Black hole interiors and phase transitions

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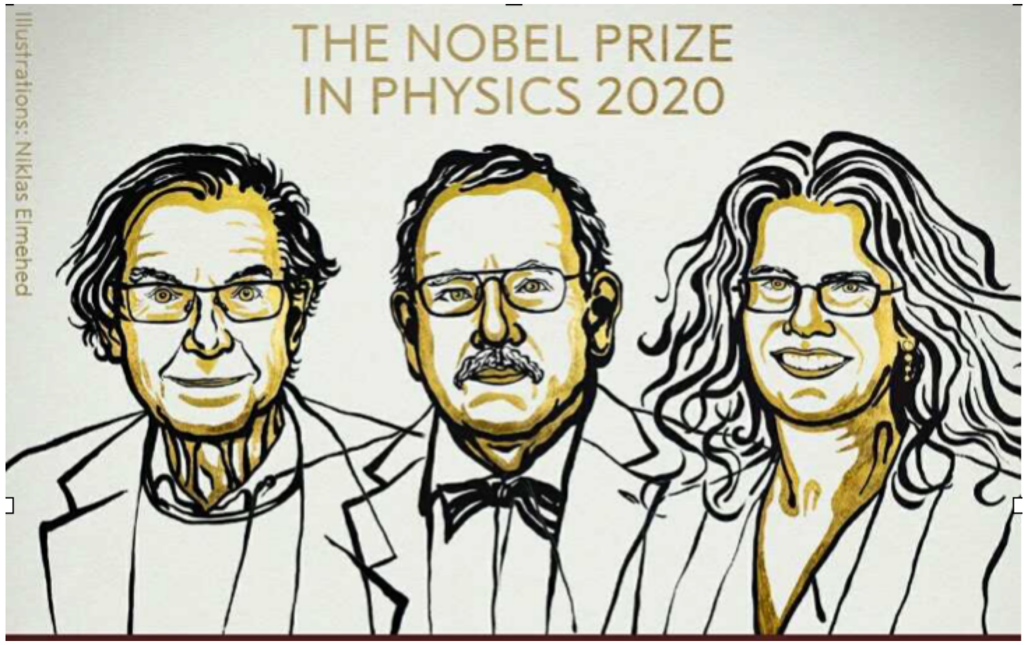
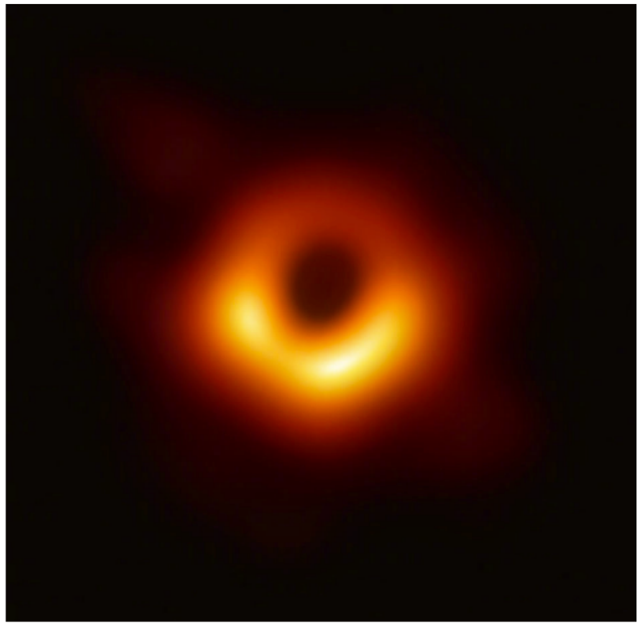
“2024 引力与宇宙学” 专题研讨会

Based on works with 高凌龙、吕宏达、Avinash Raju

Background and motivation

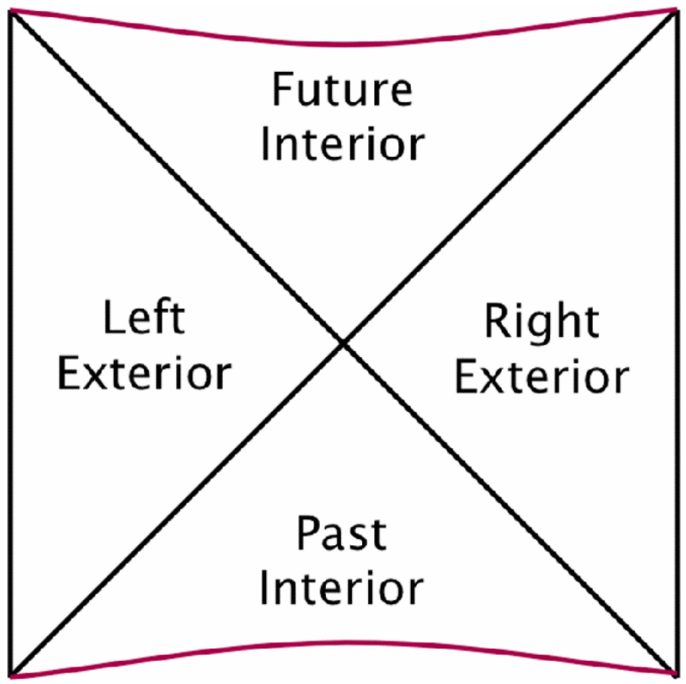
- “black hole formation is a robust prediction of the general theory of gravity”





Black hole interiors

- **black hole interior**: mysterious, classically unobservable



ER=EPR
island …

- horizon: causally disconnect

Black hole interiors

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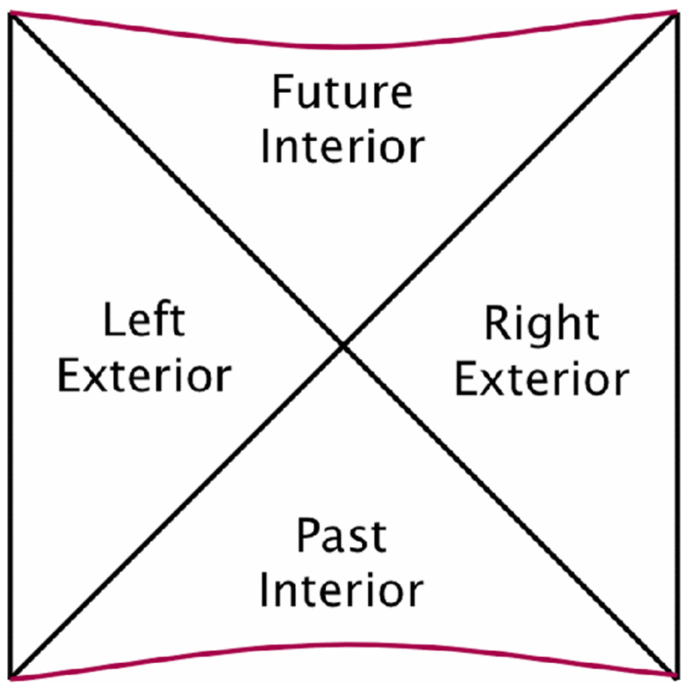


ER=EPR
island …

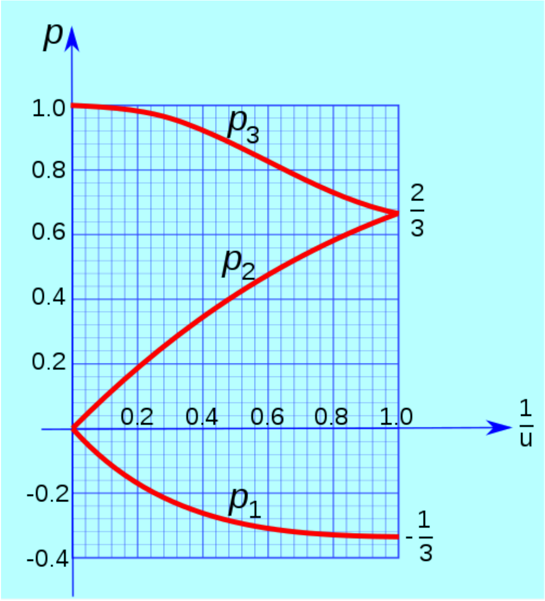
- horizon: causally disconnect [Penrose, 1970s]
- appearance of singularity [Penrose, 1970s]
- description of singularity [Belinsky, Khalatnikov, Lifshitz et al., 1970s]

- BKL singularity: a generic class of spacelike singularities in GR (coupled to matter) [Belinsky, Khalatnikov, Lifshitz, 1970s]

- Kasner singularity (homogeneous system) [Kasner, 1921]

$$ds^2 = -d\tau^2 + \tau^{2p_1} dx^2 + \tau^{2p_2} dy^2 + \tau^{2p_3} dz^2$$

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$$



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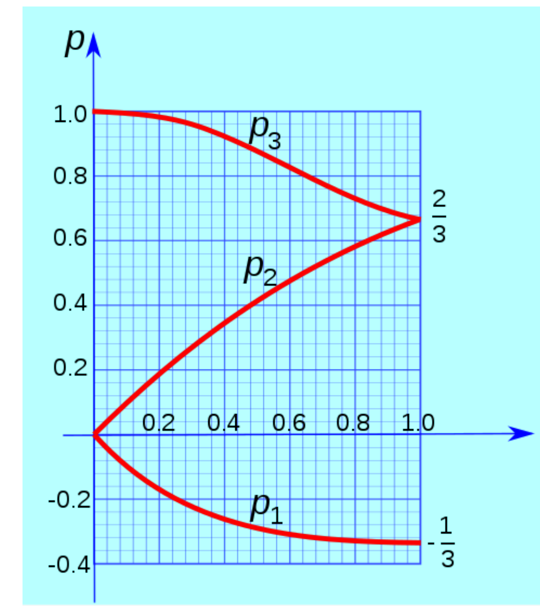
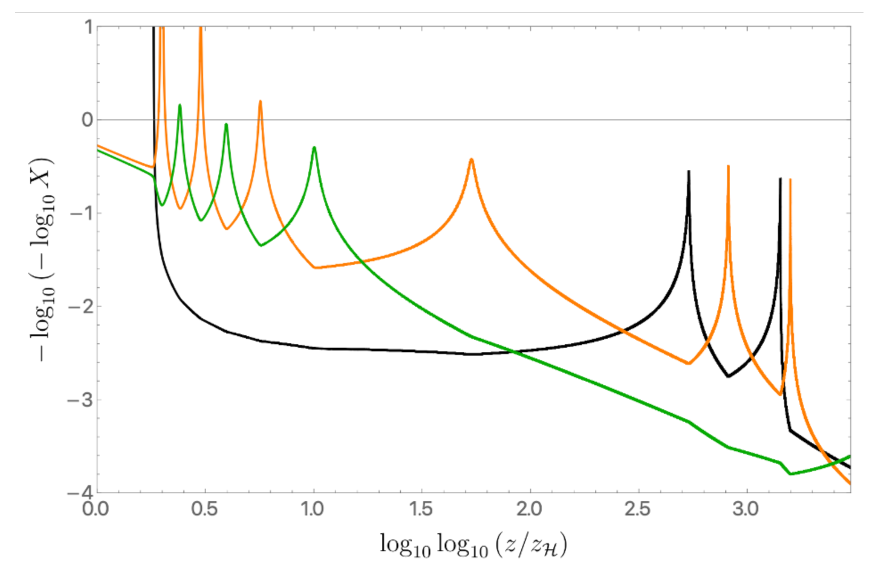


- quasi-Kasner spacetime (asymptotic metric)

$$ds^2 = -d\tau^2 + \left(\tau^{2p_l(\tau)} l_i l_j + \tau^{2p_m(\tau)} m_i m_j + \tau^{2p_n(\tau)} n_i n_j\right) dx^i dx^j$$



