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Interpreting PTA data with bouncing cosmology

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Based on: 2408.06582 (with Mian Zhu, submitted to Phys. Rev. D)

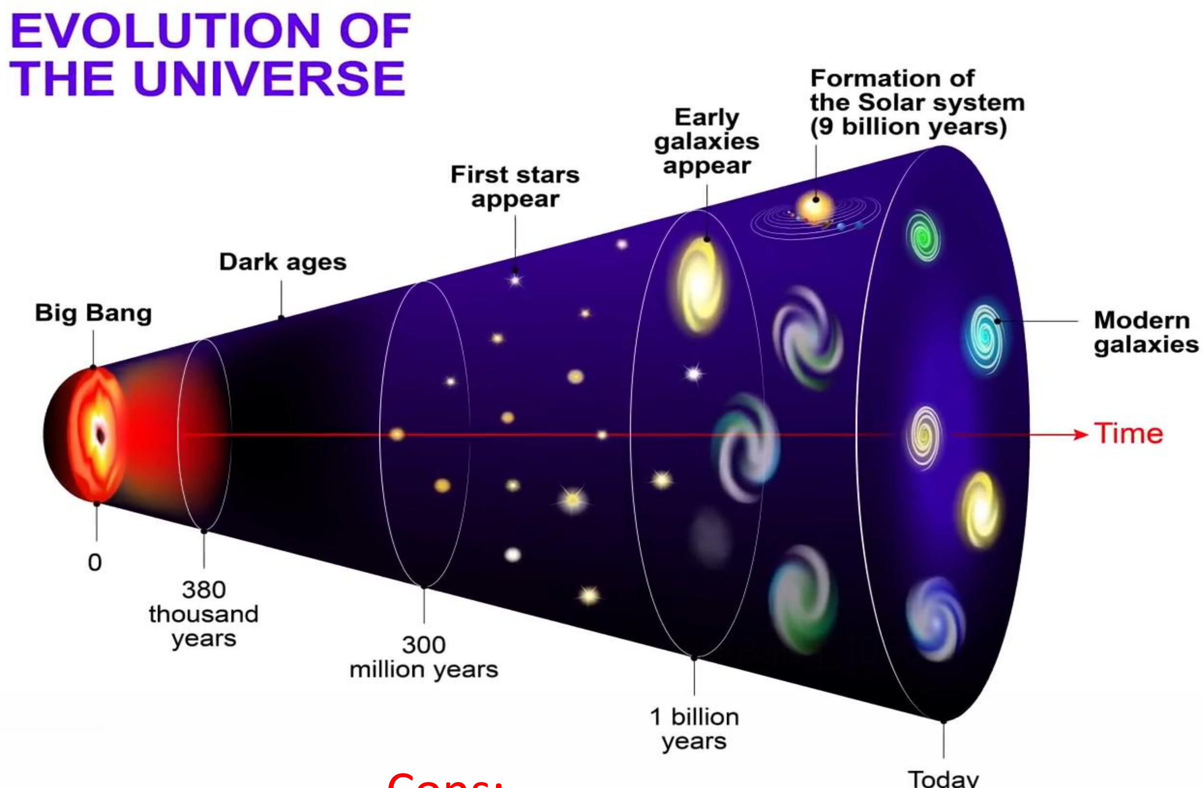
See also: 1303.2372, 1501.03568, 1610.03400, 1701.04330, 2307. 16211 (by M. Zhu et al.)

The Big Bang theory

Ralph Alpher

Hans Bethe

George Gamow



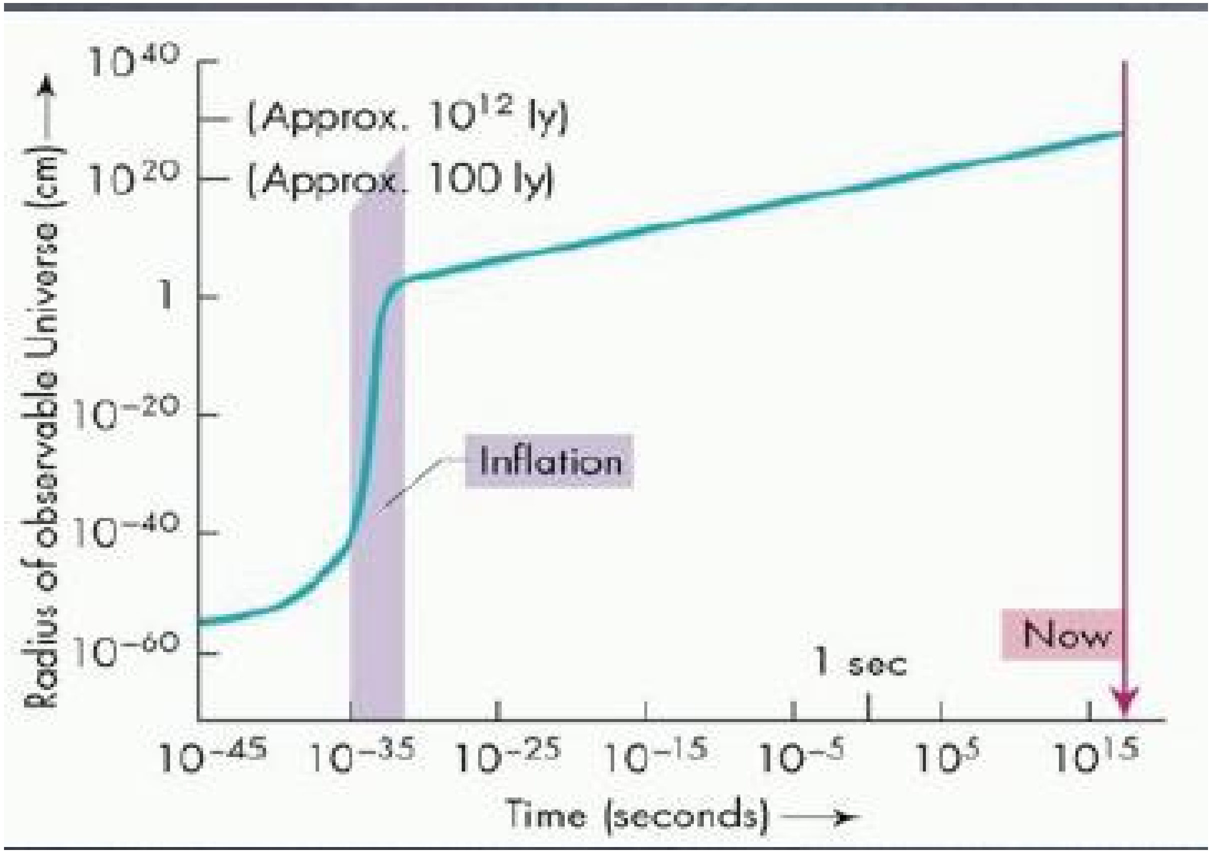
Pros:

