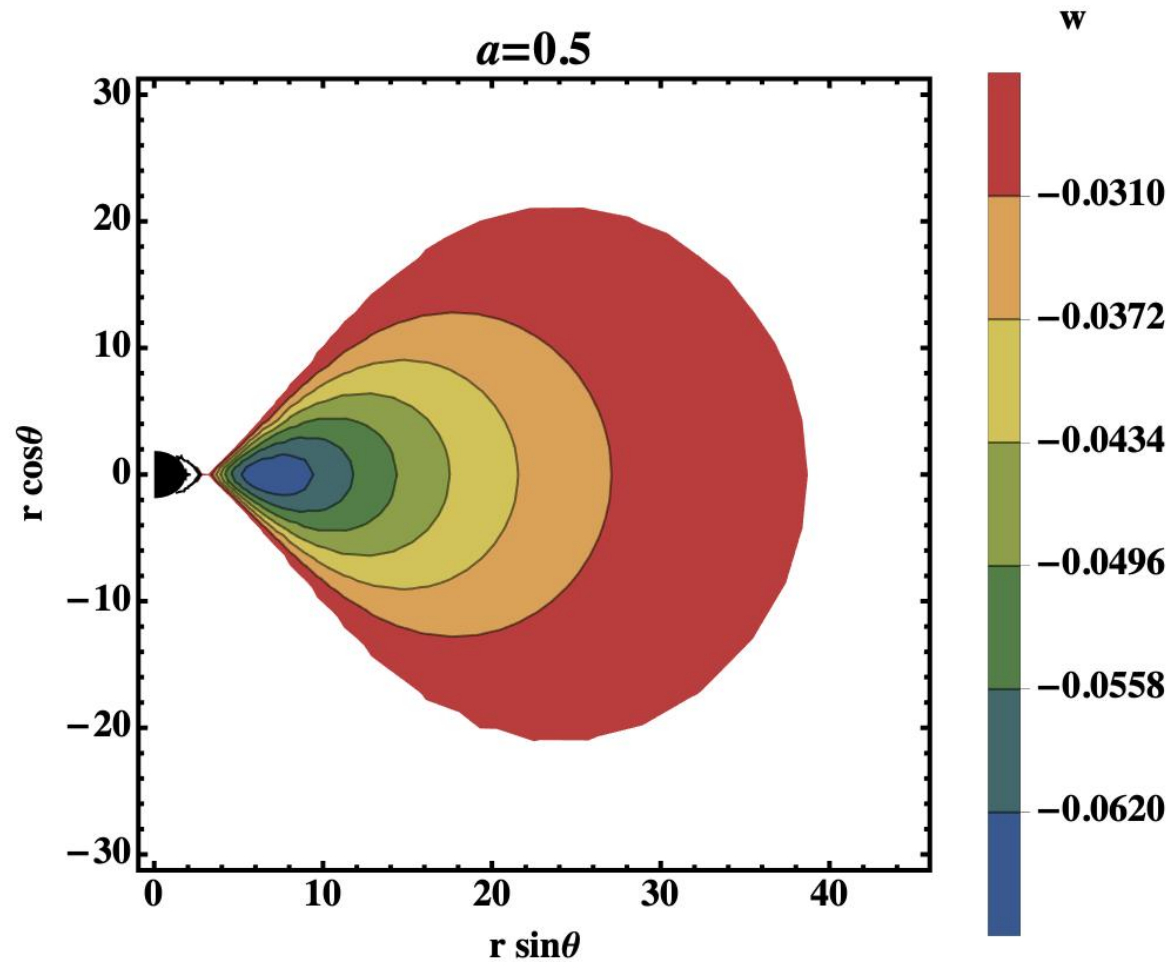
- Dimensionless parameter
- Specific angular momentum

$$\lambda \equiv \frac{l_0 - l_{ms}}{l_{mb} - l_{ms}}, \quad 0 \leq \lambda \leq 1$$

- Potential

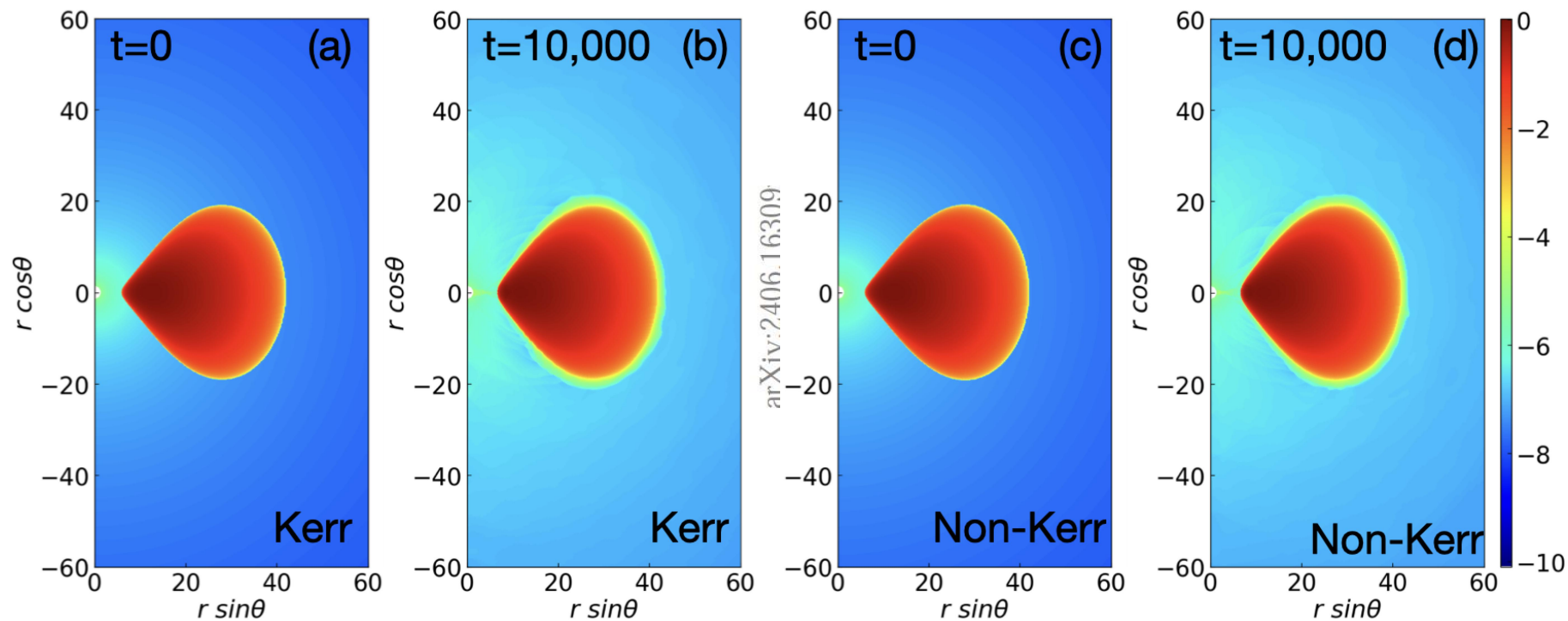
$$w(r, \theta) \equiv \frac{W(r, \theta) - W_{cusp}}{W_c - W_{cusp}}$$

# Magnetized Tori



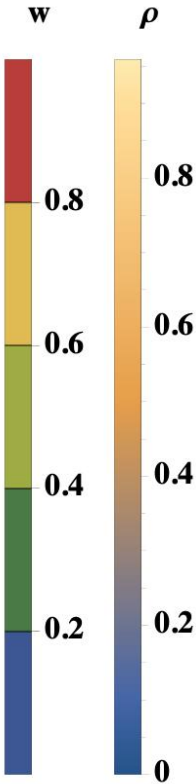
Parameters	Values
$l_0$	3.36
$\beta = p/p_m$	10
$\kappa, \eta$	4/3
$r_{cusp}$	3.01
$r_c$	6.74
$W_{cusp}$	-0.023
$W_c$	-0.067