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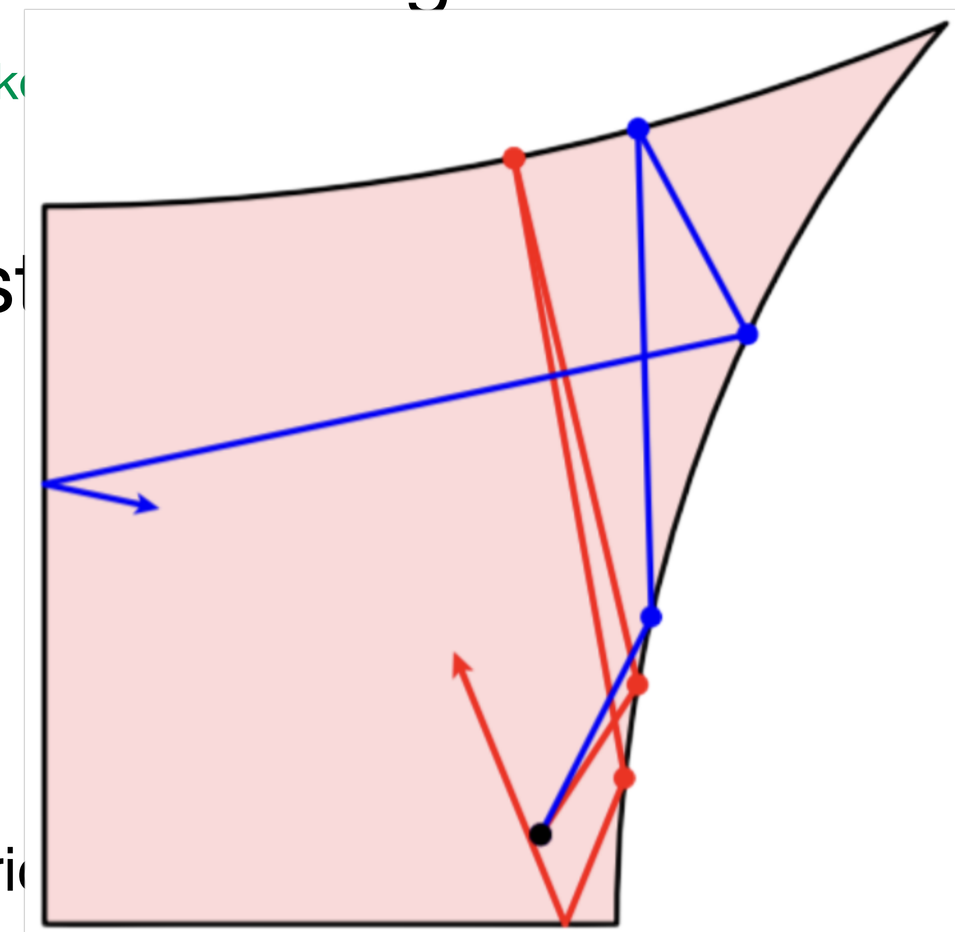

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- the billiard description: a ball in (a portion of) hyperbolic space

[Damour, Henneaux, Nicolai, 2002]

Motivation (1)

- For specific black holes, what are the black hole internal structures?

- Any possible connection between the physics inside and outside the horizon?

- What happens to the singularities during the black hole phase transitions?

Recent studies: static black holes

- Schwarzschild black hole
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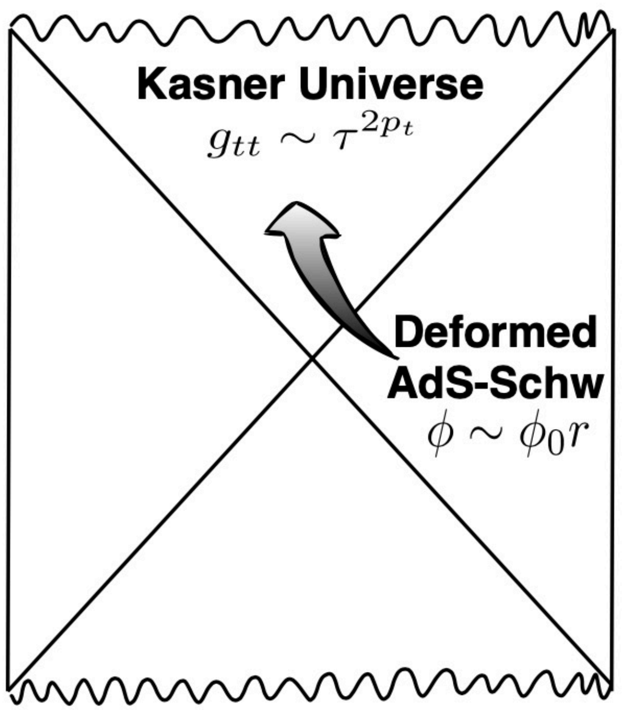
Recent studies: static black holes

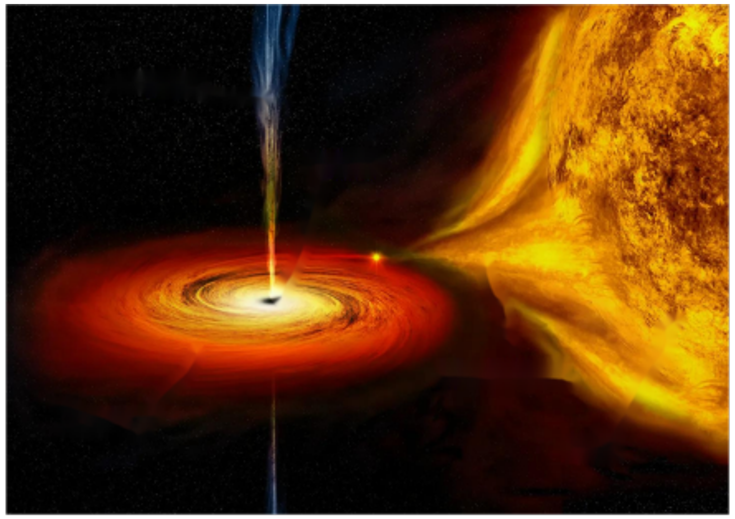
- Schwarzschild black hole
- static hairy black holes (asymptotic AdS)
- holographic superconductors
- helical black holes
- …



[Hartnoll et al. 2020, Horowitz et al. 2020; Cai, Li, Yang, et al., Vegh et al., Gao, YL, Lyu et al.,…]

stationary black holes

- Rotating black hole



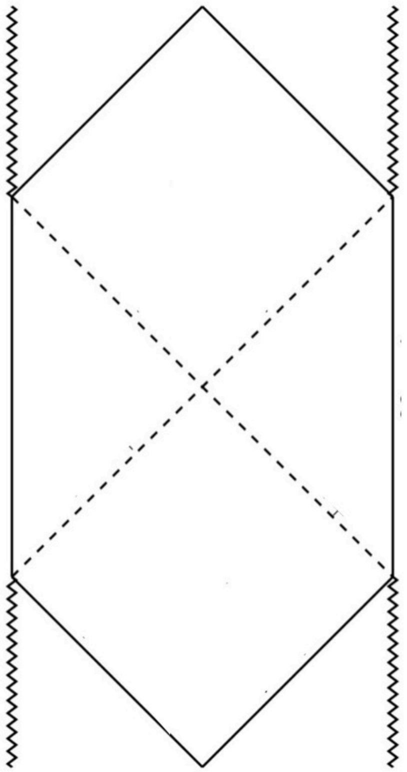
stationary black holes

- Rotating black hole
- 3D hairy rotating black holes

$$S = \int d^3x \sqrt{-g} \left(R + 2 - \partial_a \varphi \partial^a \varphi^* - m^2 \varphi \varphi^*\right)$$

$$ds^2 = \frac{1}{z^2}\left(-f e^{-\chi} dt^2 + \frac{dz^2}{f} + (N dt + dx)^2\right) ,$$

$$\varphi = \phi(z) e^{-i\omega t + i n x} .$$



- BTZ black hole
- black hole solution vs star solution
- **Case 1： real φ** $n = 0, \omega = 0$
- **Case 2： complex φ** $n \neq 0$

a periodic source

less symmetry, rich interior

[Gao, YL, Lyu, 2024]

No inner horizon

- consider $m^2 < 0$, asymptotic AdS3
  - Real scalar

Assuming more than one horizon

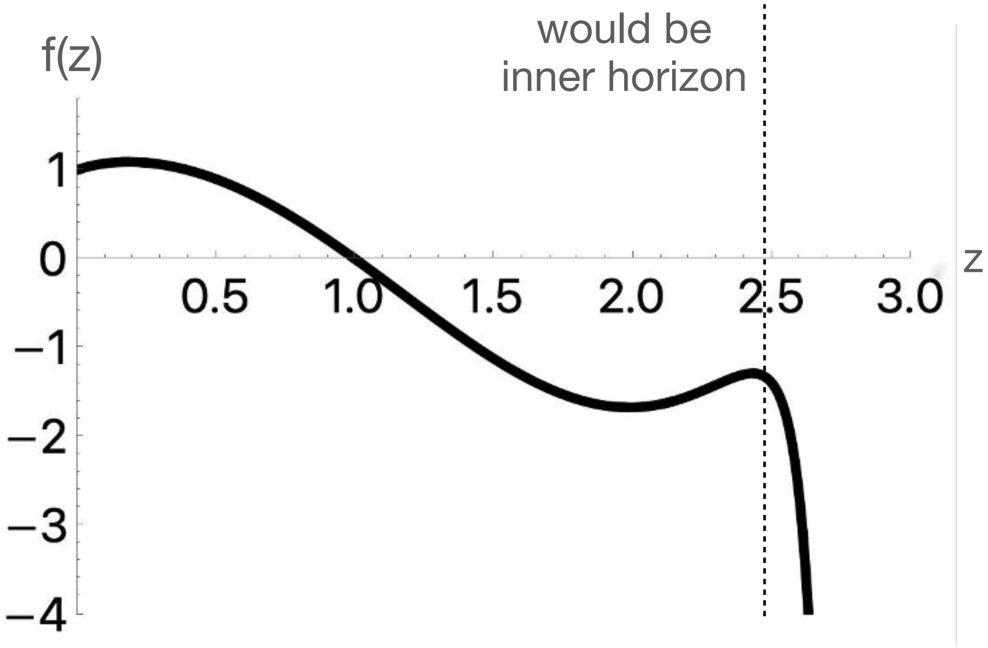
$$0 = \int_{z_h}^{z_i} \left(\frac{f e^{-\chi/2} \phi \phi'}{z} \right)' dz = \int_{z_h}^{z_i} \frac{e^{-\chi/2}}{z^3} \left(z^2 f \phi'^2 + m^2 \phi^2 \right) dz$$

  - Complex scalar (probe limit)

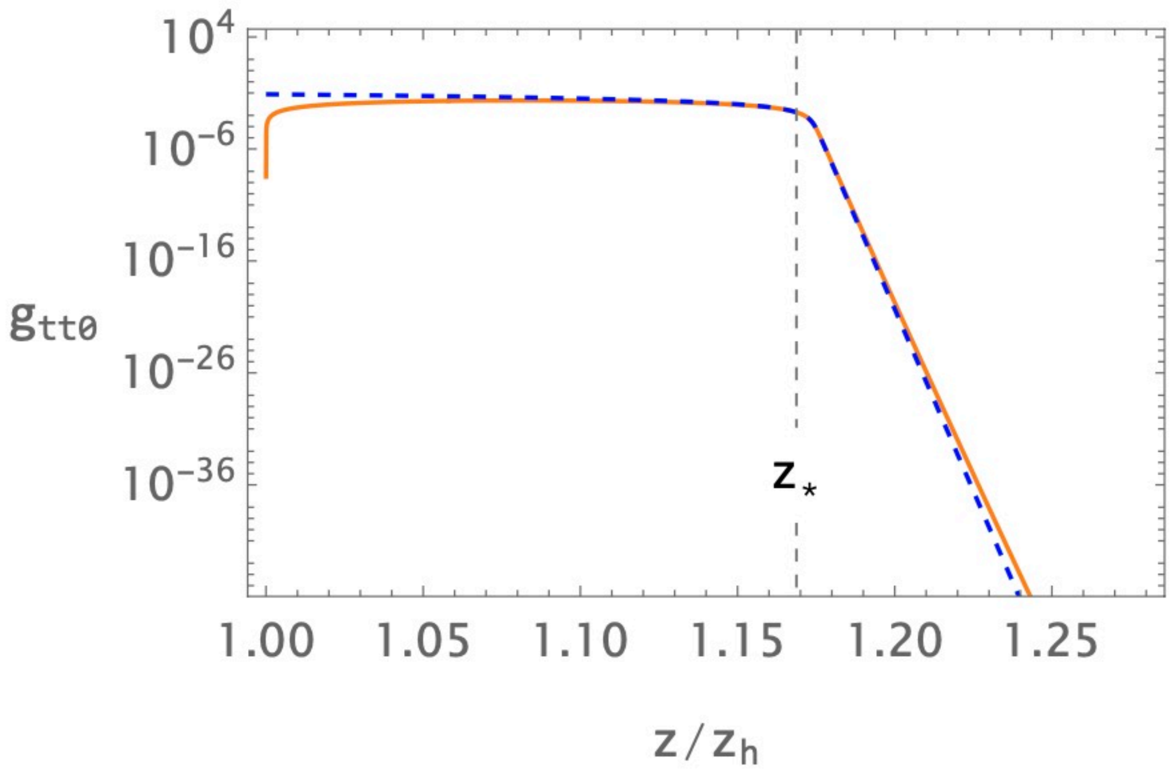
$$\phi \sim \phi_h \frac{\pi \csc(c\pi)}{\Gamma(a)\Gamma(b)\Gamma(2-c)} \cos\left(\frac{\omega - n\Omega_i}{2\kappa_i} \log(\tilde{z}) \right) \Rightarrow T_{VV} \sim \frac{1}{V^2}$$