- ✓ The age of galaxies
- ✓ The redshift of the galactic spectrum
- ✓ The He abundance
- ✓ The prediction of CMB temperature

Cons:

- ? Flatness problem
- ? Horizon problem
- ? Unwanted relics problem
- ? Singularity problem

The Inflationary Scenario



Horizon problem

Flatness problem

Unwanted relics problem

Singularity problem

Alan Guth et al., 1980

The Singularity Problem

SINGULARITY THEOREM

The universe will meet a singularity when

(1) it is described by General Relativity;

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + L_m\right]$$

(2) it satisfies Null Energy Condition;

$$T_{\mu\nu}n^\mu n^\nu = (\rho + P) \geq 0$$

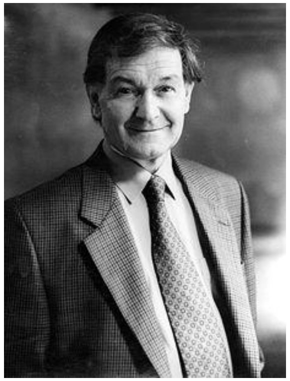
where at finite time point

$$a(t) \to 0,\ \rho(t) \to \infty$$

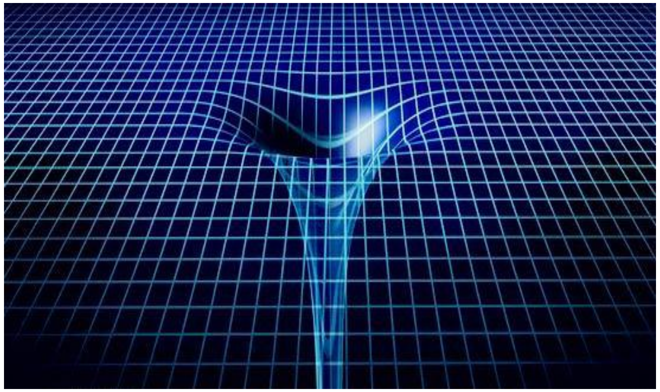
for any null vector n^μ: $n_\mu n^\mu = 0$