# 该模型动力学演化稳定性好



arXiv:2406.16309

# Equipotential surfaces and density distribution diagram



平衡环吸积盘的辐射：热电子，同步辐射

假设盘为低密度高温盘，离子无碰撞

电子温度

$$T_e = \frac{2m_p u}{3k_B \rho (2 + R)},$$

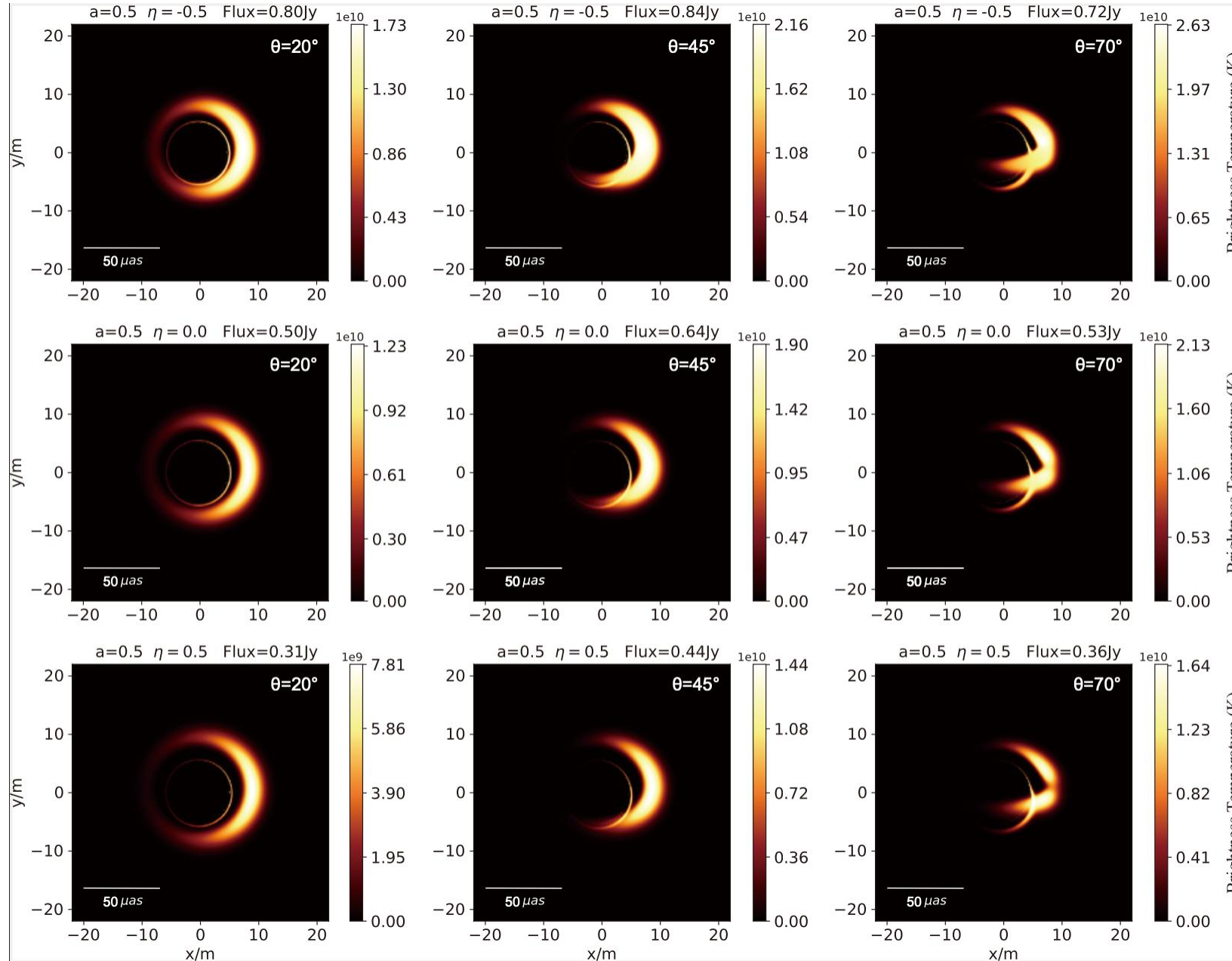
离子与电子温度比

$$R \equiv T_i / T_e :$$

$$R = R_{high} \frac{\beta^2}{1 + \beta^2} + \frac{1}{1 + \beta^2},$$

$\beta$  等离子磁化参数

# Images at different inclinations and parameters

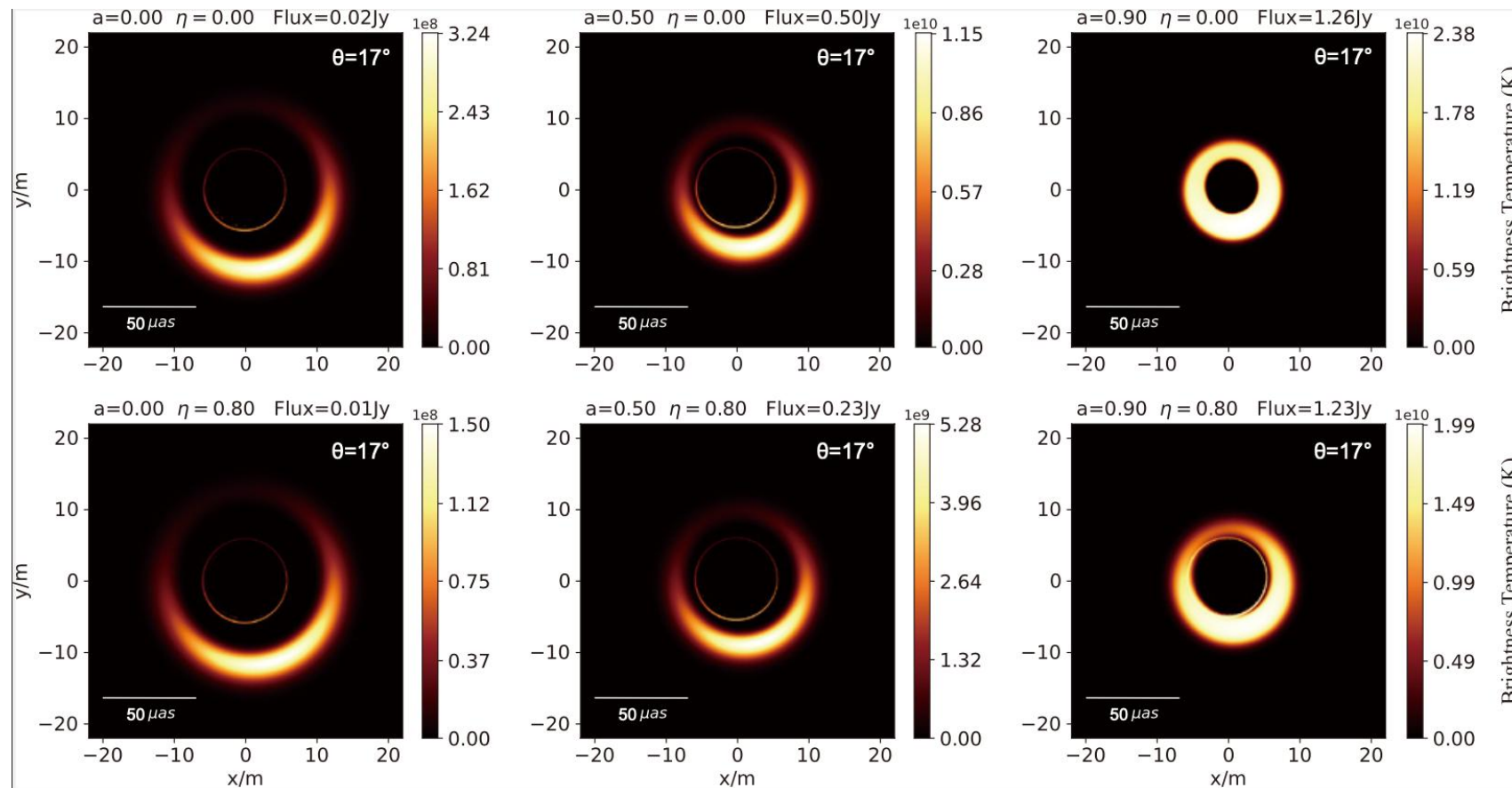


亮温

$$T_b = S\lambda^2 / (2k_B\Omega),$$

ARCMANCER

Images with different  $\eta$  parameters at  $\theta = 17^\circ$  and  $PA = 288^\circ$