- For relevant deformations, inner horizon never form

ER bridge collapse

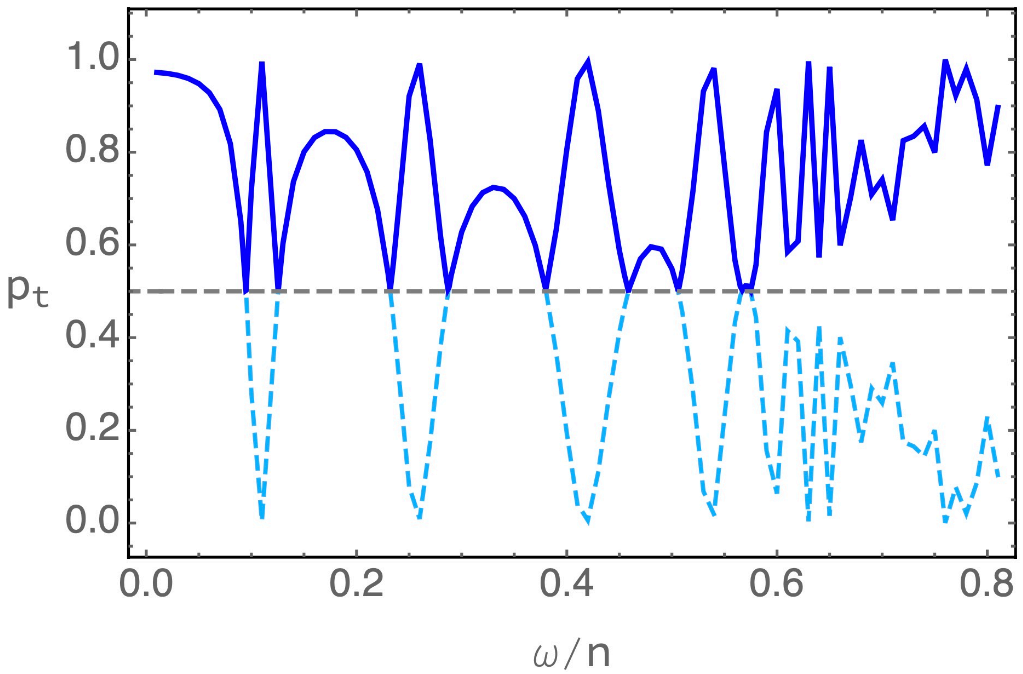


Size of ER bridge $c_2^2|\delta z| \to g_{tt0} = e^{-(c_2/c_1)^2 \delta z}$

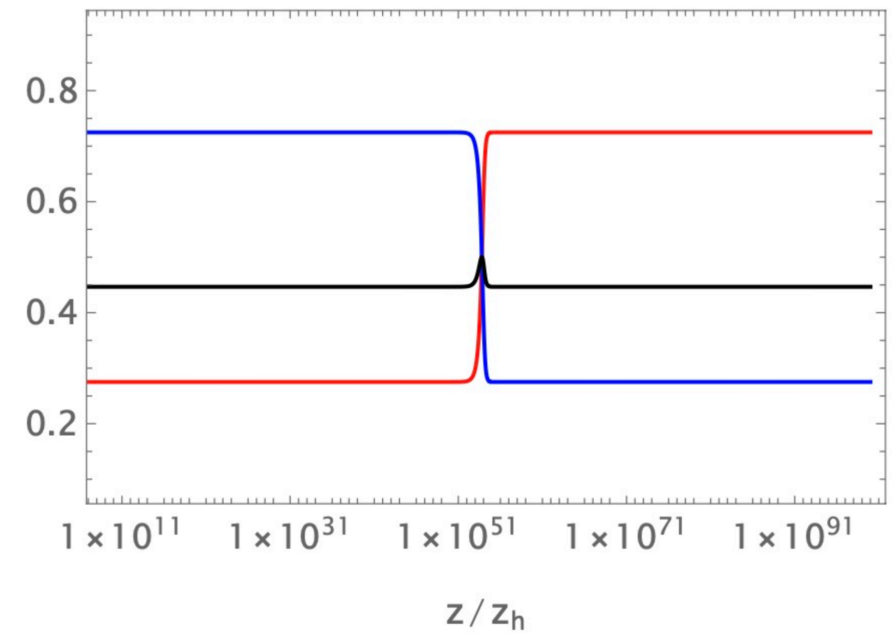


Singularity: Kasner transition

Only for complex scalar



at most one transition



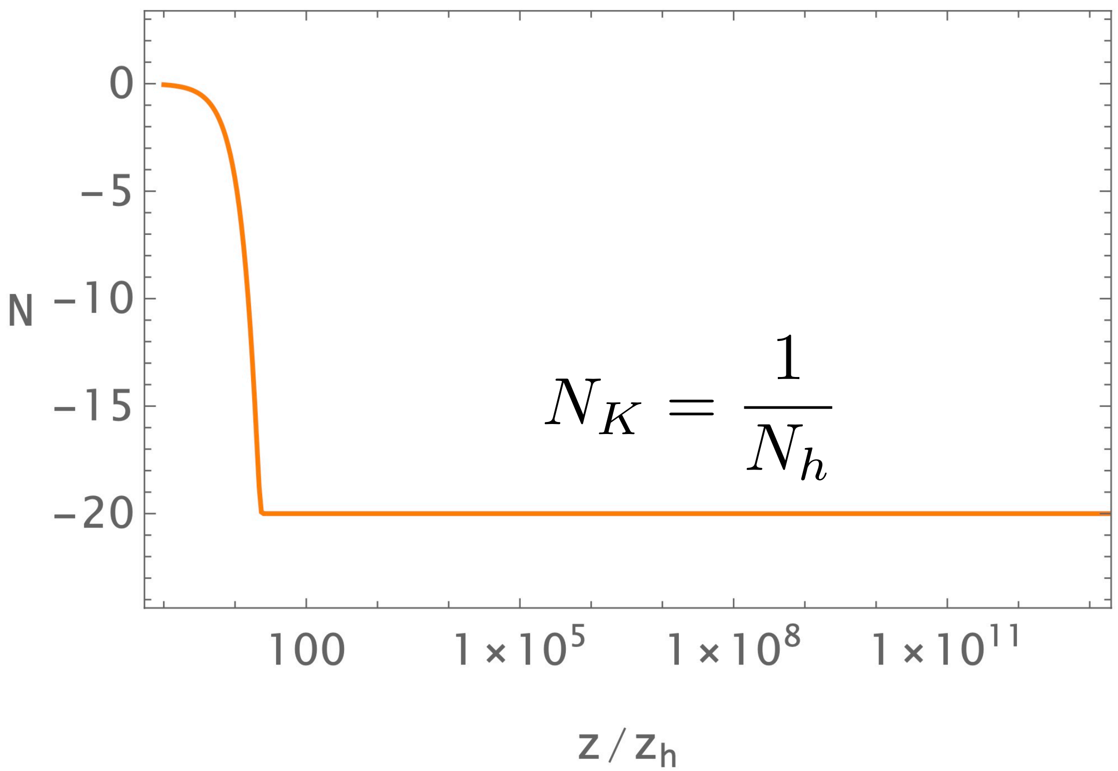
$p_t \rightarrow p_x \,, \quad p_x \rightarrow p_t \,, \quad p_\phi \rightarrow p_\phi$

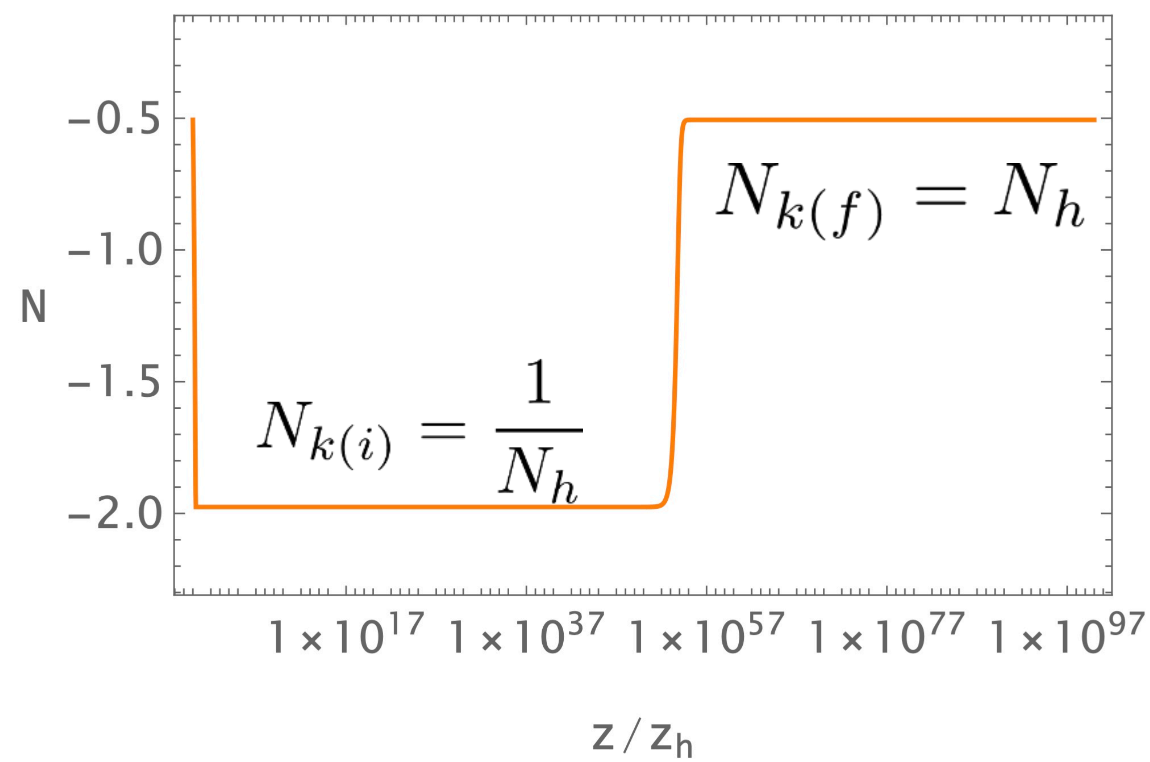
Relation of N

$$ds^2 = \frac{1}{z^2}\left(-f e^{-\chi} dt^2 + \frac{dz^2}{f} + (N dt + dx)^2\right)$$

real & complex φ : **No transition**

complex: **one transition**





$$\textbf{boundary}\ \frac{\omega}{n} = \textbf{horizon}\ N_h = \textbf{singularity}\ \frac{1}{N_k}\ or\ N_k$$