Hawking et al., 1973; Borde et al., 1994

S. Hawking



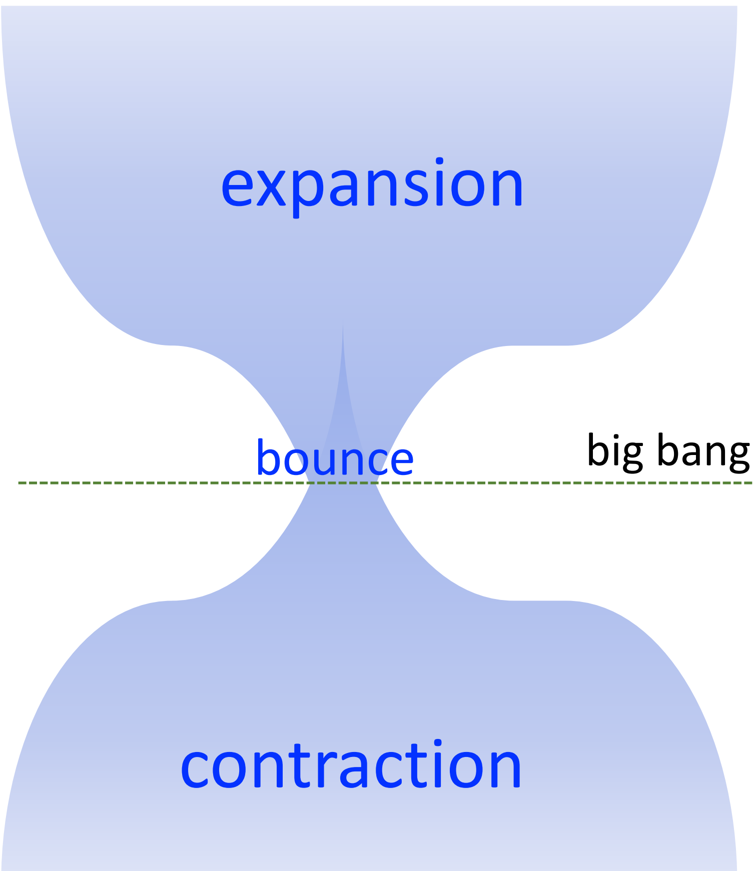
R. Penrose



Spacetime singularity

Non-Singular Cosmology

4D BASED SCENARIOS



Bounce Scenario

<u>*Contraction:*</u> $H < 0$

<u>*Expansion*</u>*:* $H > 0$

<u>*Bouncing Point*</u>*:*

$$H = 0 \; \dot{H} > 0 \Rightarrow \rho + P < 0$$

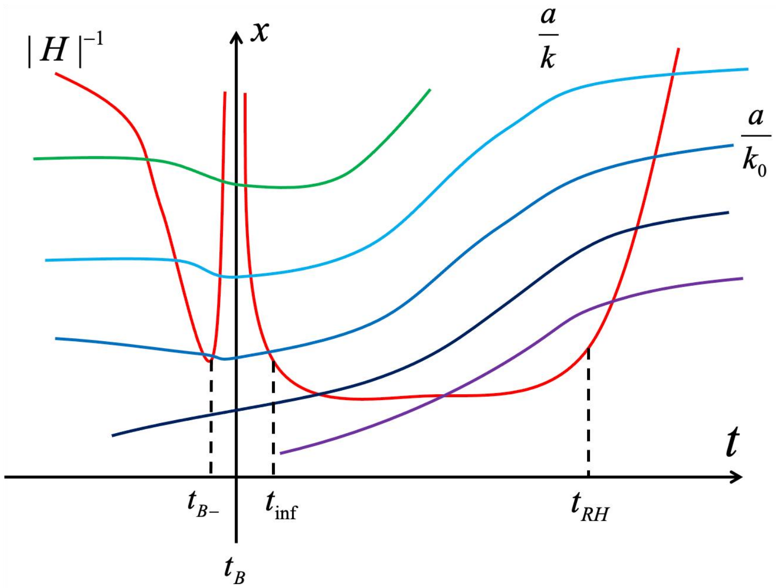
Violating the Null Energy Condition (NEC)!

Cai, **TQ**, Piao, Li, Zhang, JHEP 2007

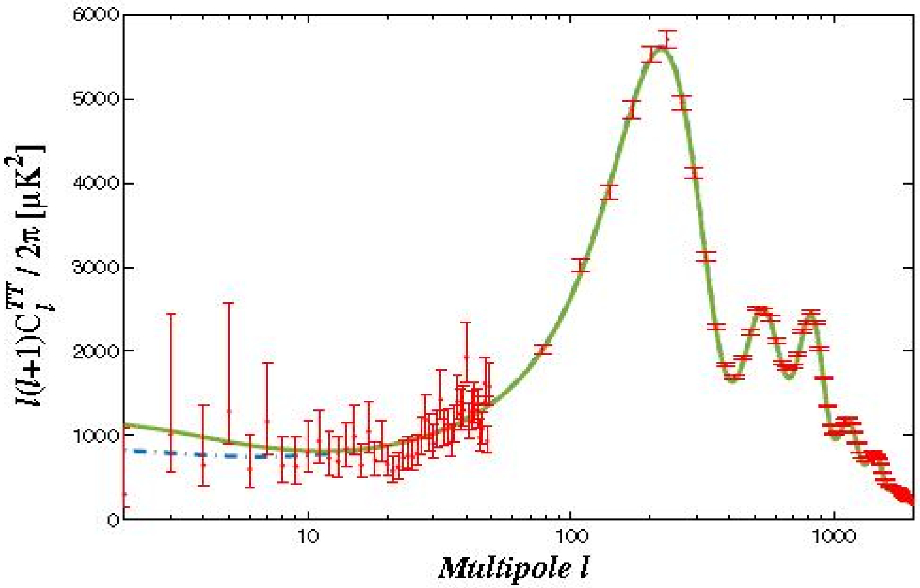
How to Test Bouncing Process?

LARGE SCALE TEST: CMB POWER SPECTRUM SUPPRESSION

Perturbations vs. bounce horizon



CMB low-l suppression

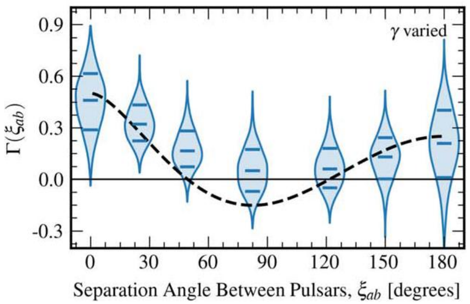


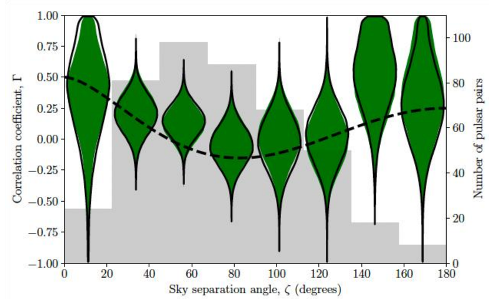
TQ, Y. T. Wang, JHEP 2015

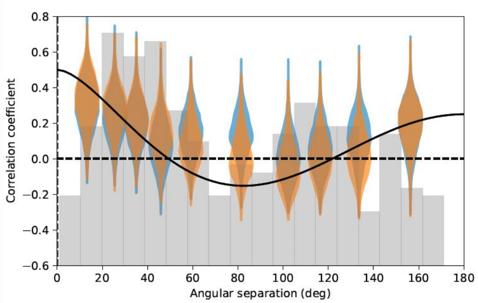
Can bounce models also have signals on small scales?

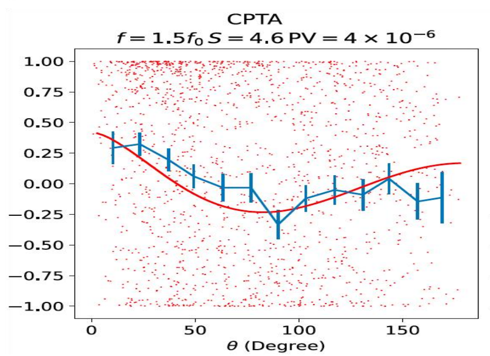
Small-scale test: Gravitational Waves

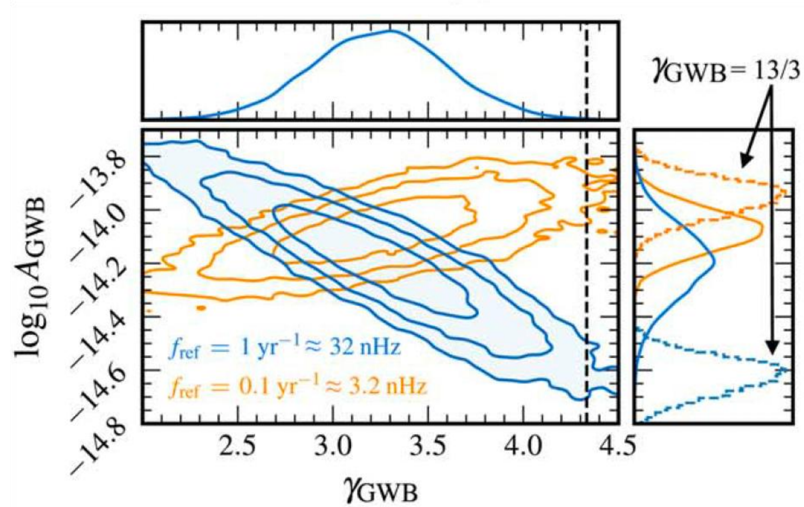
THE RECENT PTA DATA: ($f \sim 10^{-9}$Hz, $k \sim 10^{-6}$Mpc^{-1})