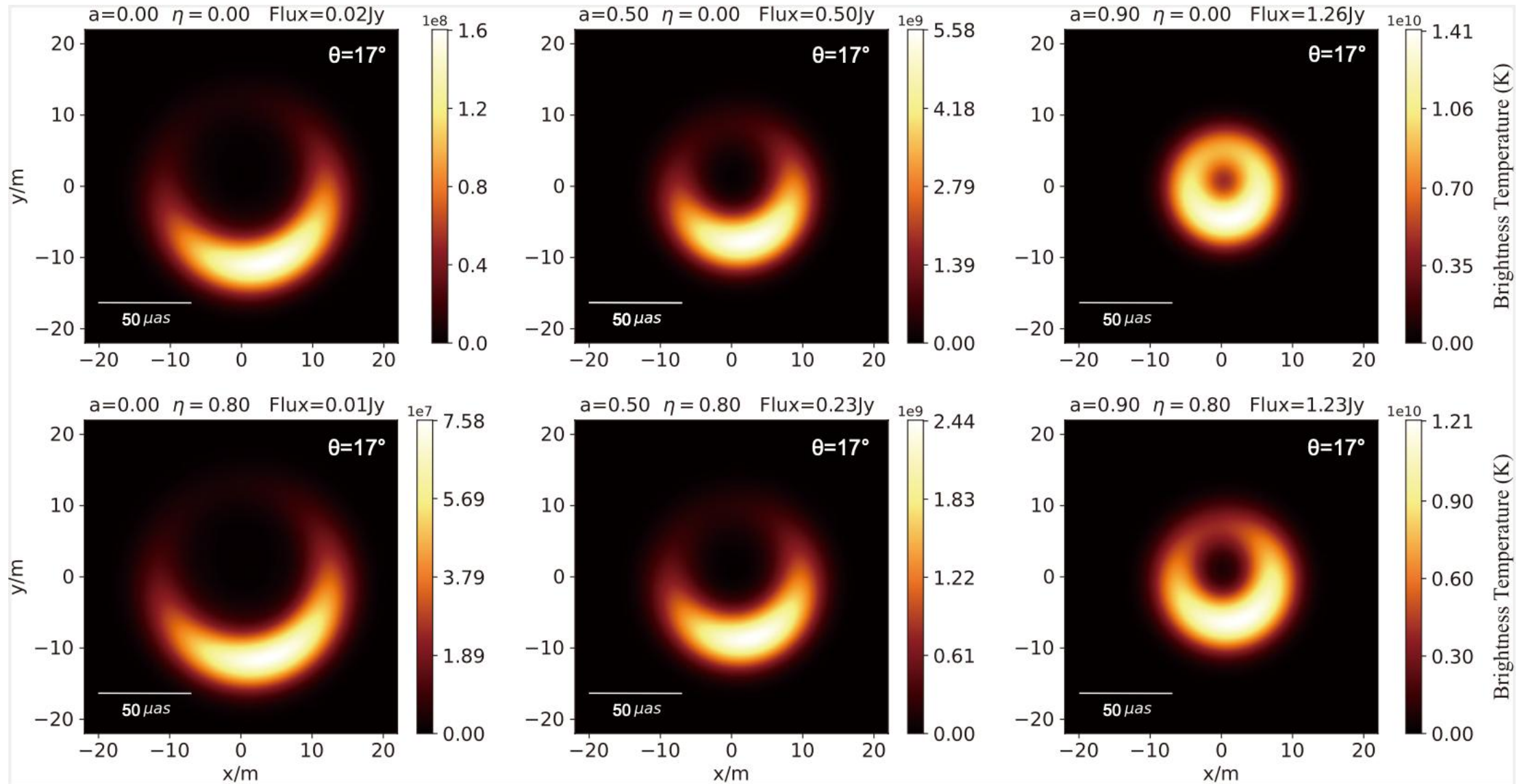


**图像特征：**

**偏离参数使  
亮环变大**

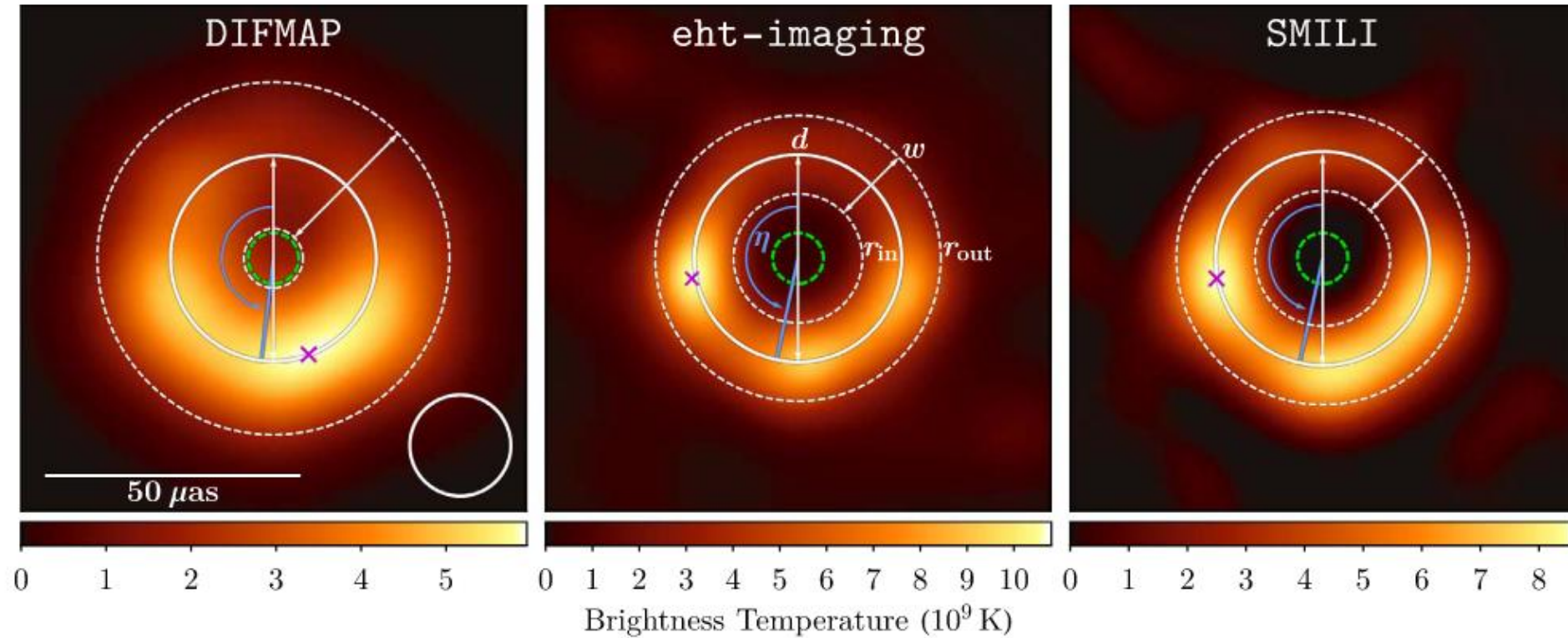
**明亮区域位  
于光子环外  
面。主要原  
因为平衡环  
位于ISCO外  
面。**

# Images convolved with a $20\mu\text{as}$ FWHM Gaussian beam





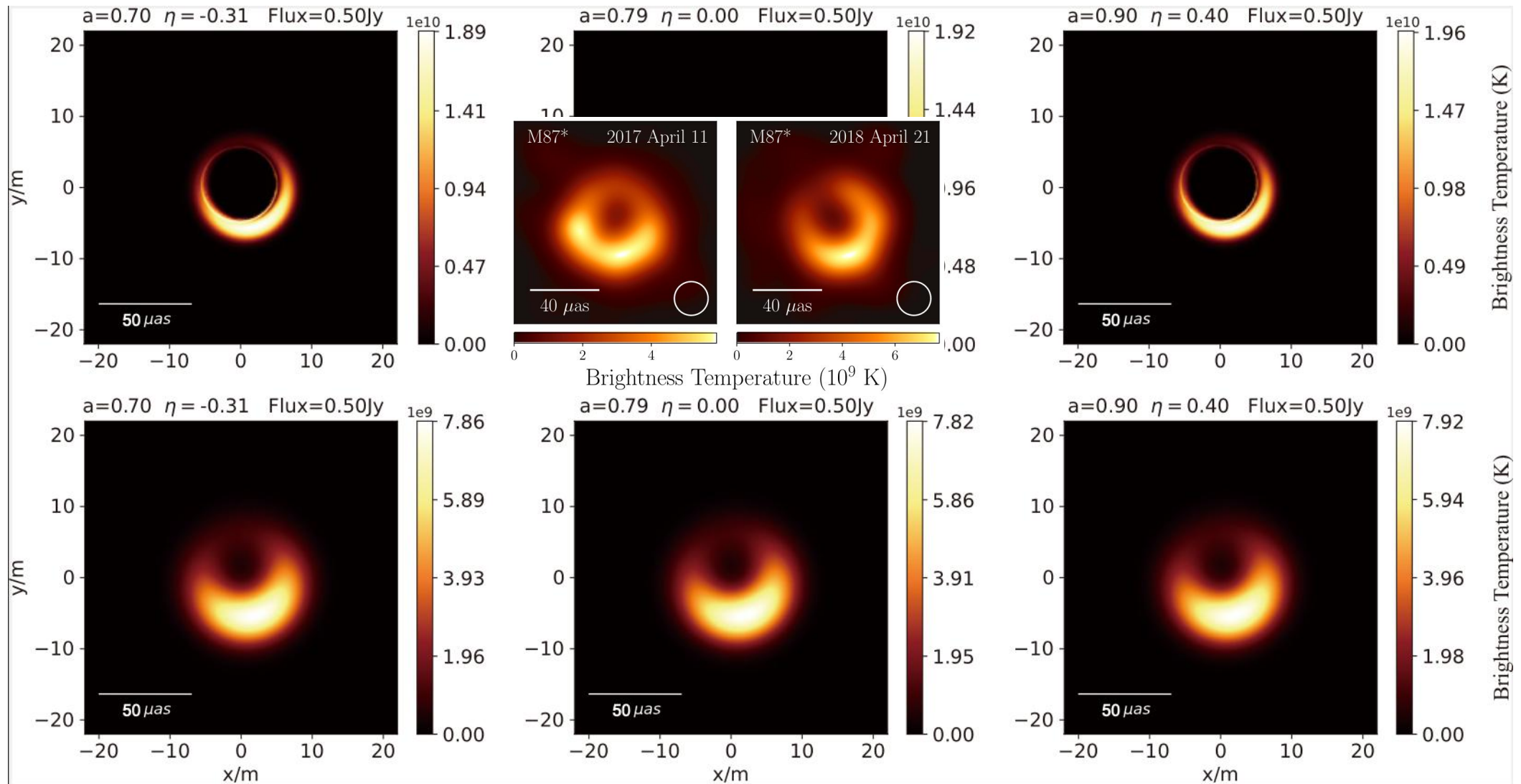
# Estimated ring properties overlaid on the fiducial images



Event Horizon Telescope Collaboration, et al., 2019, ApJL, 875, L4.

Ring diameter  $d$ :  $42 \pm 3 \mu\text{as}$  (2017)     $43.3_{-3.1}^{+1.5} \mu\text{as}$  (2018)

# Images with different $\alpha$ parameters at $d = 43.3\mu\text{ms}$

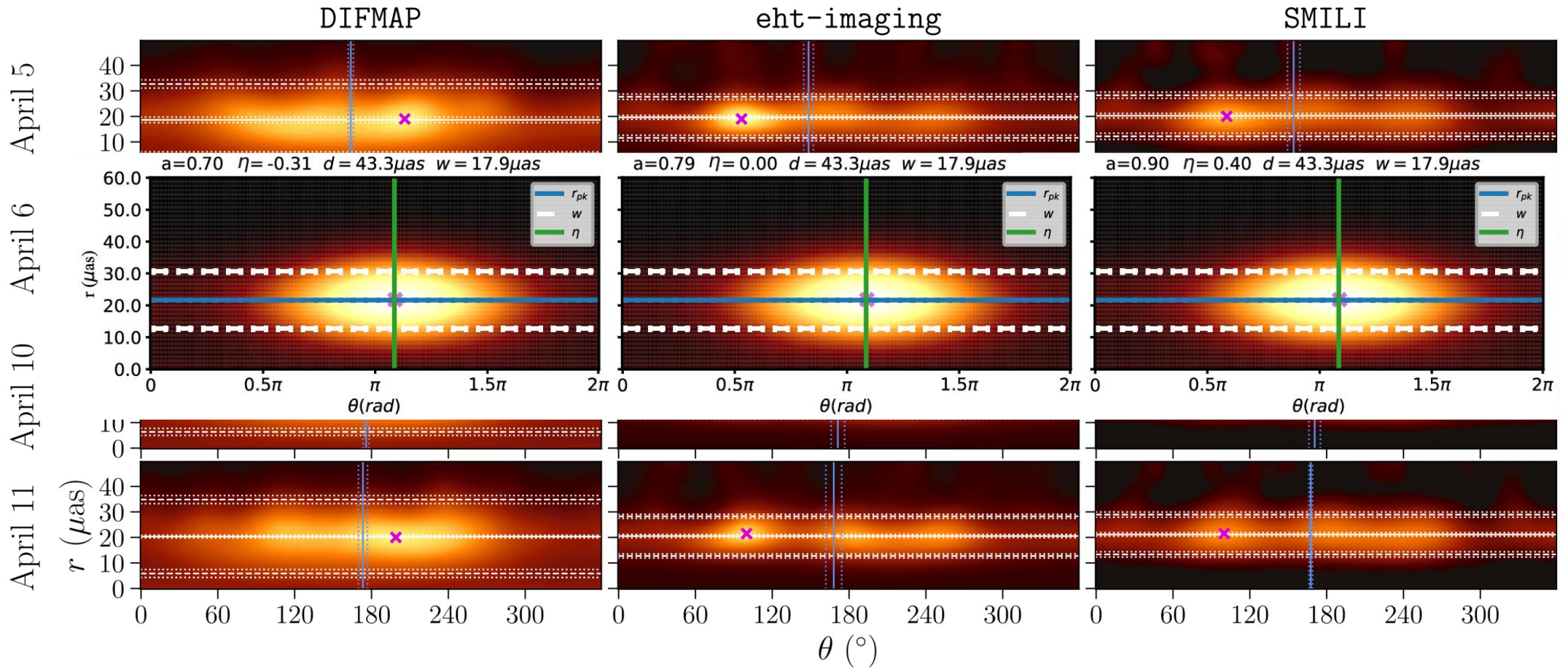


$$n_e \sim 2 \times 10^6 \text{ cm}^{-3}$$

$$T_e \sim 3 \times 10^{10} \text{ K}$$

$$B \sim 11 \text{ G}$$

# Unwrapped ring profiles of the images



Event Horizon Telescope Collaboration, et al., 2019, ApJL, 875, L4.