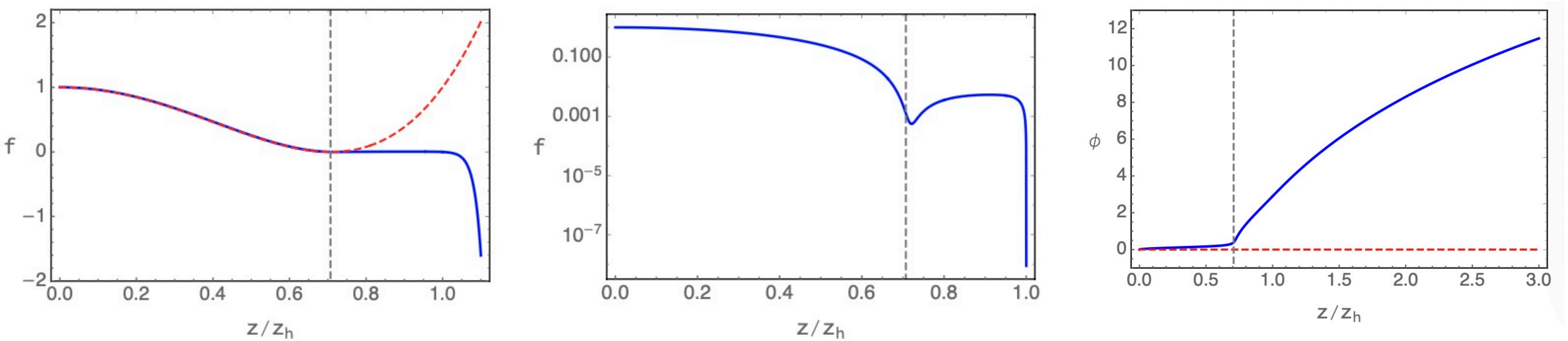
valid for general mass

not obvious in 5D

Comment (1)

- The low temperature solution: at any value of $\phi_0/\sqrt{J}$,

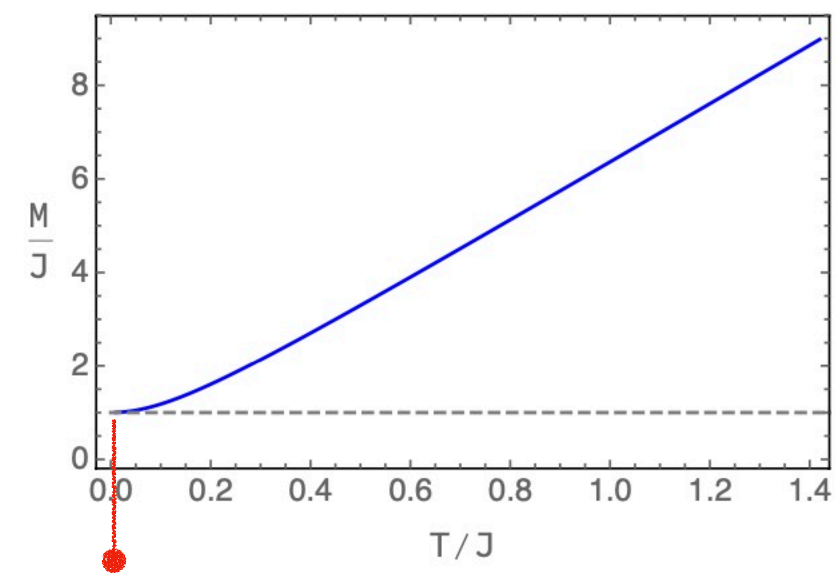
  $M/J \to 1$



- Boson star solution at zero temperature

  [Stotyn, Chanona, Mann, 2014]

- Dynamical formation?



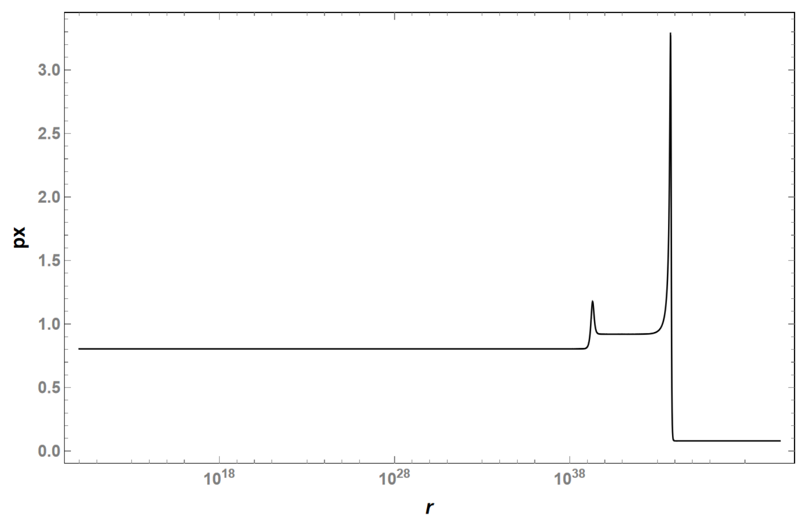
Comment (2)

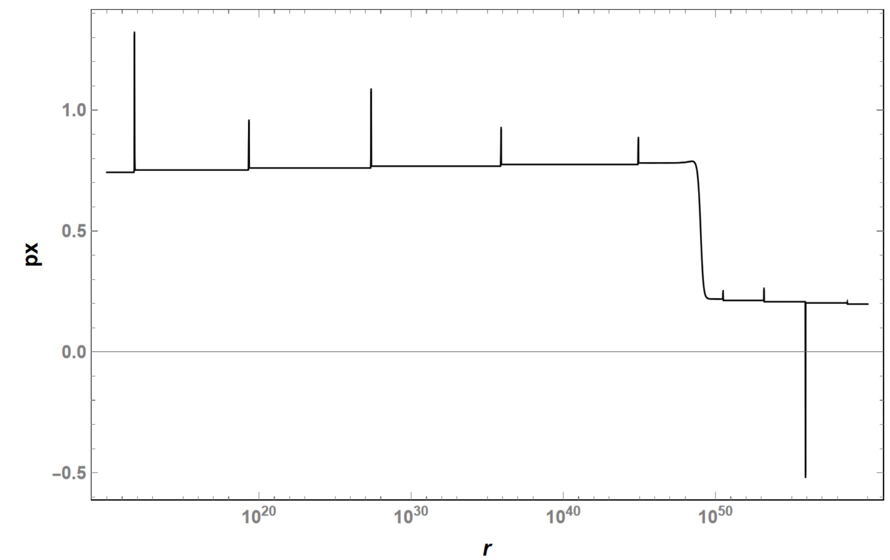
- More general scalar potential

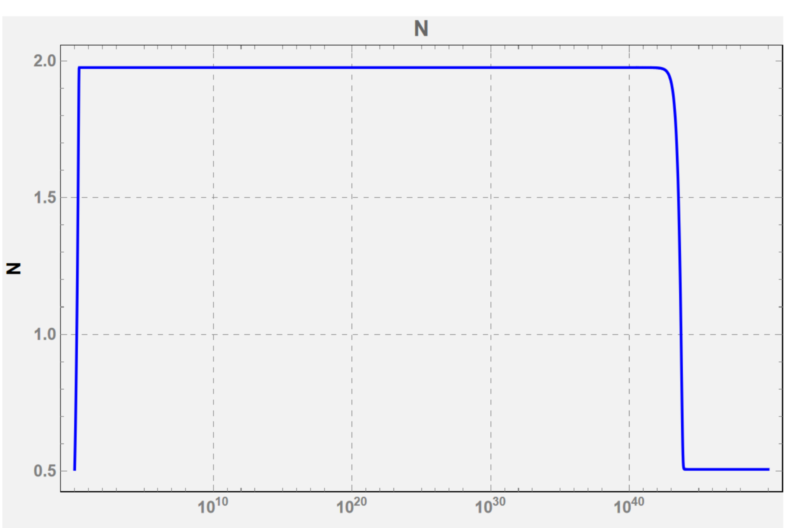
$$V(\varphi) = -m^2\varphi^*\varphi + \lambda(\varphi^*\varphi)^2 + a_n e^{(\varphi^*\varphi)^n}$$

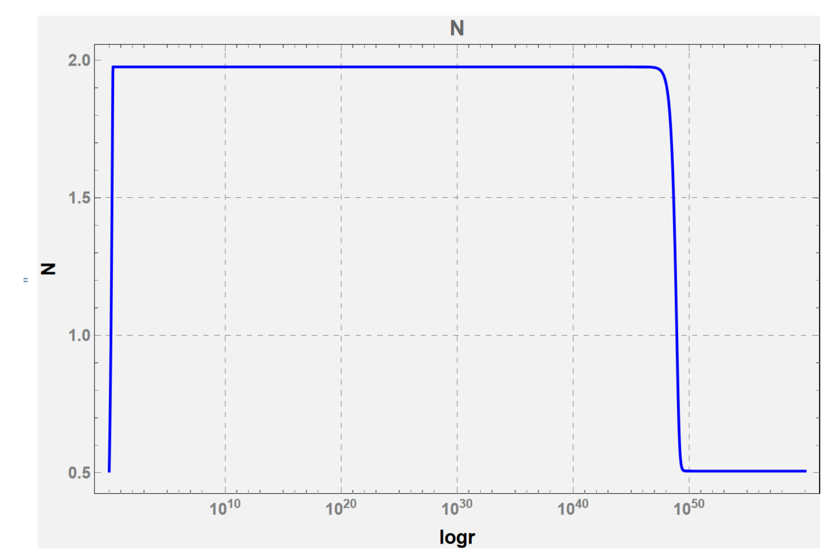
(1) effects of rotation

(2) Kasner transitions vs inversion









[Gao, Liu, Zhao, in progress]

Motivation (2)

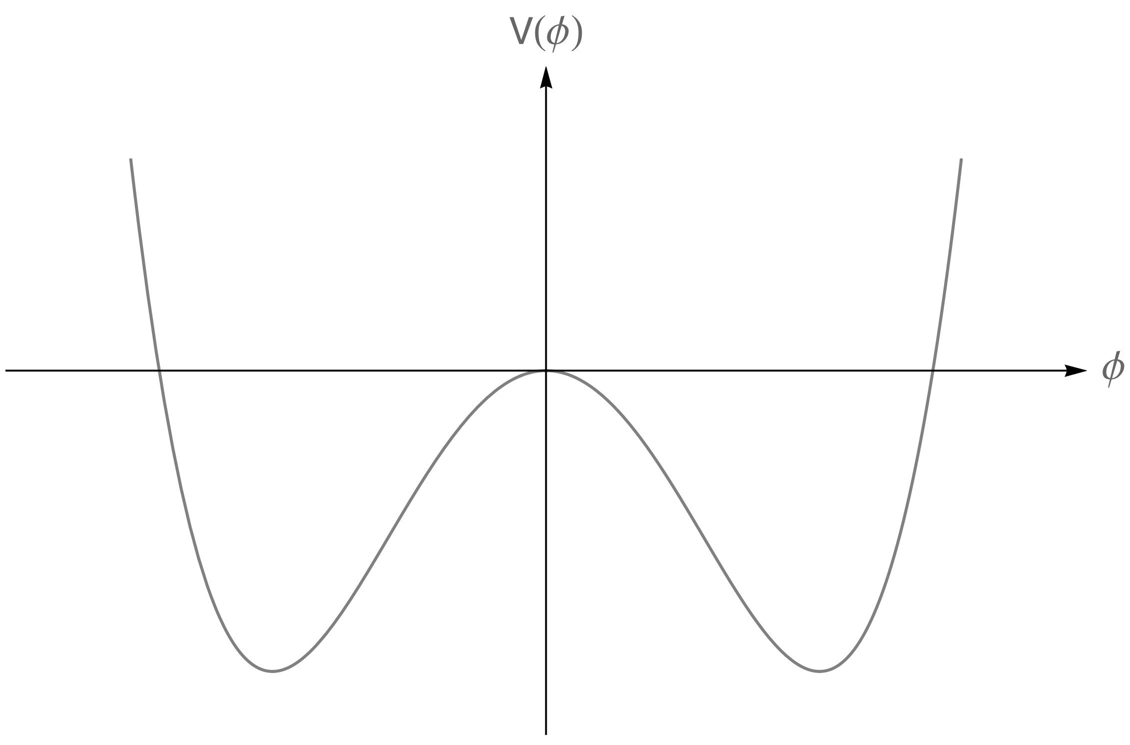
- For specific black holes, what are the black hole internal structures?