$$\Omega_{\rm gw}(f) = \frac{2\pi^2 A^2}{3H_0^2}\frac{f^{5-\gamma}}{{\rm yr}^{\gamma-3}}$$

$$A = 6.4^{+4.2}_{-2.7} \times 10^{-15}, \gamma = 3.2 \pm 0.6 (2\sigma)$$

S. Vagnozzi, JHEAp 2023

NANOGrav Collaborations, Astrophys. J. Lett., 2023
PPTA Collaborations, Astrophys. J. Lett., 2023
EPTA & INPTA Collaborations, Astron. Astrophys., 2023
CPTA Collaborations, Res. Astron. Astrophys., 2023

Possible for bounce models?

Bounce model for PTA data

OUR MODEL: BASIC FORMULATION

Action: $$S = \int d^4x\sqrt{-g}\left[M_p^2\frac{R}{2} + K(\phi, X) + L_{EFT} + L_\sigma\right]$$

where $$K(\phi, X) = [1 - g(\phi)]M_p^2 X + \beta_2 X^2 - M_p^4 V(\phi)$$

$$g(\phi) = \beta_1\left[\frac{1+\tanh\lambda_1(\phi-\phi_-)}{2}\right]\left[\frac{1+\tanh\lambda_2(\phi_+-\phi)}{2}\right] + \frac{1+\tanh\lambda_4(\phi-\phi_+)}{2} \qquad (\phi_- < \phi_+)$$

$$V(\phi) = -2V_0 e^{\sqrt{2/q}\phi}\frac{1-\tanh\lambda_3(\phi-\phi_-)}{2}$$

Approximate behavior of $K(\phi, X)$ in limits of $\phi \to -\infty$ and $\phi \to \infty$:

$$\lim_{\phi\to-\infty} K(\phi, X) = X + 2V_0 e^{\sqrt{2/q}\phi} \qquad \lim_{\phi\to\infty} K(\phi, X) = X^2$$ ($w_\phi = 1/3$, radiation-like!)

Bounce model for PTA data

OUR MODEL: BASIC FORMULATION

The curvaton Lagrangian:

$$L_\sigma = -M_p^2 f^2 \Upsilon(\phi)(\partial\sigma)^2 \qquad \text{where} \qquad \Upsilon(\phi) = \frac{1+\phi e^{\sqrt{q/2}\phi}}{e^{-\sqrt{q/2}\phi} + \phi^2 e^{\sqrt{q/2}\phi}}$$

which gives rise to the energy density:

$$\rho_\sigma = \frac{M_p^2 f^2}{2}\Upsilon(\phi)\dot{\sigma}^2$$

and EoM:

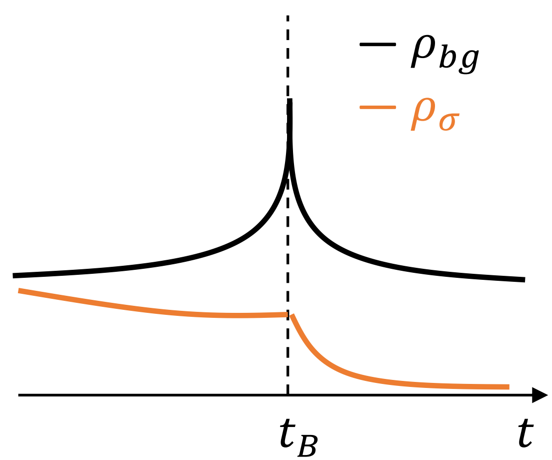
$$\ddot{\sigma} + \dot{\sigma}\frac{d}{dt}\left[\ln\left(a^3\frac{M_p^2 f^2}{2}\Upsilon(\phi)\right)\right] = 0 \Longrightarrow a^3\Upsilon(\phi)\dot{\sigma} = \text{const.}$$

Back-reaction check:

$\rho_{bg} \sim t^{-2}$

Contracting phase: $\rho_\sigma \sim e^{\sqrt{q/2}\phi}\dot{\sigma}^2 \sim (t_* - t)^2$

Expanding phase: $\rho_\sigma \sim \phi^{-1}\dot{\sigma}^2 \sim (t - t_*)^{-5/2}$



Bounce model for PTA data

BACKGROUND EVOLUTION

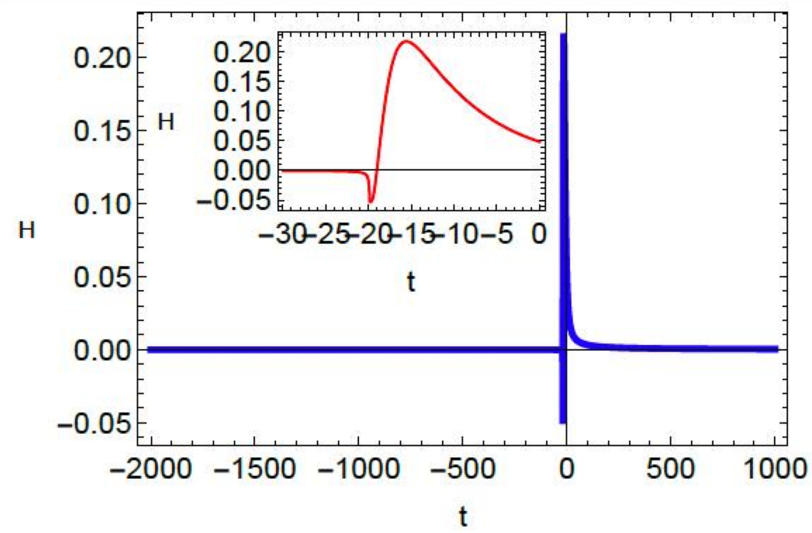
Scale factor:

$$a(\tau) = \begin{cases} a_B e^{\alpha^2 \tau_+^2/2} \left[1 + \frac{1-q}{q} \alpha^2 |\tau_-| (\tau_- - \tau)\right]^{\frac{q}{1-q}}, \ \tau < \tau_- \\ a_B e^{\alpha^2 \tau^2/2}, \ \tau_- < \tau < \tau_+ \\ a_B e^{\alpha^2 \tau_+^2/2} [1 + \mathcal{H}_+ (\tau - \tau_+)], \ \tau > \tau_+ \end{cases}$$