# b) 动态吸积盘

## GRMHD simulation

- Basic Equations

➤  $\nabla_\alpha T^{\alpha\beta} = 0, \quad \nabla_\alpha {}^*F^{\alpha\beta} = 0, \quad \nabla_\alpha \rho u^\alpha = 0$

- Ideal relativistic MHD:

$$T^{\alpha\beta} = (h + b^2)u^\alpha u^\beta + \left(p + \frac{1}{2}b^2\right)g^{\alpha\beta} - b^\alpha b^\beta$$

- Conservation form

$$\partial_t(\sqrt{-g} \rho u^t) = -\partial_i(\sqrt{-g} \rho u^i)$$

$$\partial_t(\sqrt{-g} T^t_\nu) = -\partial_i(\sqrt{-g} T^i_\nu) + \sqrt{-g} T^\kappa_\lambda \Gamma^\lambda_{\nu\kappa}$$

$$\partial_t(\sqrt{-g} B^i) = -\partial_j[\sqrt{-g} (b^j u^i - b^i u^j)]$$

$$\partial_i(\sqrt{-g} B^i) = 0$$

- Initial conditions

➤ Fishbone-Moncrief torus

➤ Parameter:  $a = 0.5/0.9375, \eta = 0/0.5$

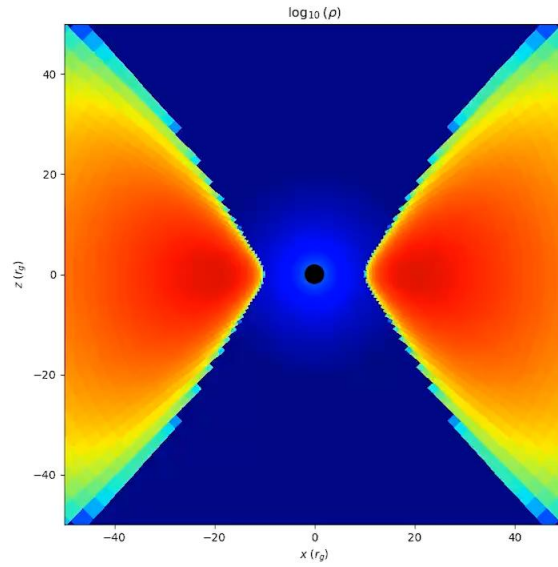
➤  $r_{in} = 10, r_{max} = 20, \gamma = 4/3$

➤ SANE:  $A_\phi = \max\left[\frac{\rho}{\rho_{max}} - 0.2, 0\right]$

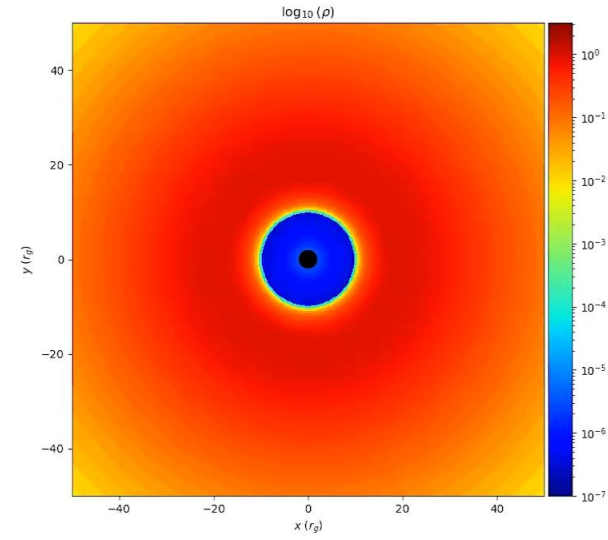
➤ Resolution: 128\*96\*96

$$\log_{10} \rho$$

$$a = 0.5, \eta = 0$$

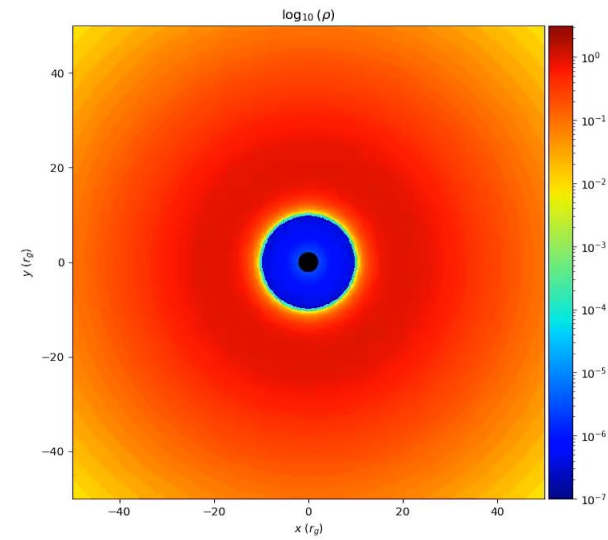
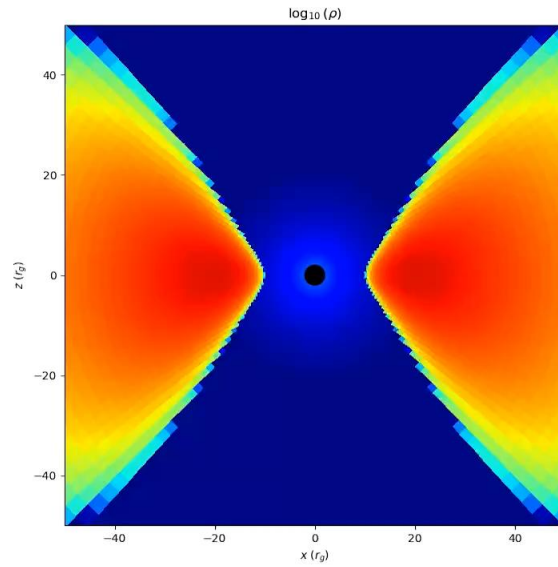