- Any possible connection between the physics inside and outside the horizon?

  - What happens to the singularities during the black hole phase transitions?

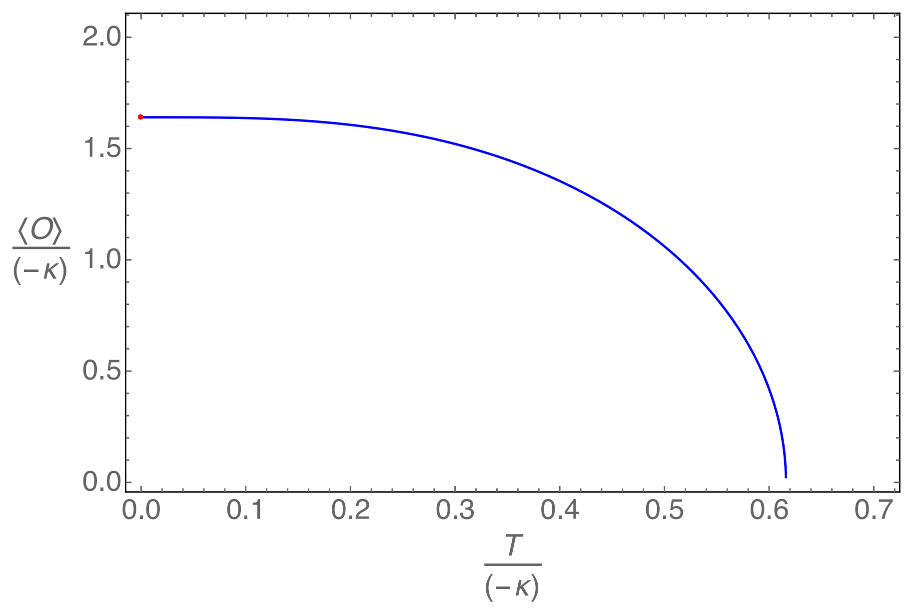
Holographic 2nd order phase transition

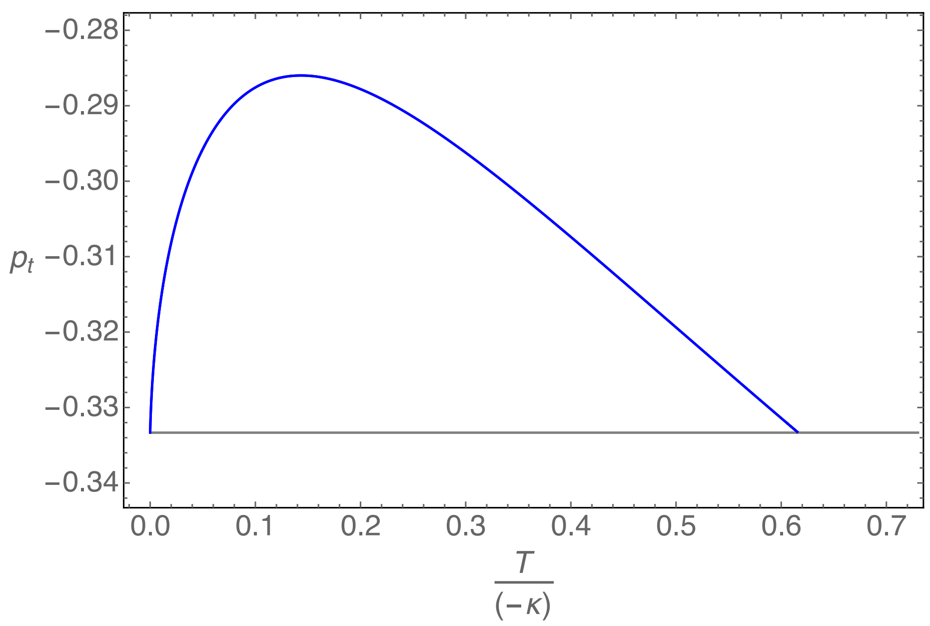
Einstein-scalar gravity + double trace deformation (mixed bnd)



[YL, Lyu, Raju, 2021]

Holographic 2nd order phase transition



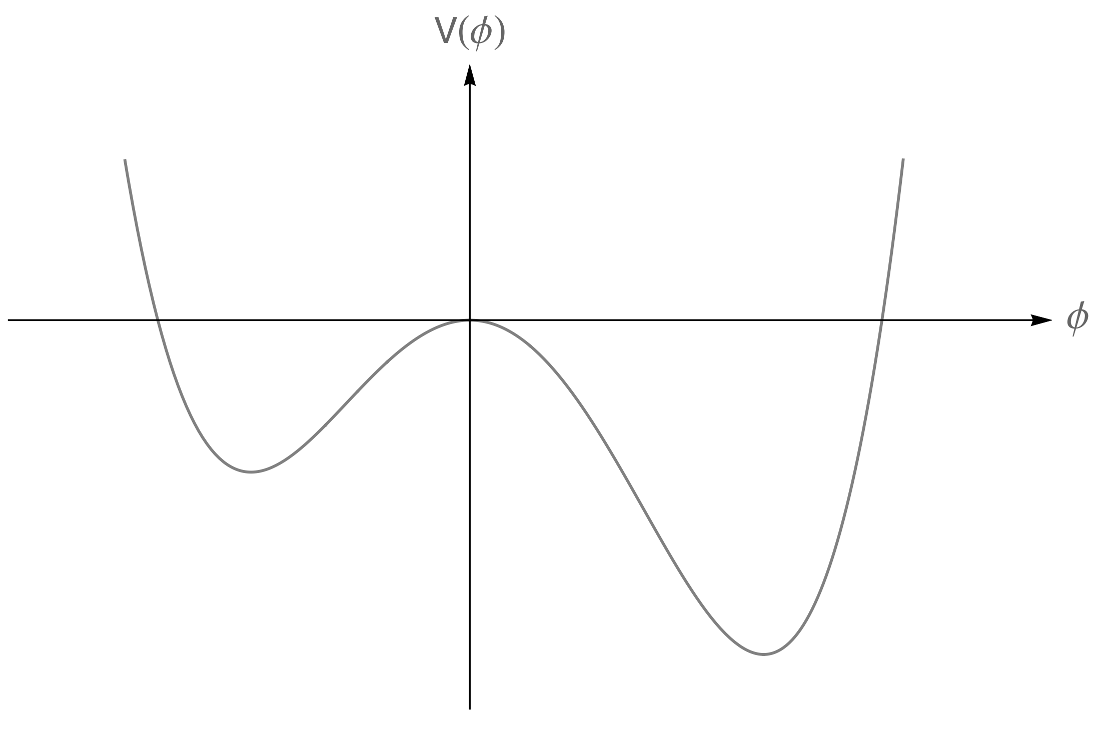


$$\phi \propto \left(1 - \frac{T}{T_c}\right)^{1/2} \propto p_\phi$$

$$p_t + \frac{1}{3} \propto 1 - \frac{T}{T_c}$$

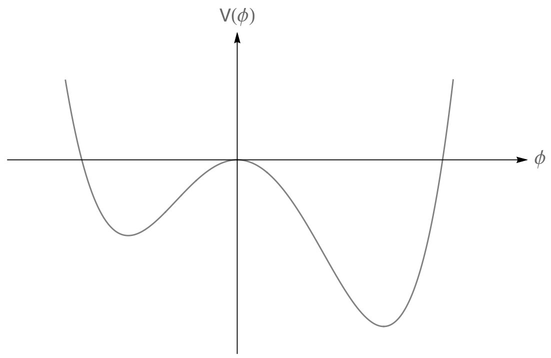
Holographic 1st order phase transition

Einstein-scalar gravity + double trace deformation (mixed bnd)

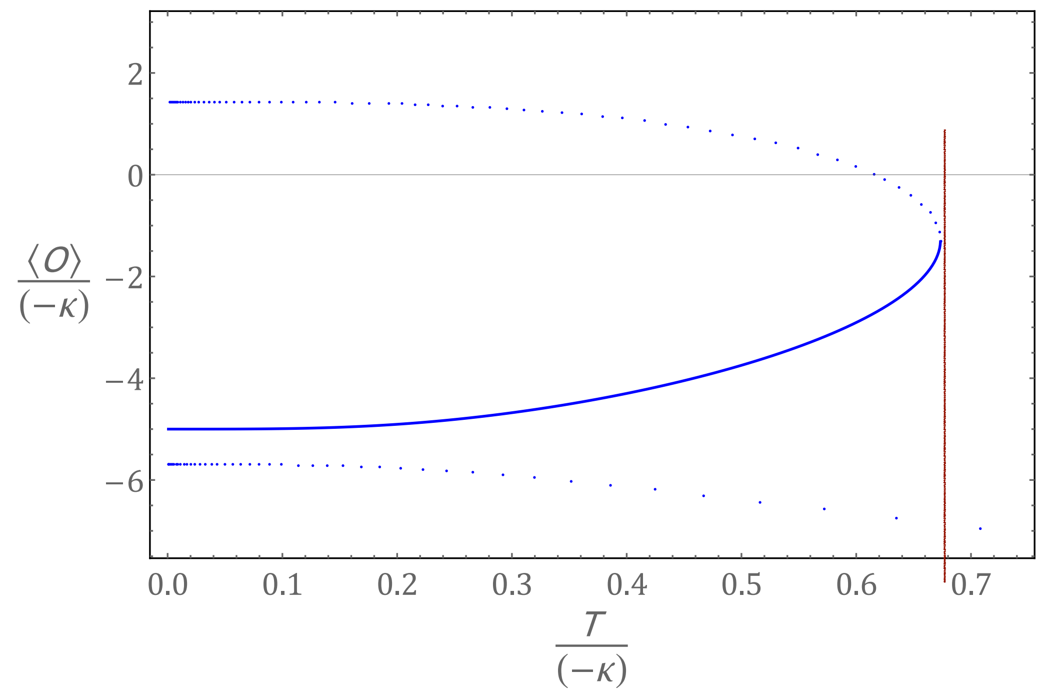


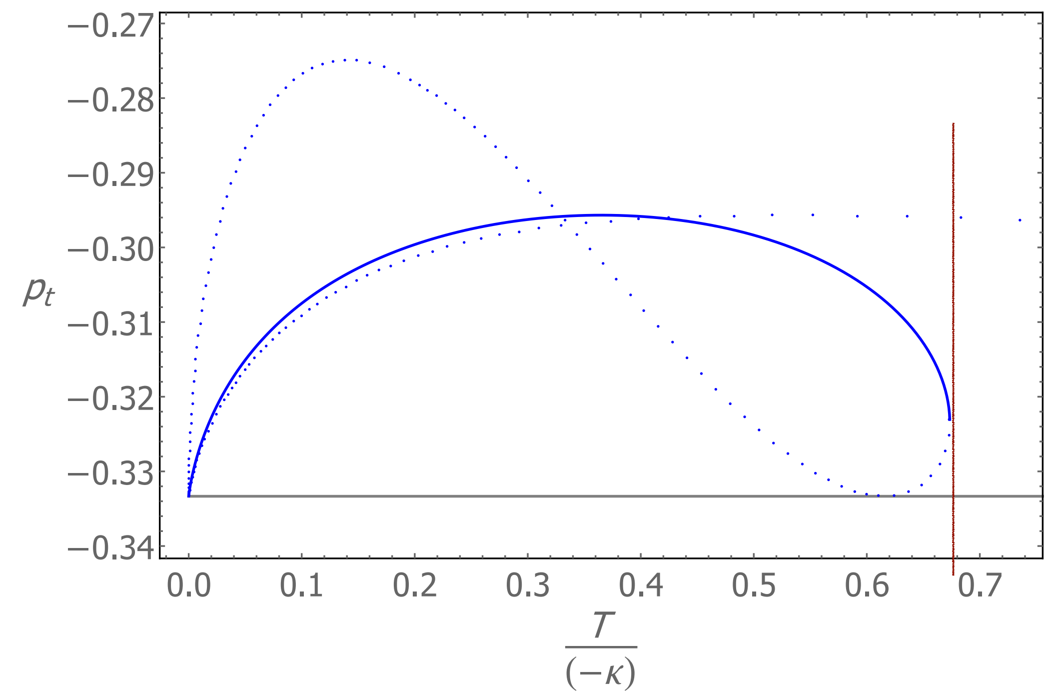
[YL, Lyu, Raju, 2021]