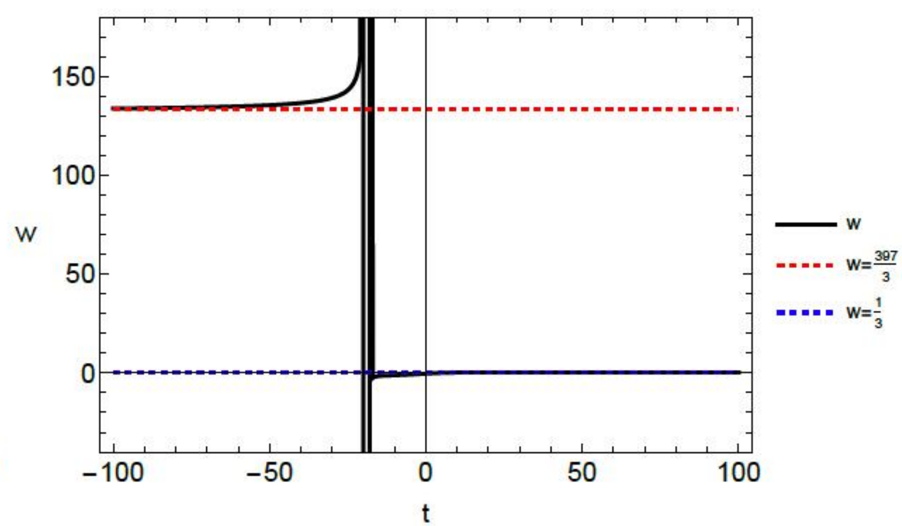
Hubble parameter:

$$\mathcal{H}(\tau) = \begin{cases} \alpha^2 \tau_- \left[1 + \frac{1-q}{q} \alpha^2 |\tau_-| (\tau_- - \tau)\right]^{-1}, \ \tau < \tau_- \\ \alpha^2 \tau, \ \tau_- < \tau < \tau_+ \\ \alpha^2 \tau_+ [1 + \alpha^2 \tau_+ (\tau - \tau_+)]^{-1}, \ \tau > \tau_+ \end{cases}$$

Numerical results:



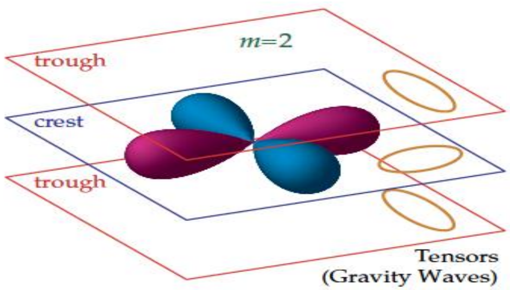
$$w = P/\rho$$

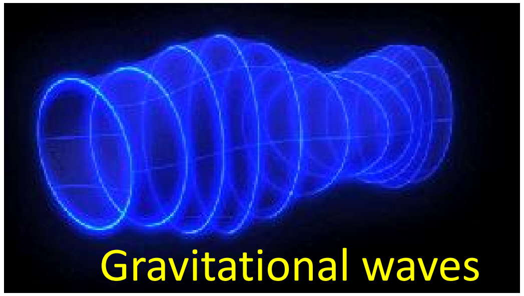


Bounce model for PTA data

TENSOR PERTURBATIONS

Why tensor perturbation?





Perturbed tensorial action:

$$S_2^T = \int d\tau d^3x \frac{a^2}{8} M_p^2 \left[\gamma_{ij}'^2 - \left(\partial\gamma_{ij}\right)^2\right]$$

γ_{ij}: tensor perturbation

Equation of motion (Mukhanov-Sasaki equation):

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$v_k \equiv a\gamma_k/(2\sqrt{M_p})$

Tensorial power spectrum: $$P_T \equiv 2 \times \frac{k^3}{2\pi^2 M_p^3} |\gamma_k|^2 = \frac{4k^3}{\pi^2 M_p^2} \left|\frac{v_k}{a}\right|^2$$

Bounce model for PTA data

SOLUTIONS

In Contracting phase ($\tau < \tau_-$):

EoM: $$v''_{1k} + \left[k^2 - \frac{q(2q-1)}{(1-q)^2(\tau_e - \tau)^2}\right] v_{1k} = 0$$

Sol: $$v_{1k}(\tau) \simeq \frac{\sqrt{\pi(\tau_e - \tau)}}{2} H_\nu^{(1)}[k(\tau_e - \tau)]$$