t = 0



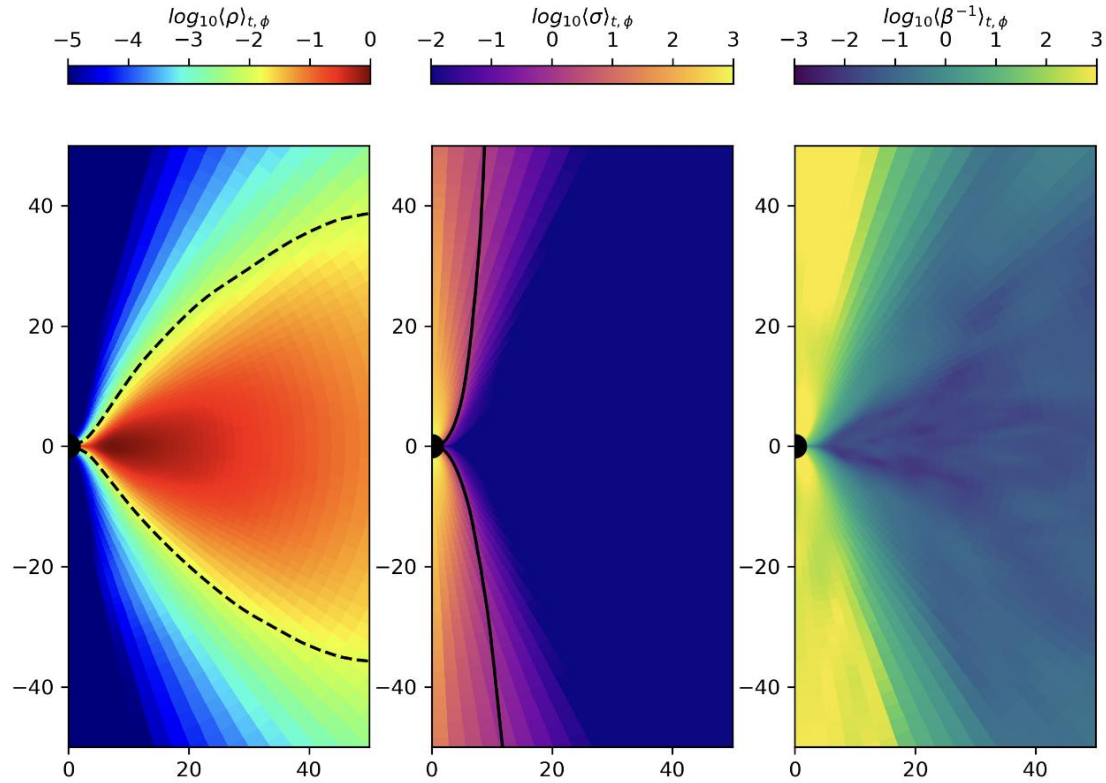
t = 0

$$a = 0.5, \eta = 0.5$$

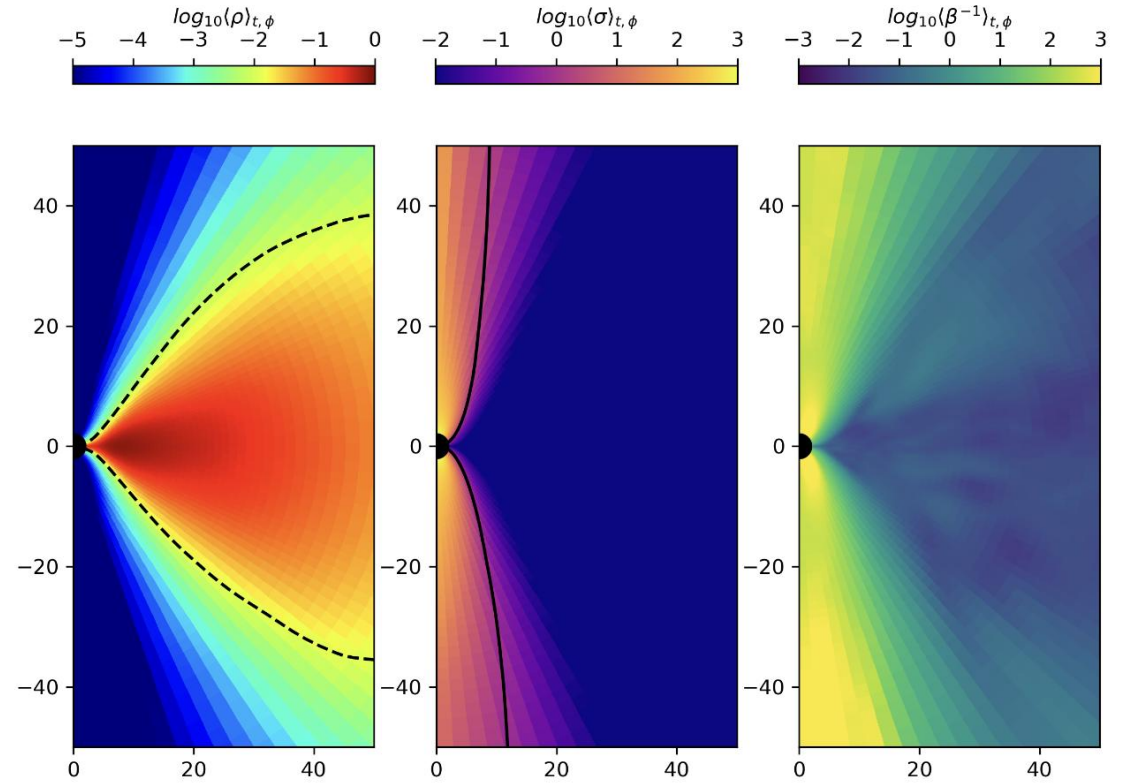


# Image averaged in $t$ and $\varphi$ directions

$$\sigma = b^2/\rho \quad \beta = P_{gas}/P_{mag}$$



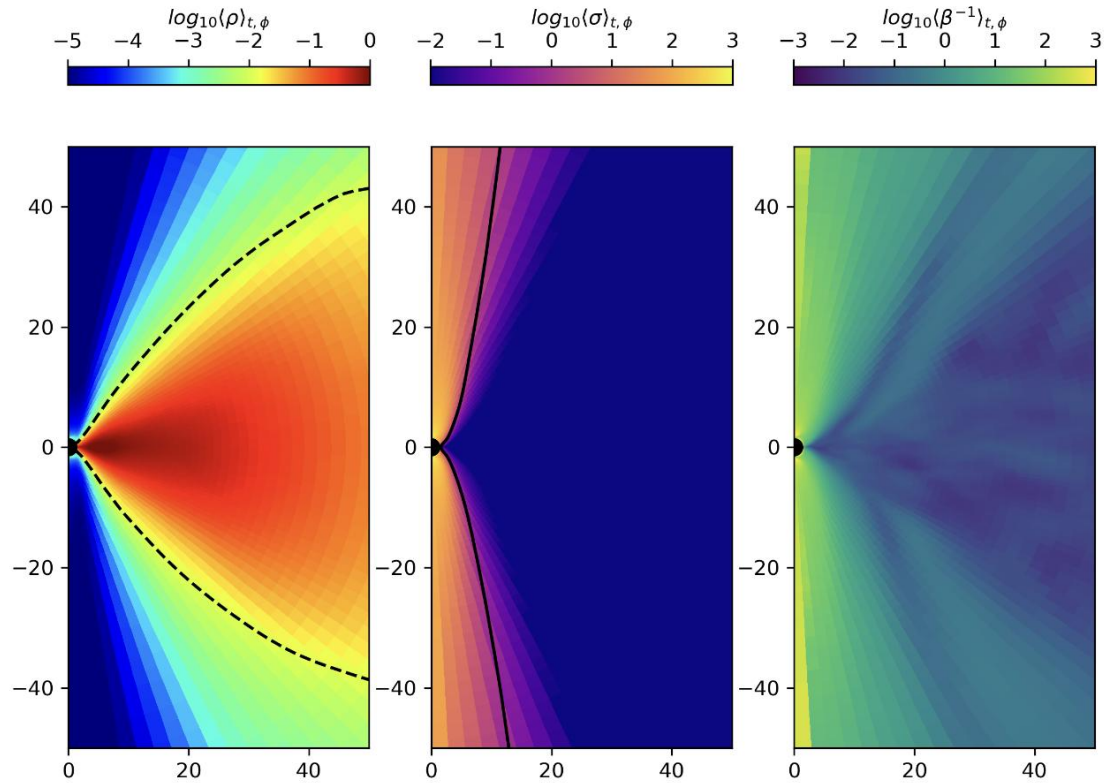
$a = 0.5, \eta = 0$



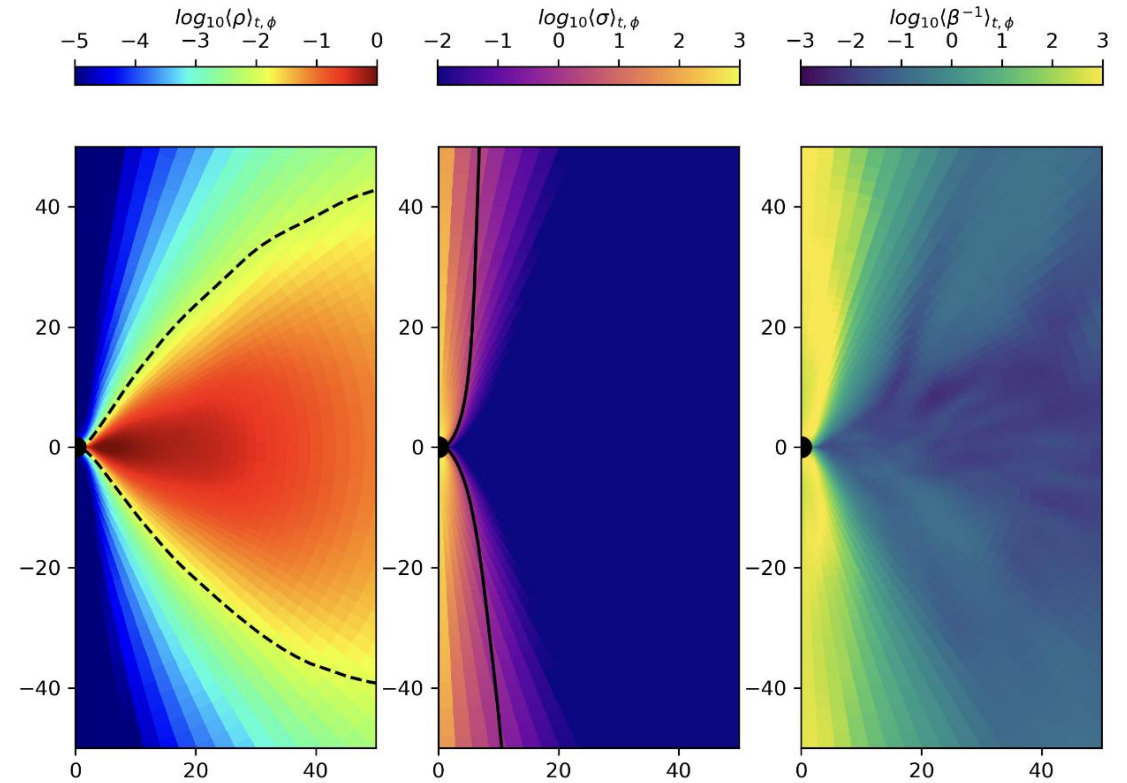
$a = 0.5, \eta = 0.5$

# Image averaged in $t$ and $\varphi$ directions

$$\sigma = b^2 / \rho \quad \beta = P_{gas} / P_{mag}$$

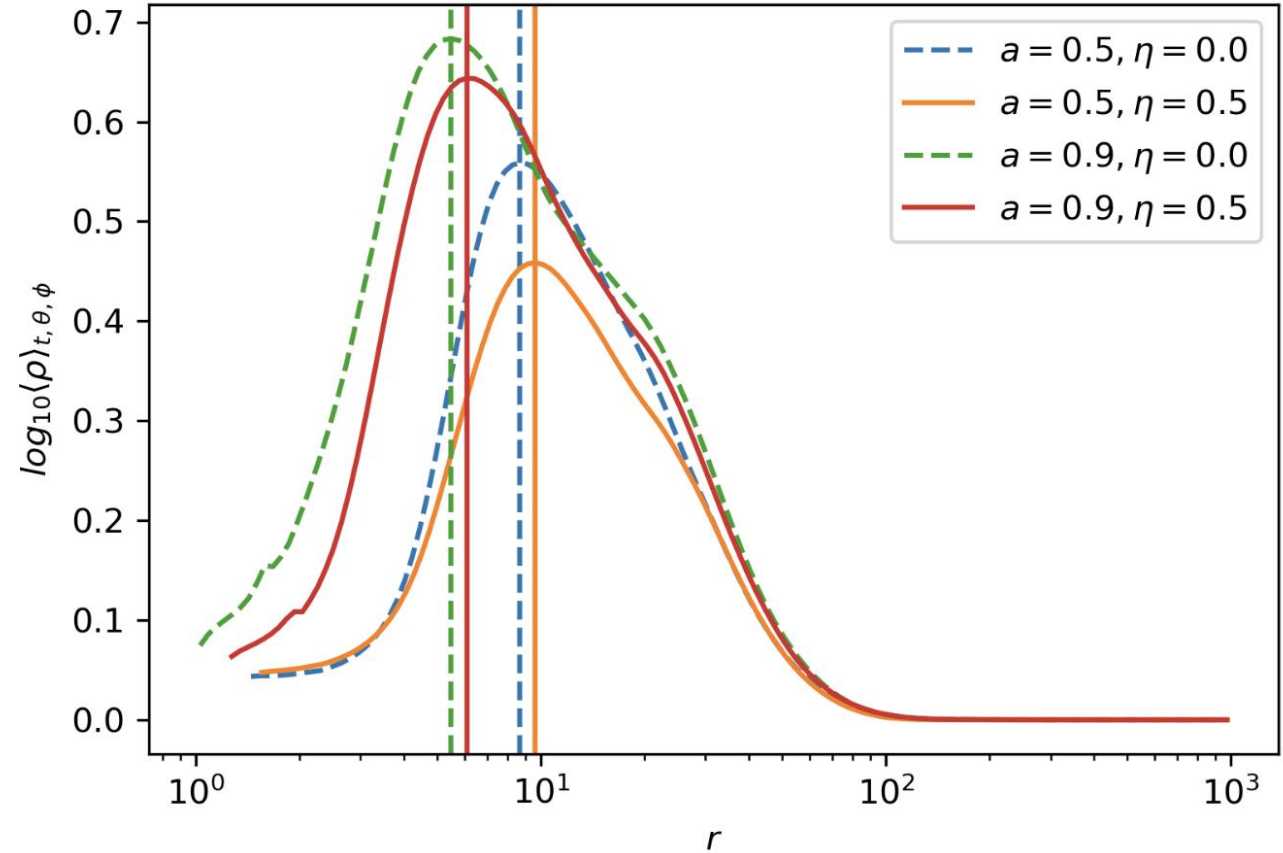