Holographic 1st order phase transition



- first order phase transition





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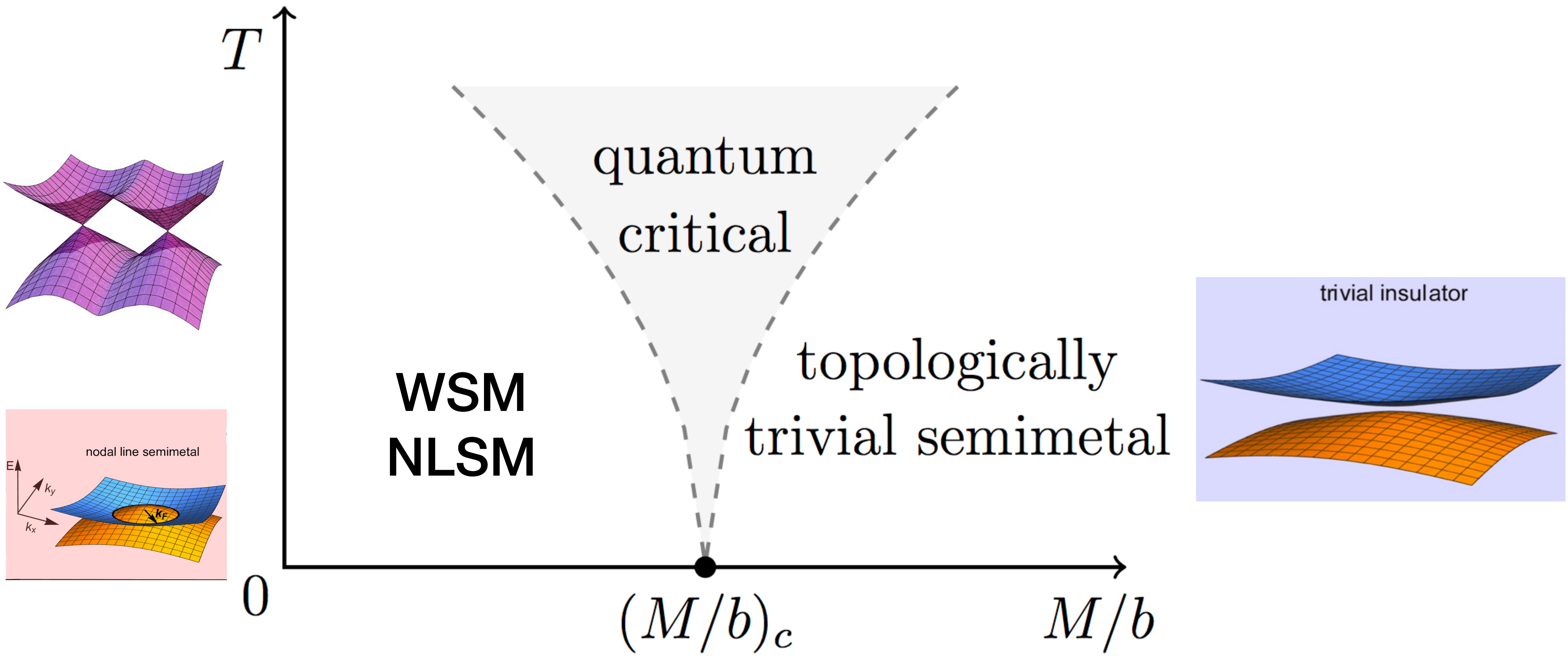
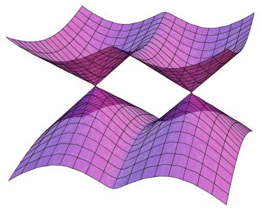
Topological phase transitions

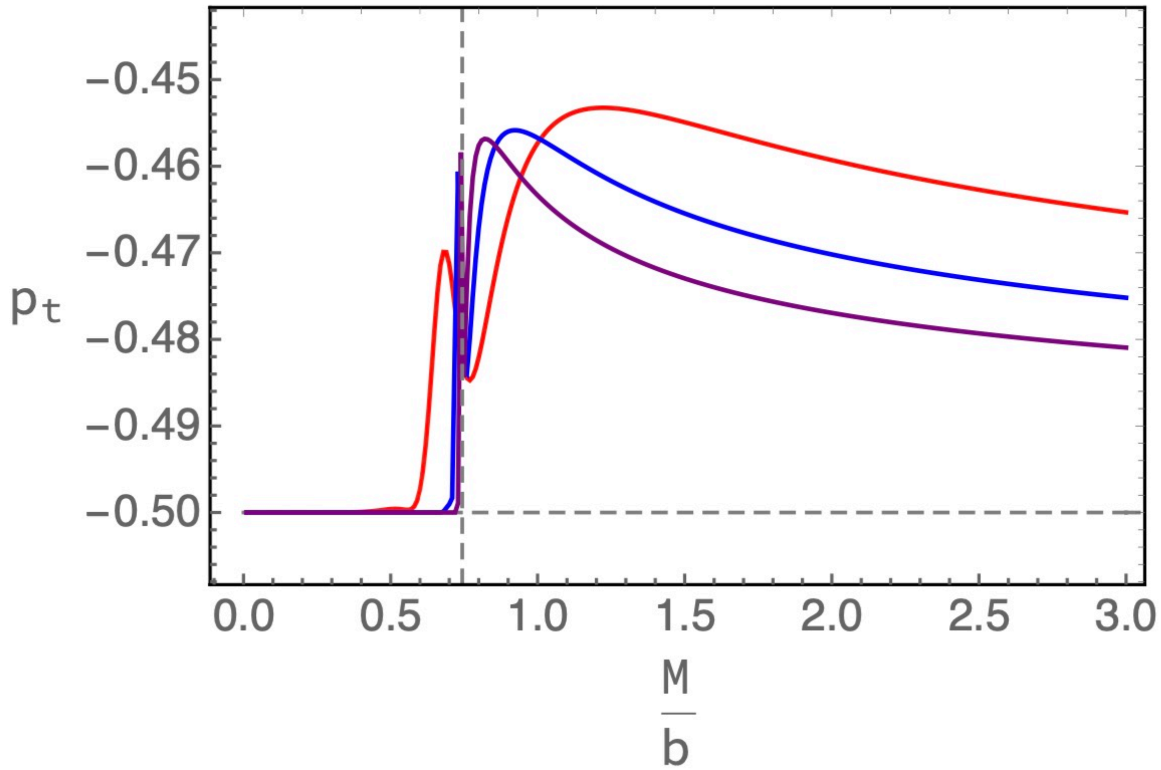


[Landsteiner, YL, Sun, 2016; YL, Wu, 2021]

Holographic topological phase transition



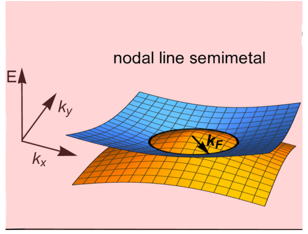
- In holographic WSM, at low temperature, the Kasner exponent is constant



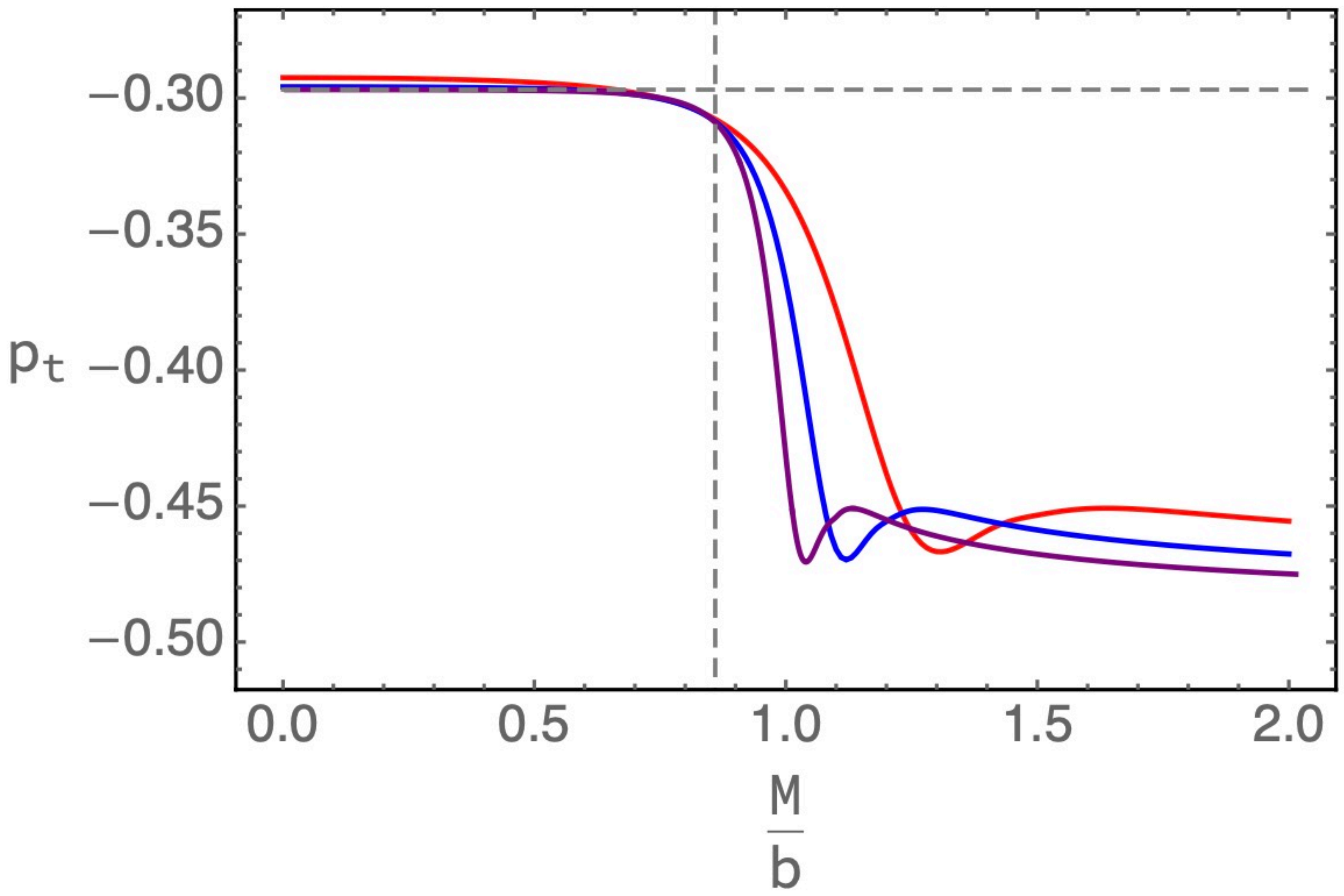
- oscillation of the scalar field in the top phase inside the horizon at low temperature

[Gao, YL, Lyu, 2023]

Holographic topological phase transition



- In holographic NLSM, at low temperature, the Kasner exponent is (almost) constant