$$\nu \equiv \frac{1-3q}{2(1-q)}$$

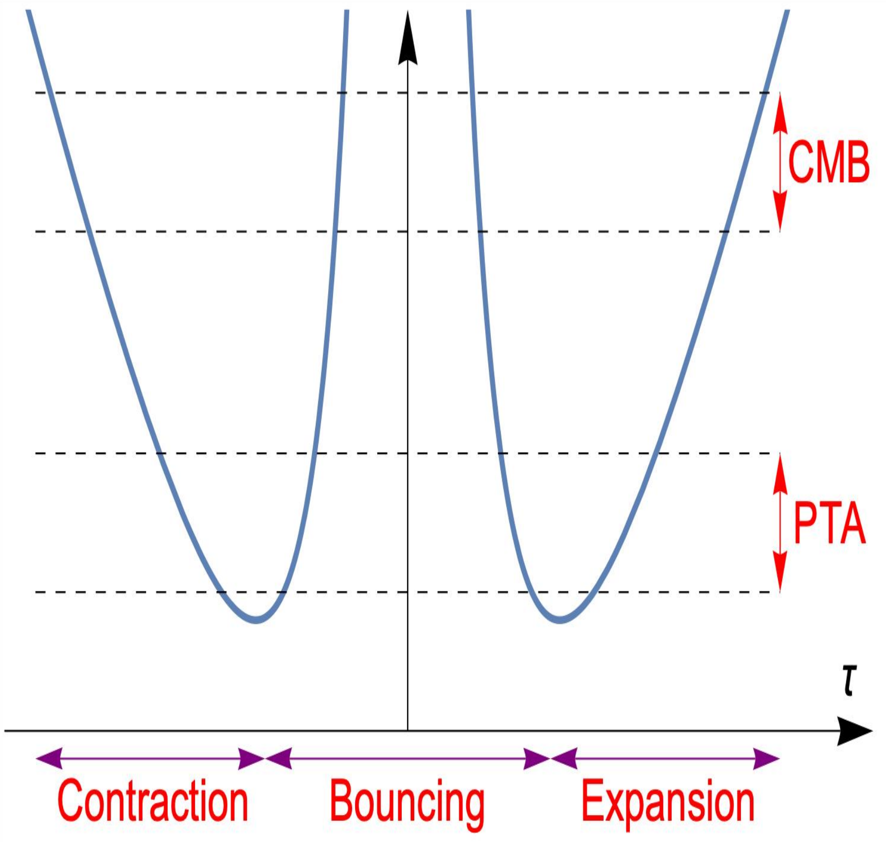
In Bouncing phase ($\tau_- < \tau < \tau_+$):

EoM: $$v''_{2k} + (k^2 - \alpha^2) v_{2k} = 0$$

Sol: $$v_{2k} = b_{T,1} e^{\sqrt{\alpha^2 - k^2}\tau} + b_{T,2} e^{-\sqrt{\alpha^2 - k^2}\tau}$$

In Expanding phase ($\tau > \tau_+$):

$w = 1/3 \Rightarrow a''/a = 0$ oscillating solution with fixed amplitude: $v_{3k} \sim b_{T,1} e^{-ik\tau}$



Bounce model for PTA data

SOLUTIONS

The junction function:

$$\nu_{1k}(\tau_-) = \nu_{2k}(\tau_-) \qquad \nu_{2k}(\tau_+) = \nu_{3k}(\tau_+)$$

$$\nu'_{1k}(\tau_-) = \nu'_{2k}(\tau_-) \qquad \nu'_{2k}(\tau_-) = \nu'_{3k}(\tau_+)$$

which gives rise to

$$b_{T,1} = e^{-\sqrt{\alpha^2 - k^2}\tau_-} \frac{2^{\nu-2} i}{\sqrt{\pi}} \Gamma(\nu) k^{-\nu} \left[\frac{q}{\alpha^2(1-q)\tau_+}\right]^{\frac{q}{1-q}} \left[\frac{\alpha^2(1-q)^2\tau_+}{\sqrt{\alpha^2 - k^2} q^2} - 1\right]$$

The tensor power spectrum will be:

$$P_T \simeq 2^{-\frac{1+q}{1-q}} \frac{\Gamma^2(\nu)}{\pi^3} \left(\frac{H_+}{M_p}\right)^2 \left(\frac{q}{1-q}\right)^{\frac{2q}{1-q}} \left(\frac{k}{a_B H_+}\right)^{\frac{2}{1-q}}$$

with $q \lesssim 1,\ 2/(1-q) > 0$

GW energy density:

$$\Omega_{GW}(k) \equiv \frac{1}{3H^2} \frac{d\rho_{GW}}{d\ln k} \simeq 10^{-6} P_T(k)$$

Bounce model for PTA data

CONSTRAINTS ON PARAMETERS

1) Validness of effective field theory ($H_+/M_p < 1$):

Not reach Planck scale if $\lambda_* = \dfrac{a_B}{k_*} > l_P$ $\qquad$ k_*: cutoff scale

2) Consistency with observations ($P_T(k = k_{PTA}) \sim 10^{-3}$, $n_T = 1.8 \pm 0.3$):

The spectrum could be parameterized as

$$P_T(k) \simeq 10^{-3}\left(\frac{k}{k_{PTA} = 10^6\mathrm{Mpc}^{-1}}\right)^{\frac{2}{1-q}} \Longrightarrow H_+ \sim 0.1 M_p$$

$$n_T = 1.8 \pm 0.3 \Longrightarrow q < 0.0476$$

3) Validness of perturbation theory ($P_T < 1$ for $k \leq k_*$):

From the parametrization we have

$$\ln\left(\frac{k}{10^6\mathrm{Mpc}^{-1}}\right) < \frac{1-q}{2}\ln 10^3 \simeq 3.5(1-q)$$