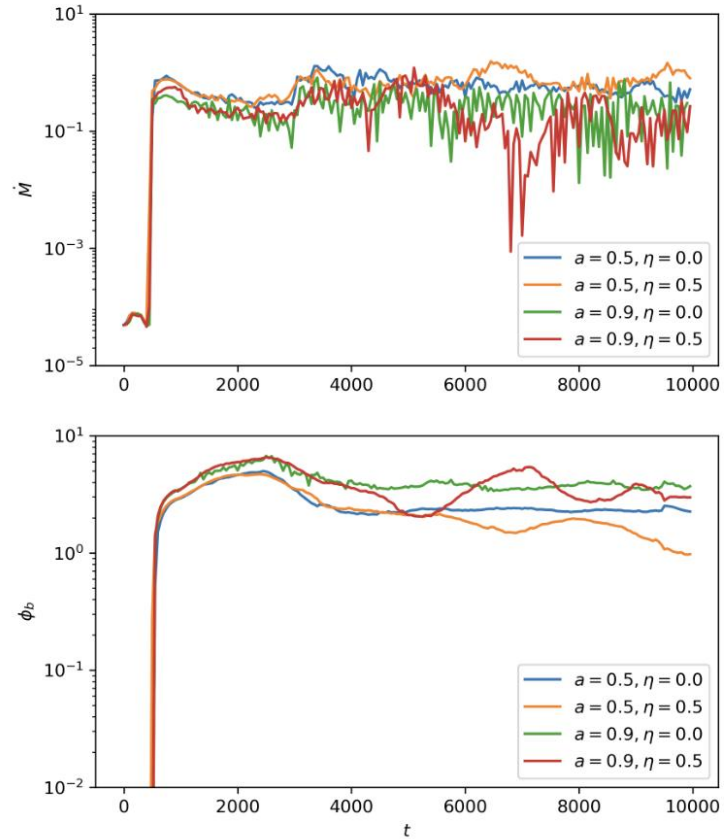


$a = 0.9, \eta = 0$



$a = 0.9, \eta = 0.5$

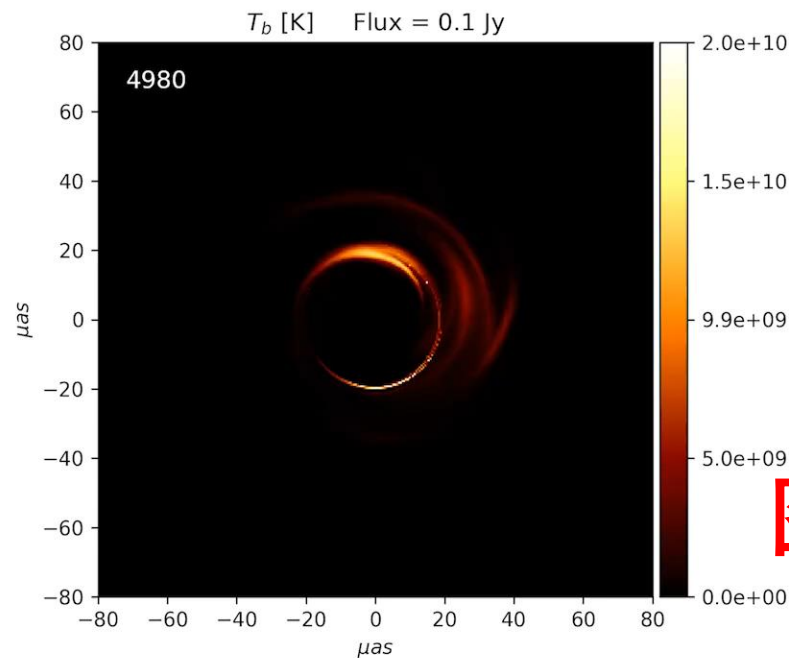
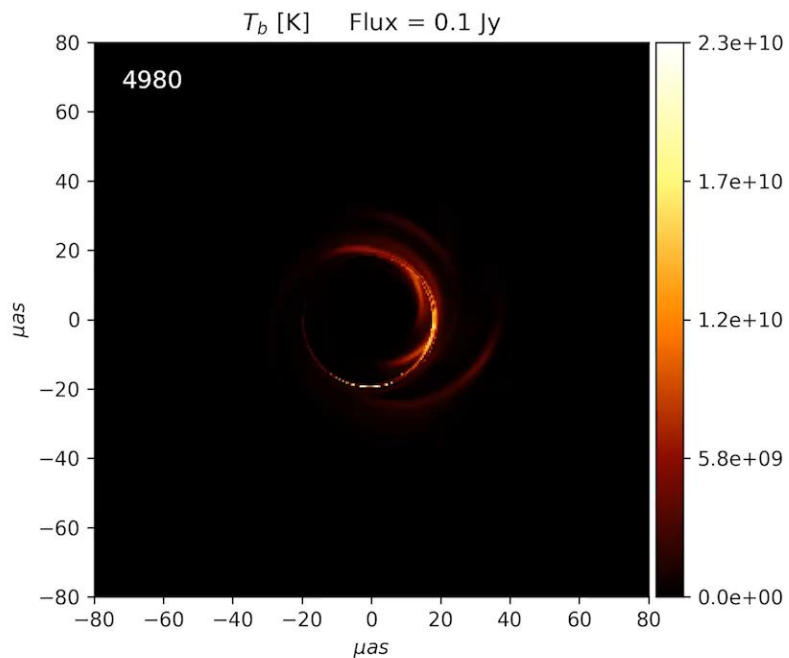
# Accretion rate, magnetic flux, and radial matter distribution



$$\dot{M} = - \int_0^{2\pi} \int_0^\pi \rho u^r \sqrt{-g} d\theta d\phi \quad \Phi_{\text{BH}} = \frac{\Phi_{\text{BH}}}{\sqrt{\dot{M}}} \quad \Phi_{\text{BH}} = \frac{1}{2} \int_0^{2\pi} \int_0^\pi |\star F^{rt}| \sqrt{-g} d\theta d\phi$$