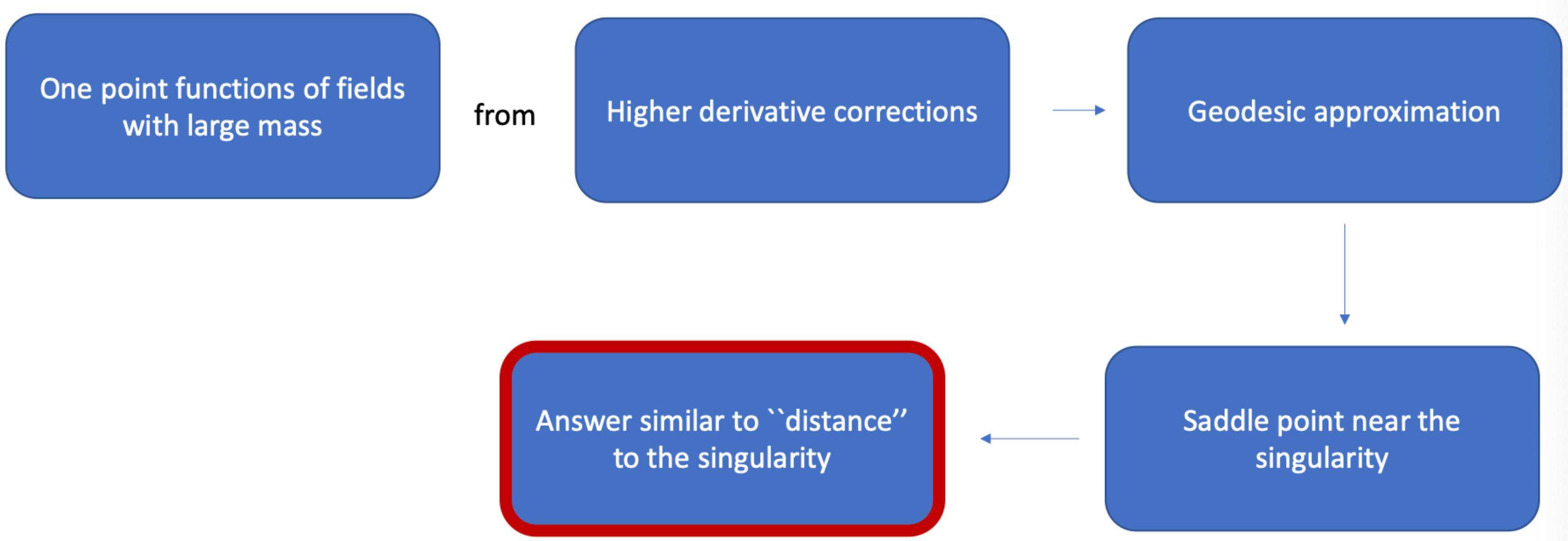


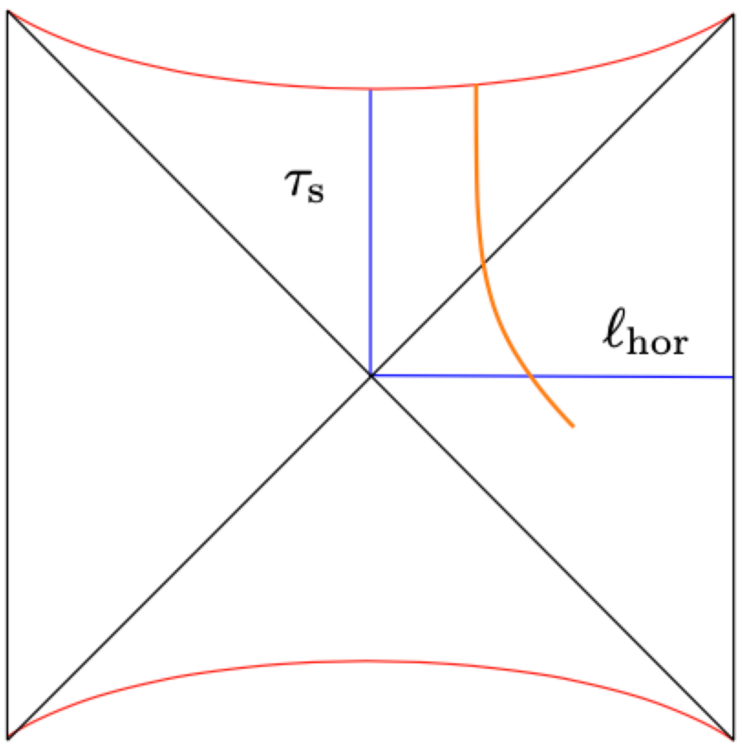
Probes of singularity by timelike geodesics

- the proper time τ_s from the event horizon to singularity

$$\langle O \rangle \sim (\text{powers of } m) \times \exp\left[-im\tau_{\rm s} - m\ell_{\rm hor}\right] \ , \qquad \text{for} \qquad {\rm Im}(m) < 0 \, ,$$

$$S = \frac{1}{16\pi G_{\rm N}} \int \left[\frac{1}{2} (\nabla\varphi)^2 + \frac{1}{2} m^2 \varphi^2 + \alpha \varphi W^2 \right]$$

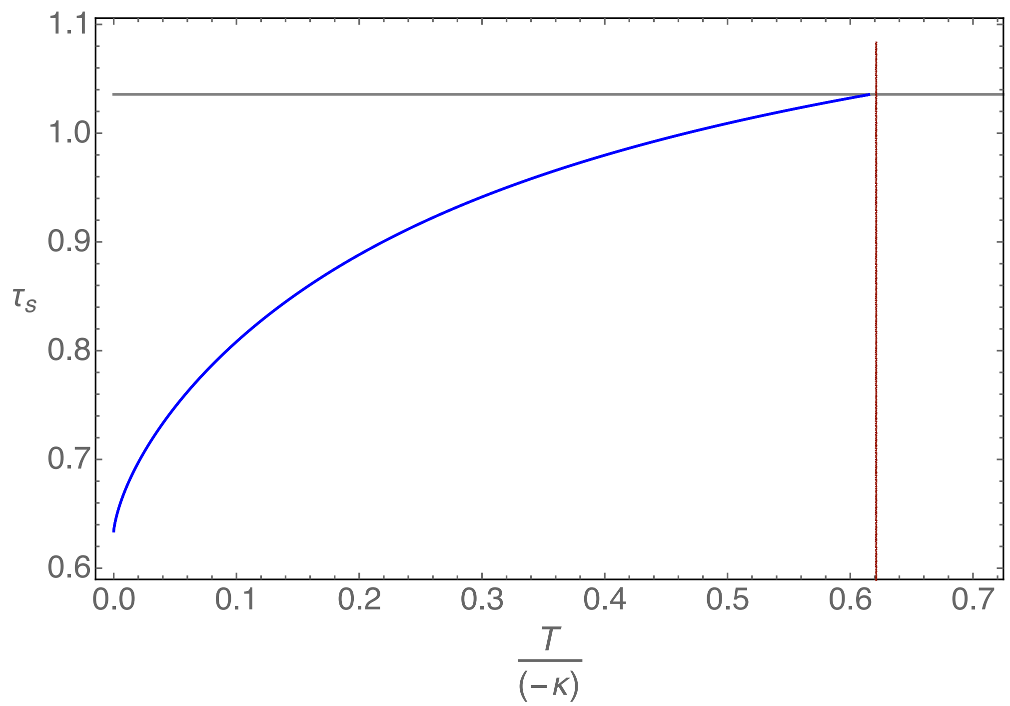


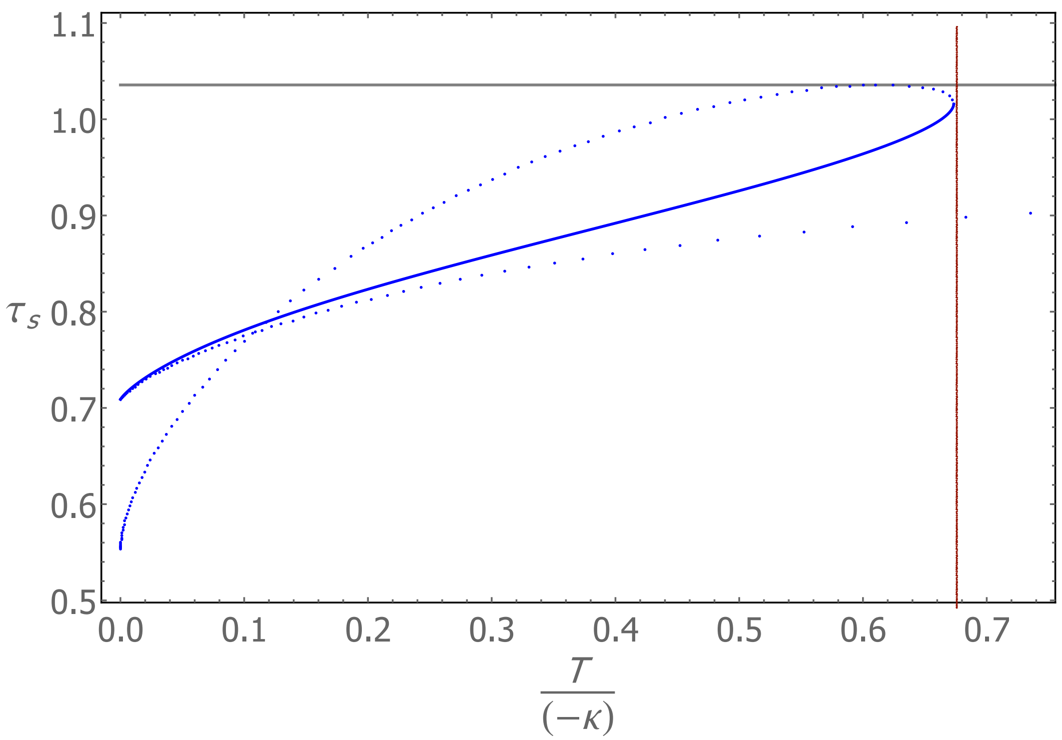


[Grinberg, Maldacena, 2020]