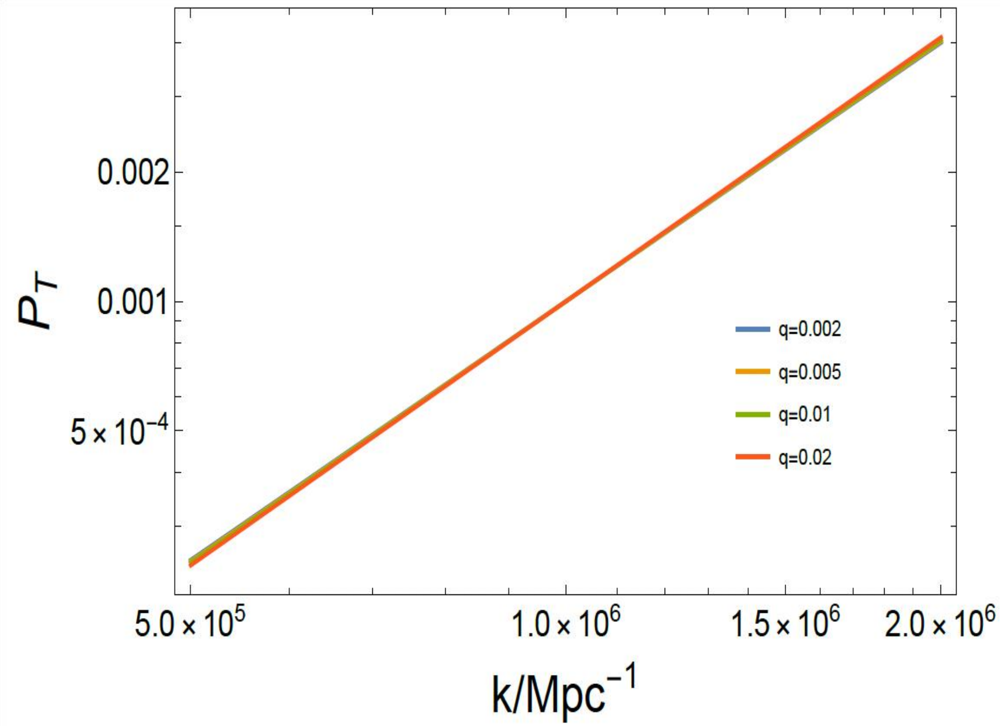
$$q < 0.0476 \Longrightarrow k_* \leq \mathcal{O}(10^7)\mathrm{Mpc}^{-1}$$

Bounce model for PTA data

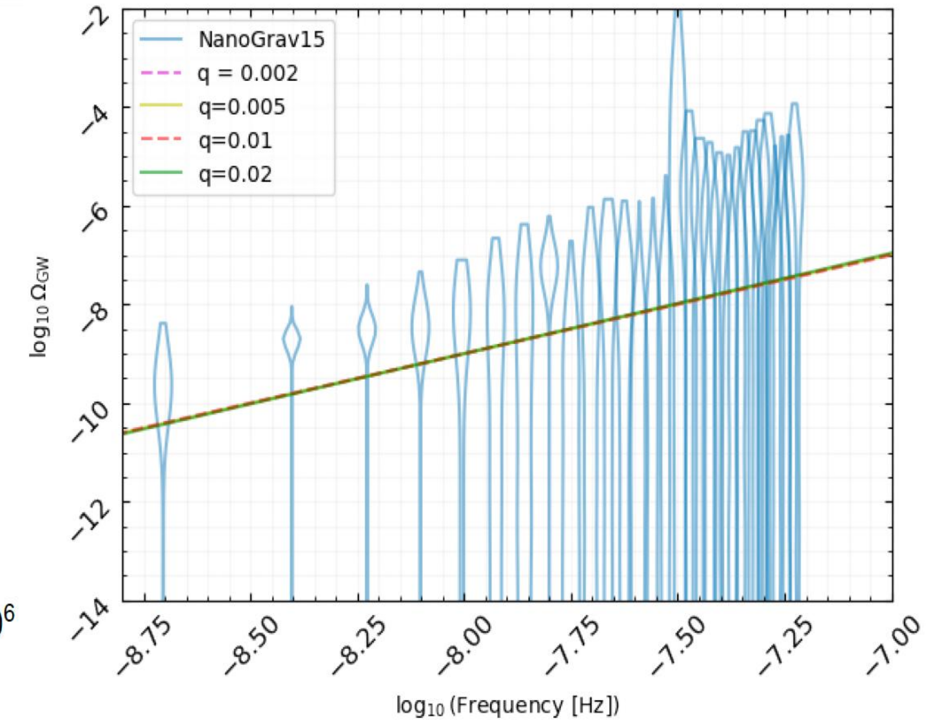
NUMERICAL RESULTS VS. PTA DATA

(using NANOGrav15 data as an example)

Tensor power spectrum:



GW energy density:



TQ, Zhu, 2408.06582

Bounce model for PTA data

SCALAR PERTURBATIONS

Perturbation from ϕ:

EoM: $$\mu_k'' + \left(k^2 - \frac{z_s''}{z_s}\right)\mu_k = 0$$

where $\mu_k \equiv z_s \delta\phi$

$$\frac{z_s^2}{a^2} \equiv \frac{\dot{\phi}^2 K_X + \dot{\phi}^4 K_{XX}}{H^2 M_p^2} = \frac{3\beta_2\dot{\phi}^4 + 2(1-\beta_1)M_p^2\dot{\phi}^2}{4H^2 M_p^2}$$

Scalar power spectrum:

$$P_\phi(k) \simeq \frac{\Gamma^2(\nu)(1-2\nu)^{1-2\nu}}{3\pi^3 2^{8-4\nu}}\left(\frac{k}{|\mathcal{H}_-|}\right)^{\frac{2}{1-q}}\frac{H_-^2}{M_p^2} \quad \nu \equiv \frac{1-3q}{2(1-q)}$$ (blue-tilted!)

Comparing with the tensor perturbation we find that

$$r_\phi(\tau = \tau_+) \equiv \frac{P_T}{P_\phi} = 96$$ (gets severely suppressed!)

Bounce model for PTA data

SCALAR PERTURBATIONS

Perturbation from σ:

EoM: $$u_k'' + \left(k^2 - \frac{z_c''}{z_c}\right) u_k = 0$$

where $u_k \equiv z_c \delta\sigma$

$$z_c \equiv a M_p f \sqrt{\frac{1+\phi e^{\sqrt{\frac{q}{2}}\phi}}{e^{-\sqrt{\frac{q}{2}}\phi} + \phi^2 e^{\sqrt{\frac{q}{2}}\phi}}} = \begin{cases} \dfrac{-f\sqrt{q(1-3q)}}{\sqrt{2V_0}(1-q)}(\tau_e - \tau)^{-1} & \text{Contracting phase} \\ \alpha^2(1+\alpha^2\tau^2) - \dfrac{4(\beta_1 - 1)}{3\beta_2 q} M_p^2 e^{-\frac{2\tau^2}{T^2} - \frac{1}{2}\alpha^2\tau^2} & \text{bouncing phase} \end{cases}$$

Scalar power spectrum:

$$P_\sigma(k) \equiv \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z_c^2} \simeq \frac{V_0(1-q)^2}{2\pi^2 f^2 q(1-3q) a_+^2}$$

(scale invariant!)