$a = 0.5, \eta = 0$

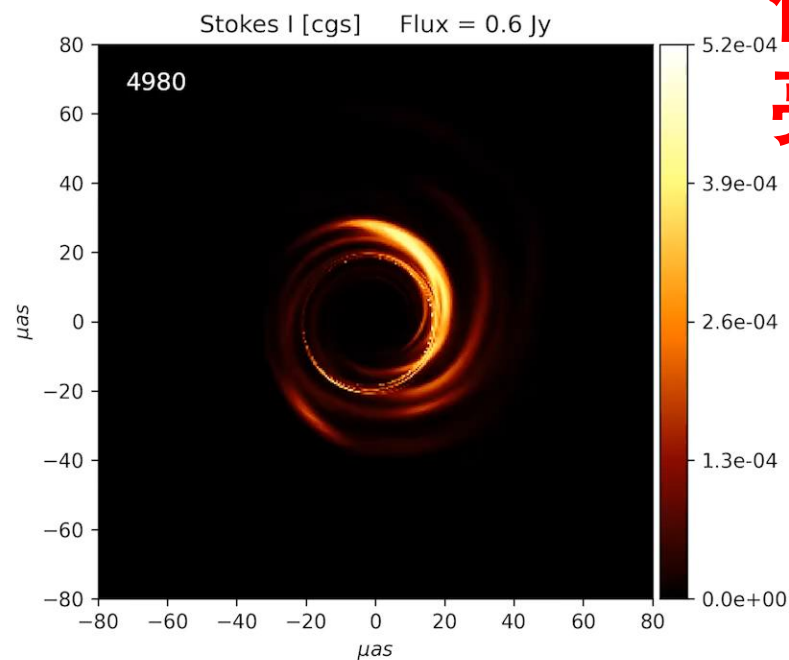
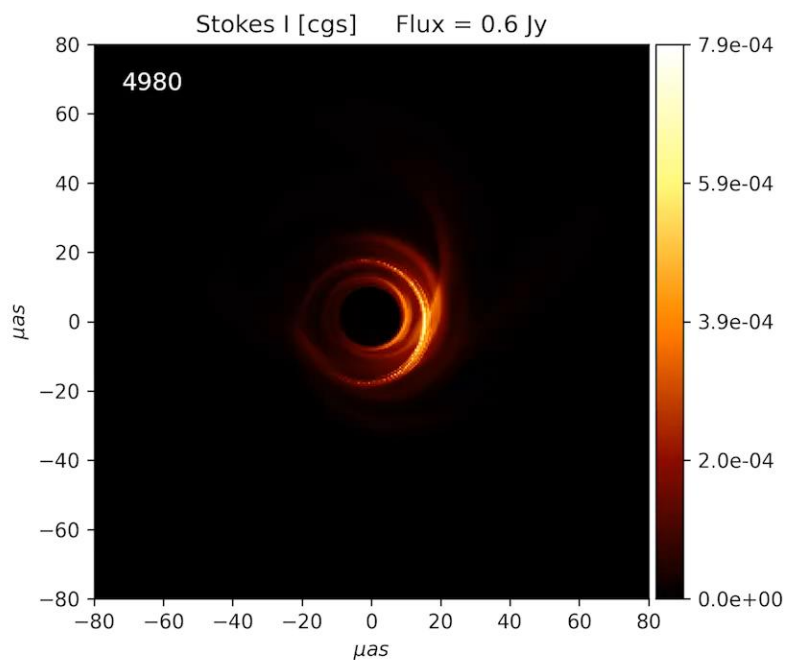


$a = 0.5, \eta = 0.5$

图像特征:

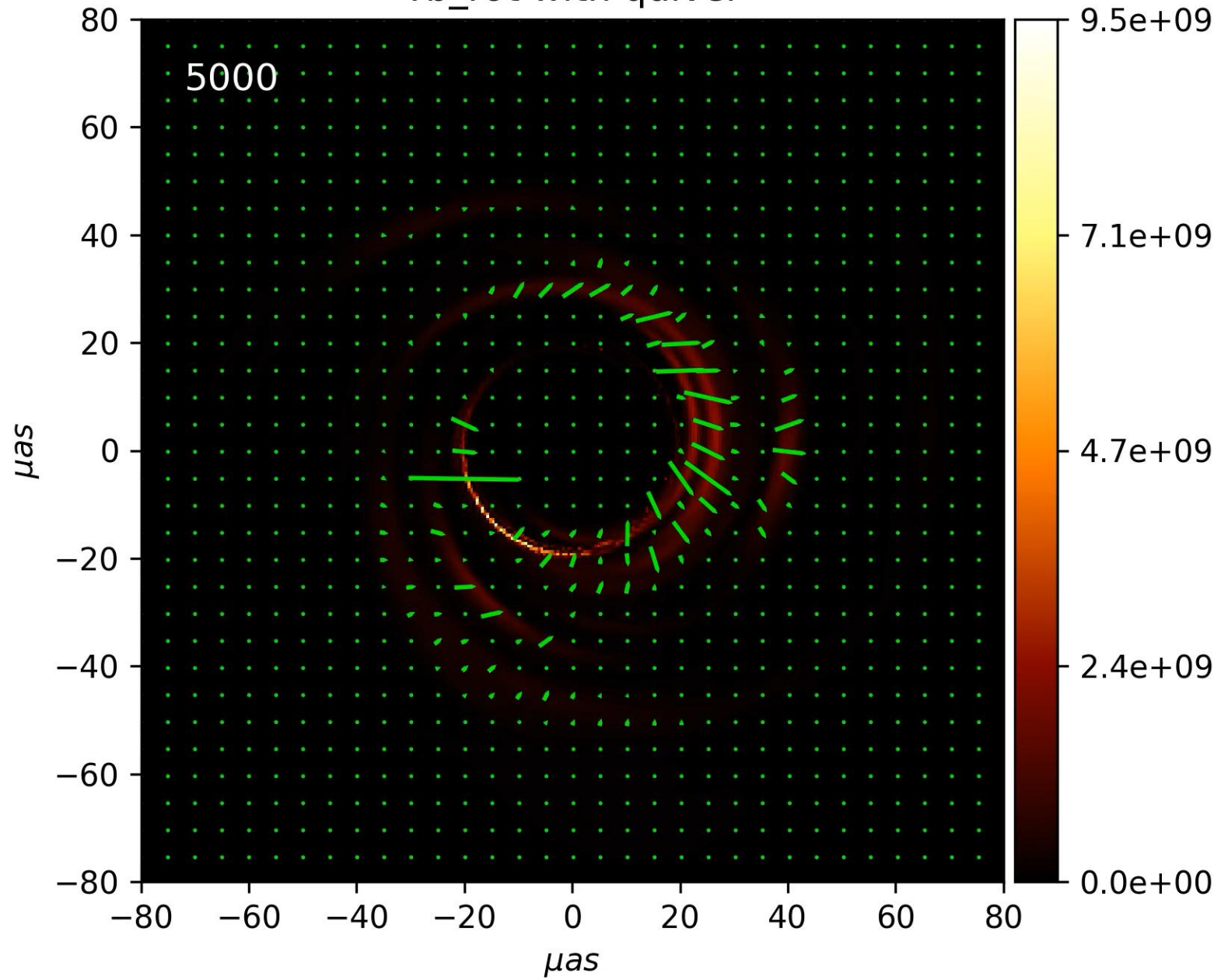
偏离参数使  
亮环变大

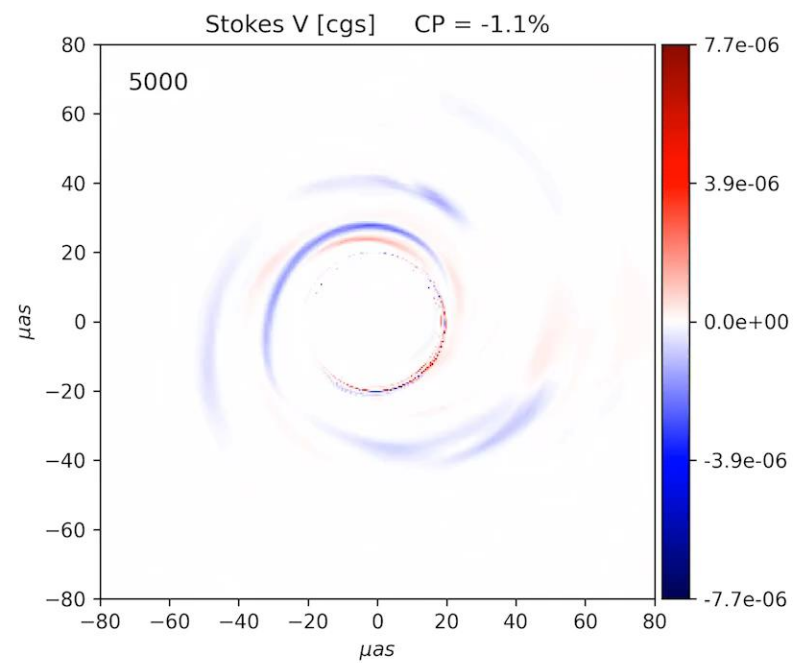
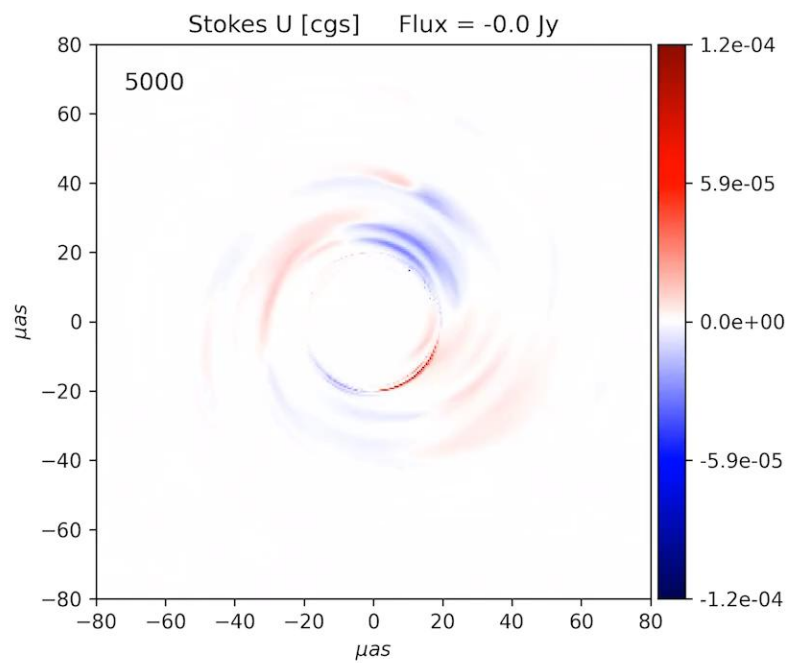
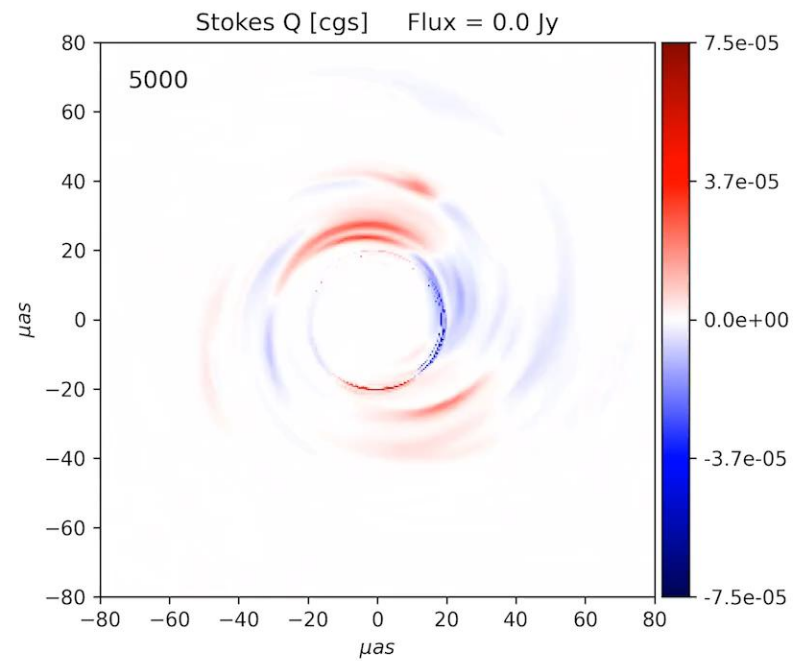
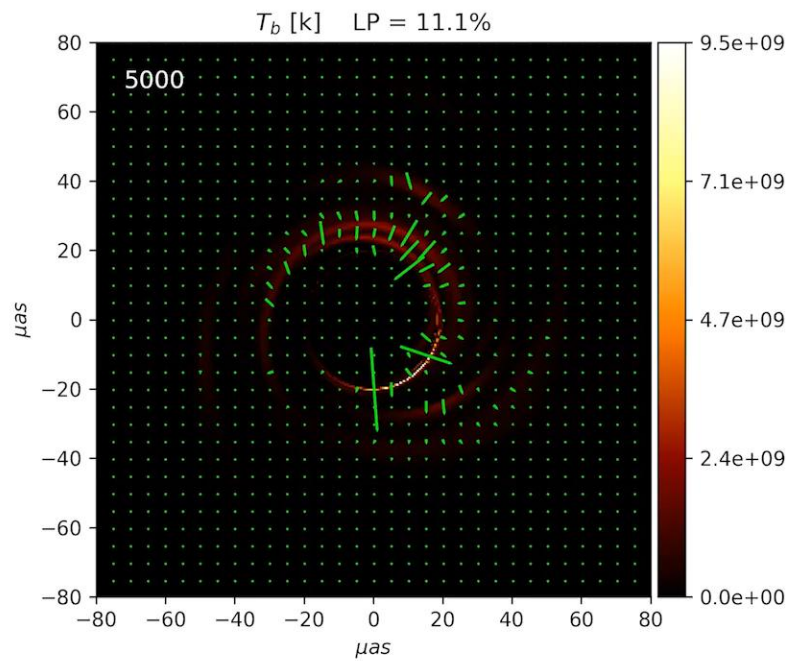
$a = 0.9, \eta = 0$