Probes of singularity by timelike geodesics

- during 2nd&1st order phase transition

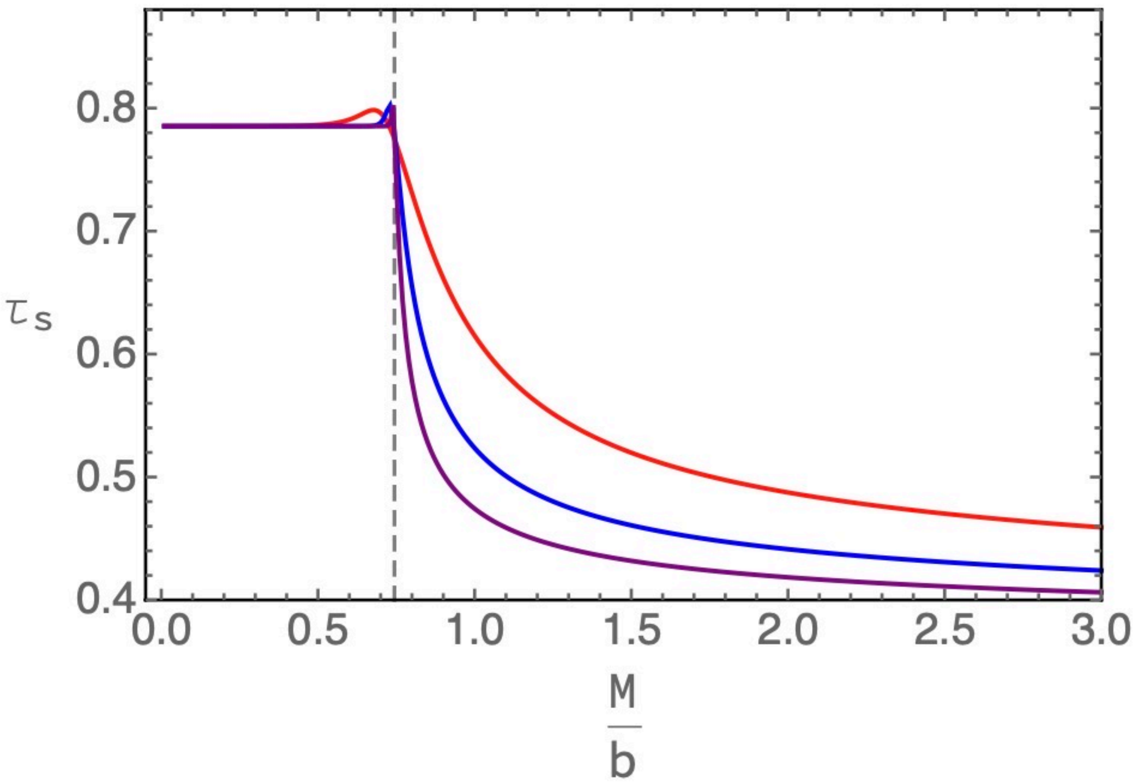




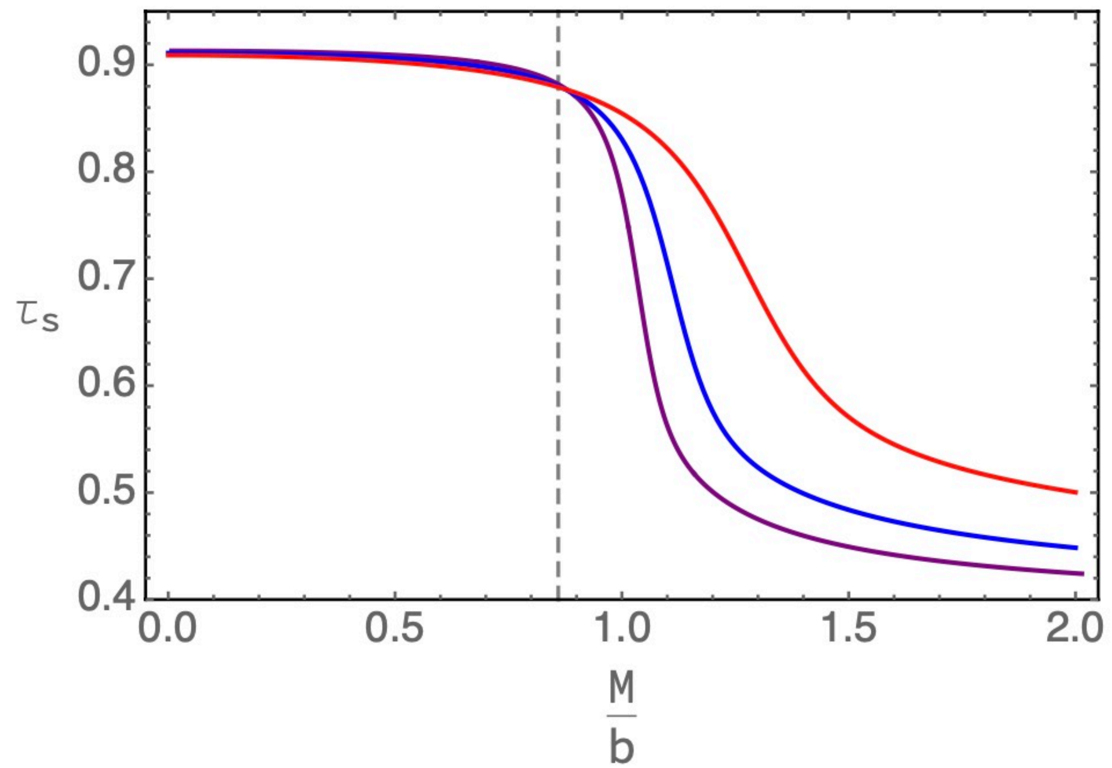
- Order parameter: VEV of the scalar operator

Probes of singularity by timelike geodesics

- during topological phase transition



Holo WSM



Holo NLSM

- Order parameter: AHE (for WSM), …

Summary

- The internal structure of 3D hairy rotating black holes: no inner horizon, ER bridge collapse (oscillation), Kasner inversion

Summary

- The internal structure of 3D hairy rotating black holes: no inner horizon, ER bridge collapse (oscillation), Kasner inversion
- The physics inside the black hole horizon is linked to the physics outside (via Euclidian gravity)
- During the 2nd order phase transition, the Kasner exponents are continuous while their derivatives w.r.t T are discontinuous
- During the 1st order phase transition, the Kasner exponents are discontinuous
- During the topological phase transition, the Kasner exponents are (almost) constant

Future work

- “relation of N” in other hairy rotating black holes?
- The black hole interiors during the black hole dynamical formation?
- evaporating black holes?
- Are the properties of singularities during the phase transition universal?
- dynamical phase transitions?
- Other probes of singularities
- …

Thank you!

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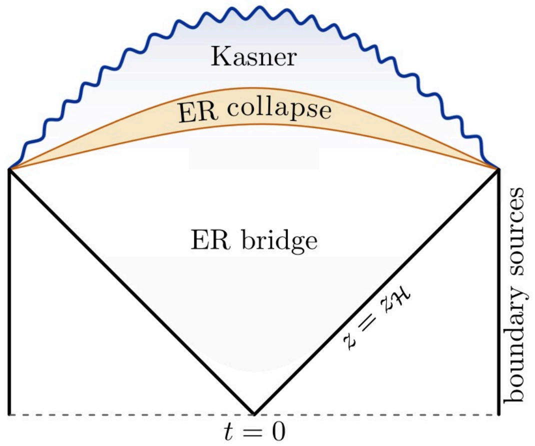
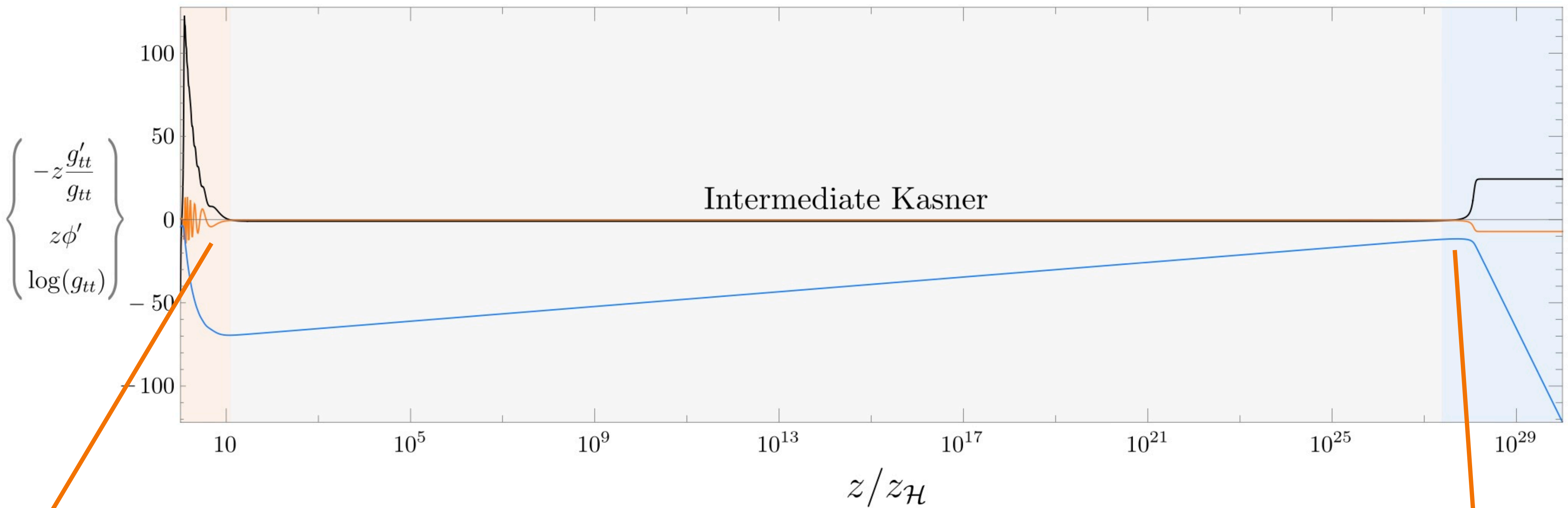
Universality



- the inside of a holographic superconductor



Oscillation of the scalar field is not necessarily connected to the collapse of the ER bridge.
[YL, Lyu, 2022]

(1) The Kasner transition might occur infinite times;
(2) Oscillation might exist after Kasner transition;

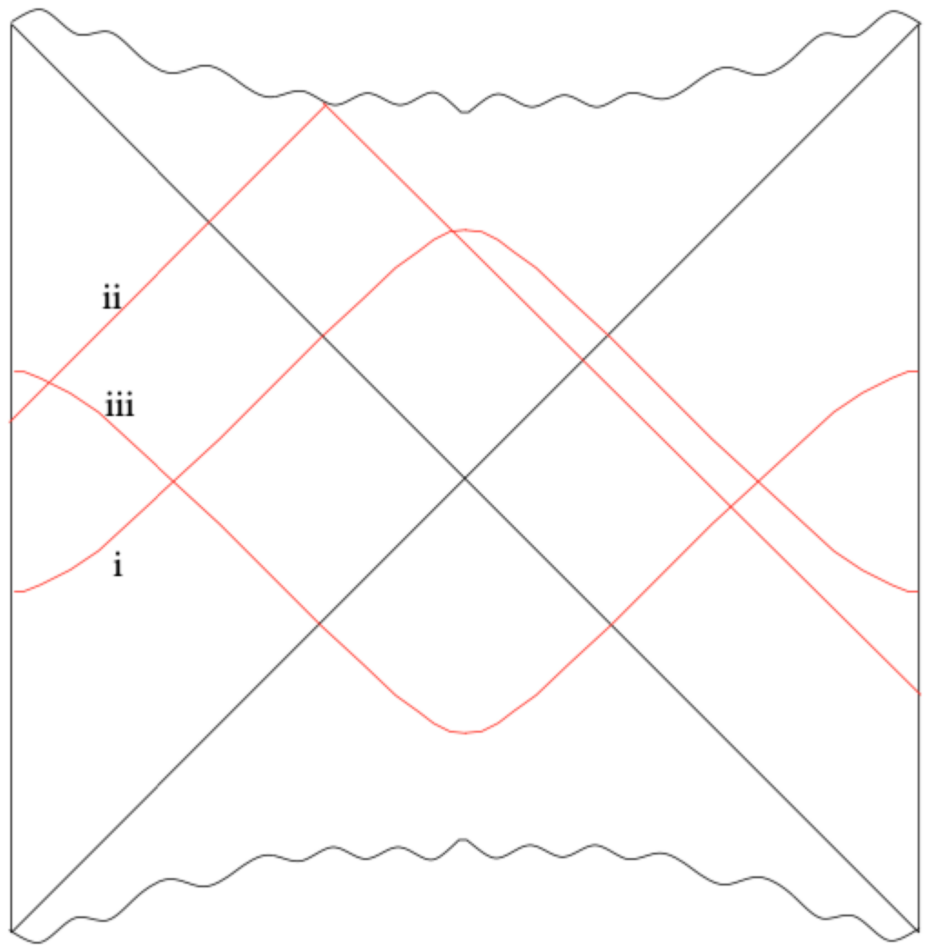
Any connection between fields inside and outside the horizon?
Any connection to thermalization?

[Hartnoll, Horowitz, Kruthoff, Santos, 2021]

Probes of singularity (I)

- For large dimension of the operator in field theory, the two point correlator is dominated by spacelike geodesics

$$G(\omega) \equiv \int_{-\infty}^{+\infty} dt e^{i\omega t} \left\langle \mathcal{O}\left(-i\frac{\beta}{2}\right) \mathcal{O}(t) \right\rangle \qquad \omega = iE$$



- Radial conserved quantity of the geodesic: E,
- In large E limit, the geodesic can approach the singularity

$$L = 2\log\frac{2}{E} + \frac{l_1}{E} + \frac{\alpha^2}{2}\frac{1}{E^2} + \frac{l_3}{E^3} + \frac{1}{2}\left(2m_T - 4\alpha\beta + 2\alpha^3\lambda_3 - 2\alpha^3\lambda_3\log 2\right)\frac{\log E}{E^3} - \frac{1}{2}\alpha^3\lambda_3\frac{\log^2 E}{E^3} + l' E^{\frac{1}{p_t}} \,,$$

[Fidkowski, Hubeny, Kleban, Shender, 2003; Festuccia, H. Liu, 2005]