Bounce model for PTA data

SCALAR PERTURBATIONS

From above, we see that

P_ϕ: blue-tilted, dominant in small scales (large k)

P_σ: scale-invariant, dominant in large scales (small k)

The curvature perturbation:

At small scales:

In Ekpyrotic contracting phase $H/\dot{\phi} = \sqrt{q/2}$:

$$P_\zeta^{(s)}(k) \equiv \left(\frac{H}{\dot{\phi}}\right)^2 P_\phi(k) \simeq 10^{-5} q \left(\frac{k}{10^6 \mathrm{Mpc}^{-1}}\right)^{\frac{2}{1-q}}$$

At large scales:

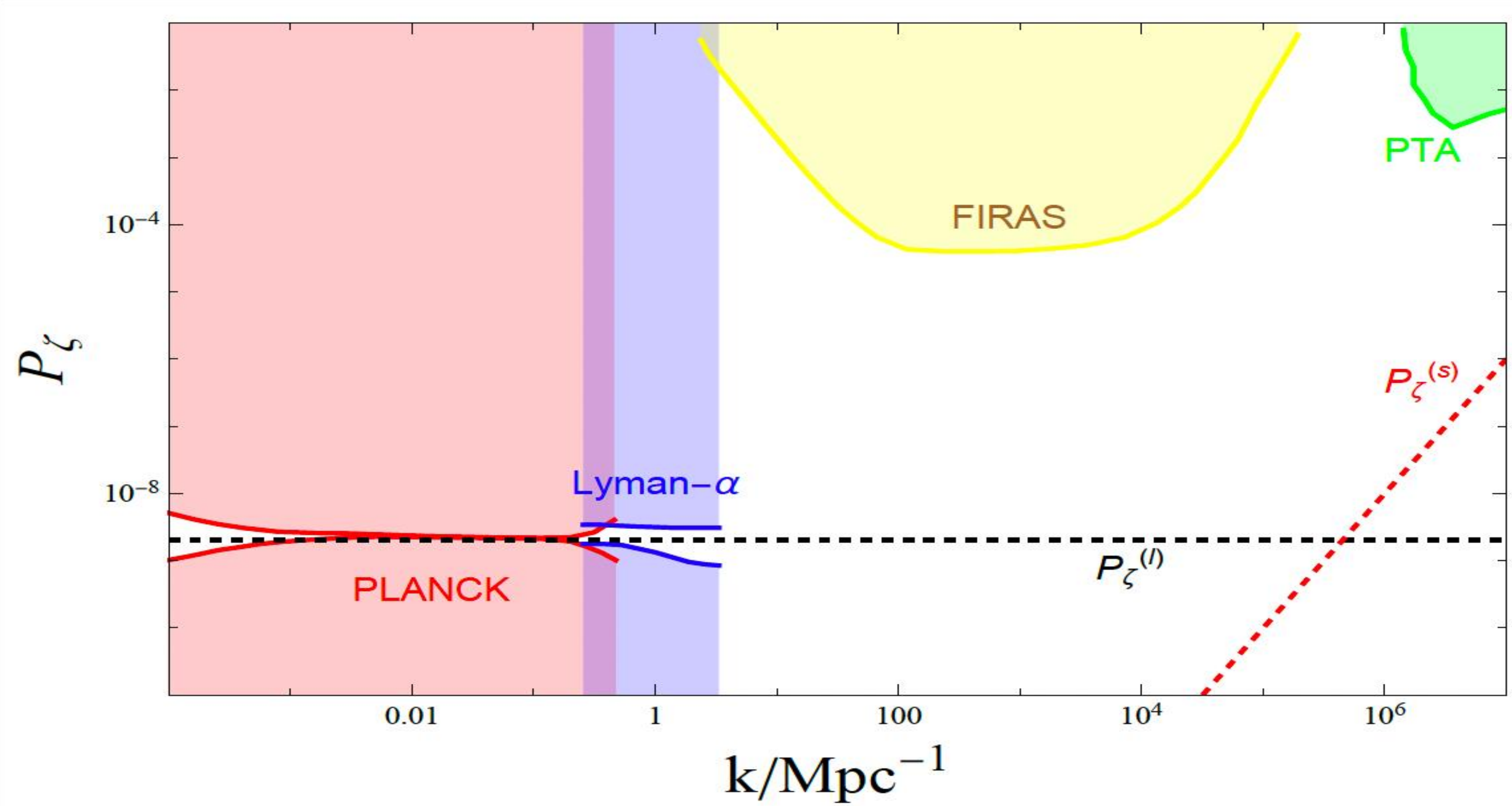
$\Upsilon(\phi)$ is chosen to make $H/\dot{\sigma} =$ "const"$(= s)$:

$$P_\zeta^{(l)}(k) \equiv \left(\frac{H}{\dot{\sigma}}\right)^2 P_\sigma(k) \simeq s^2 \frac{V_0(1-q)^2}{2\pi^2 f^2 q(1-3q)a_+^2}$$

Bounce model for PTA data

COMPARING WITH THE OBSERVATIONS

For CMB constraints, we require $P_\zeta^{(l)}(k_{CMB}) = s^2 \frac{V_0(1-q)^2}{2\pi^2 f^2 q(1-3q)a_+^2} \simeq 2.1 \times 10^{-9}$



Connecting $P_\zeta^{(l)}$ and $P_\zeta^{(s)}$ results in $k_{pivot} \simeq 10^5 \mathrm{Mpc}^{-1}$.

Conclusions

- Bounce scenario can avoid the Big Bang singularity;
- Bounce can have signals in both CMB or PTA observations;
- We proposed a model that shows:
  - ✓ Tensor perturbations: consistent with the PTA data
  - ✓ Scalar perturbations: well within the observational constraints
  - ✓ Parameter choice: consistent with Theoretical requirements
- Extensions: test bounce in all the scales?

Thanks For Attention!