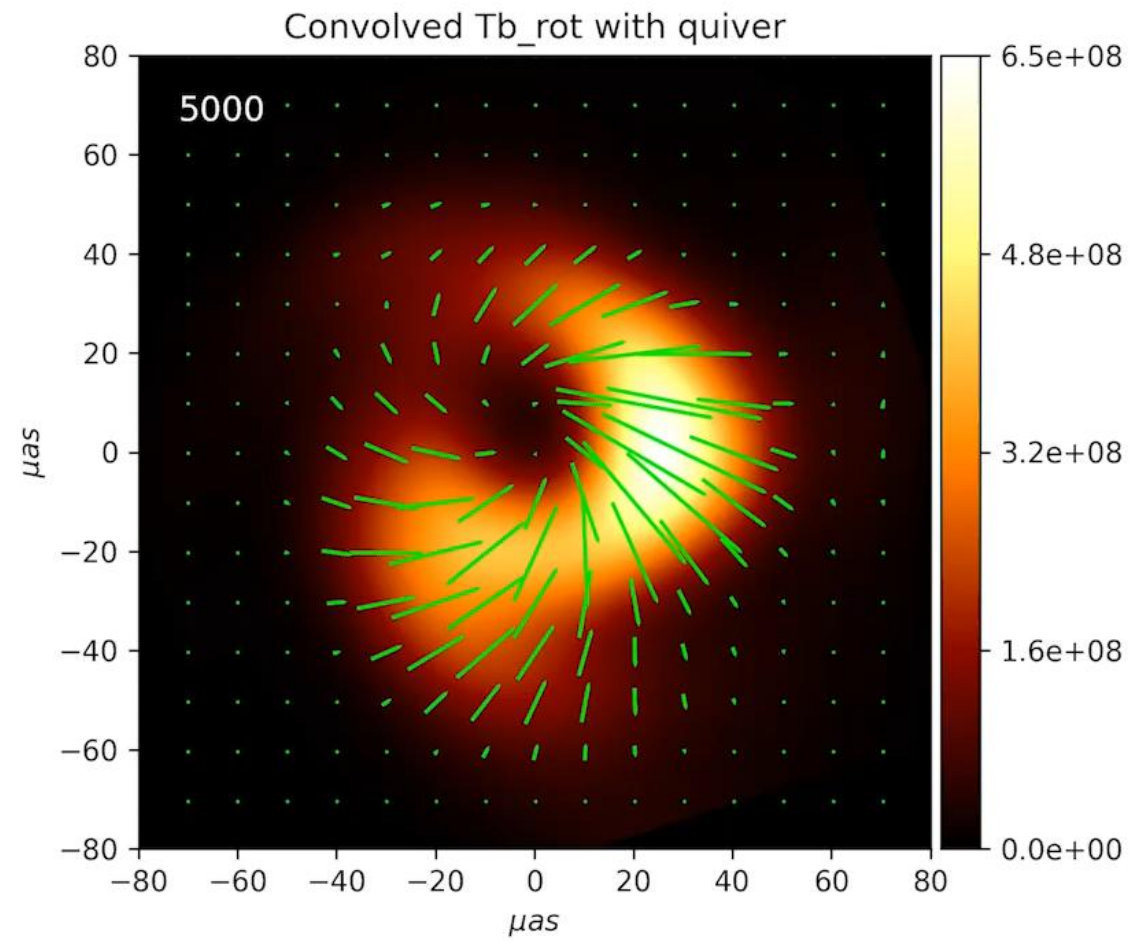
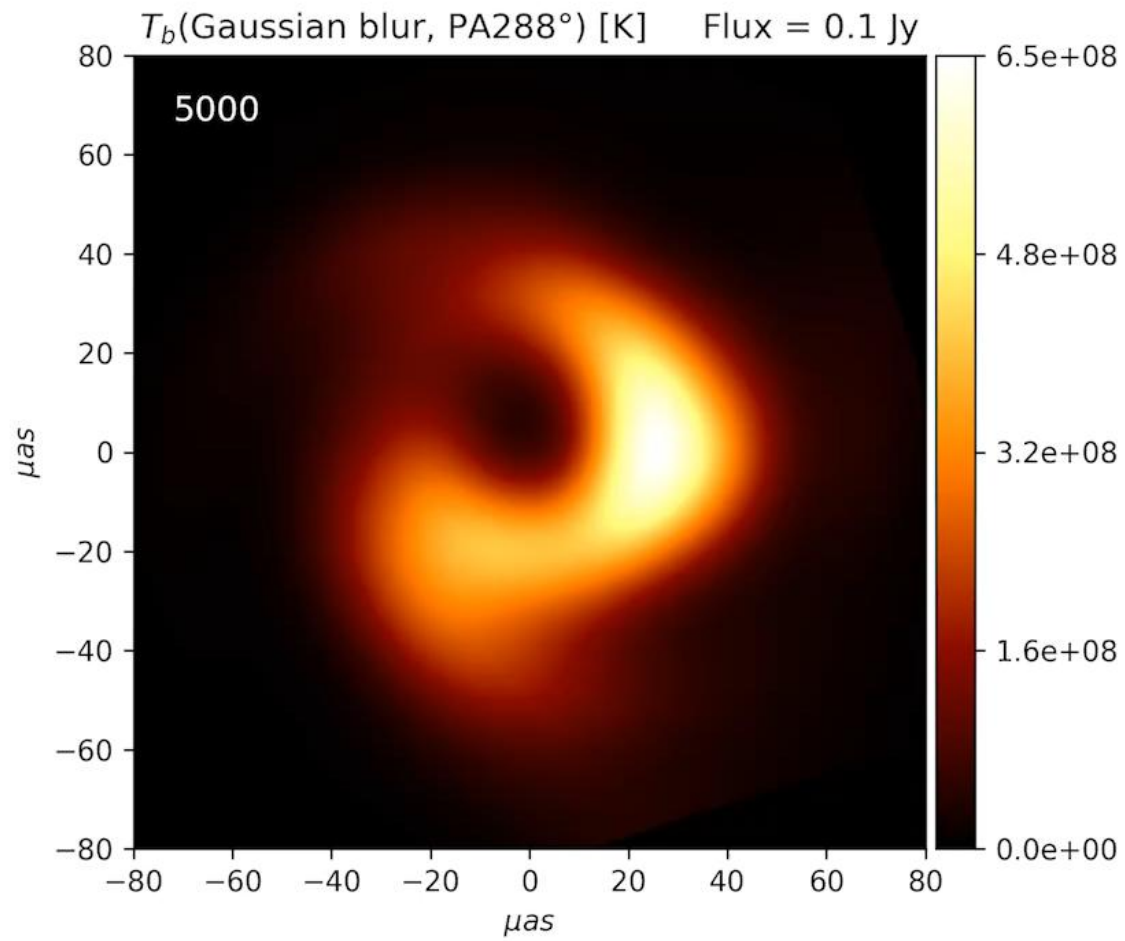


$a = 0.9, \eta = 0.5$

Tb\_rot with quiver







### 3. Summary

1. 研究了Konoplya-Azhideenko黑洞周围两种几何厚吸积盘的图像特征
2. 黑洞的旋转参数使几何厚吸积盘的图像变小。
3. 偏离参数使几何厚吸积盘的图像变大。

前路漫漫亦灿灿

Thank you!

