“2024 引力与宇宙学”专题研讨会

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Solid physics from a simple gravitational tool

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w/ Baggioli, Li, Sun, JHEP 05: 198 (2024).

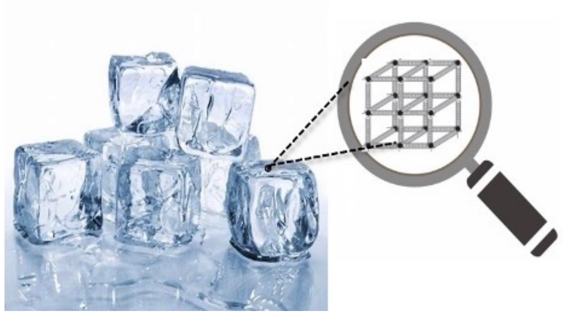
w/ Xia, 2405.17092.

w/ Xu, in preparation.

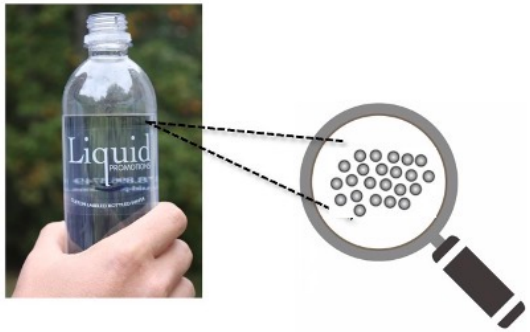
w/ Baggioli, Kim, Li, SCPMA 64, 7 (2021).

Elasticity of solids

Phases of matter can be classified and described by symmetries and their spontaneous breaking.



Solid physics focuses on properties of solid materials and their mechanisms.

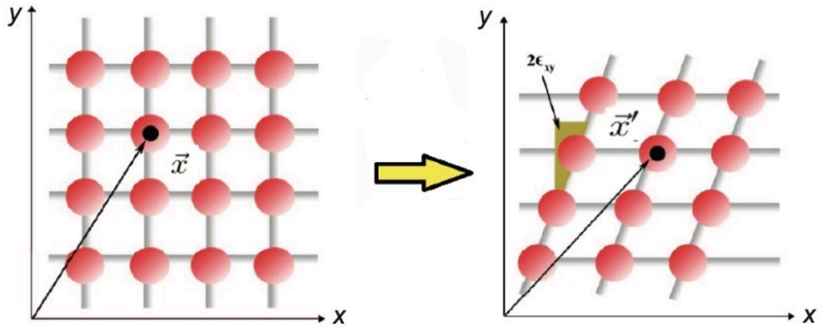


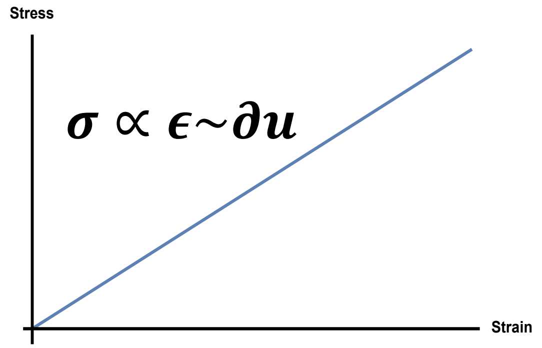
The spontaneous breaking of translations endow solids with their ability to respond elastically to mechanical deformations which is a basic aspect of these systems.

Solids have fixed shape while liquids flow and take the shape of their container.

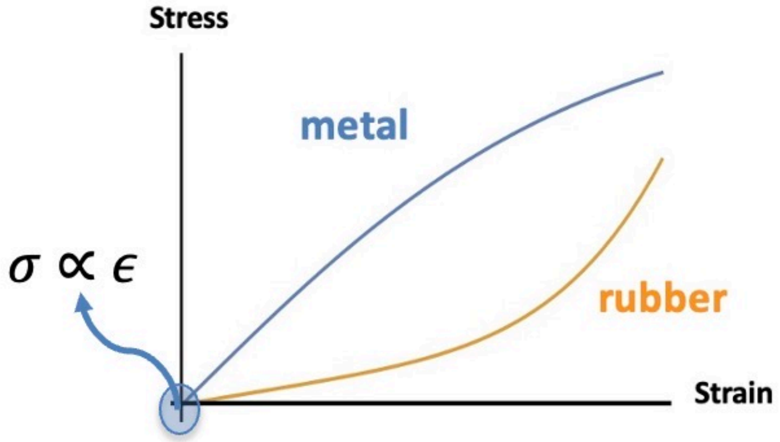
The elastic response is well understood when restricted to the linear regime (small deformations) in the conventional elasticity theory

$$u = x' - x$$





However, solids can exhibit more complicated behaviors in non-linear regime (e.g., strain softening, strain hardening, mechanical failure).



- A complete understanding of the non-linear behavior of complex solids has not been achieved yet.
- Irreversible deformations are inevitable for sufficiently large deformations (mechanical failure). A long list of criteria have been proposed.

During past decades, several fundamental theories have been rewritten in terms of modern **EFT** language (symmetry + action) :

- **Fermi's liquid theory** (Polchinski, 1992)
- **Hydrodynamics** (Martin, Siggia, Rose, Son, Nicolis, Liu, Rangamani, … , 1973-2015)
- **Elasticity theory**

Phonons as goldstone bosons

H. Leutwyler (Bern U. and CERN) (Sep, 1996)

Published in: *Helv.Phys.Acta* 70 (1997) 275-286

Elasticity bounds from Effective Field Theory

Lasma Alberte (ICTP, Trieste), Matteo Baggioli (Crete U.), Victor Cancer Castillo (Bar Barcelona), Oriol Pujolas (Barcelona, IFAE and BIST, Barcelona) (Jul 19, 2018)

Published in: *Phys.Rev.D* 100 (2019) 6, 065015, *Phys.Rev.D* 102 (2020) 6, 069901

1807.07474 [hep-th]

Relativistic Fluids, Superfluids, Solids and Supersolids from a Coset Construction

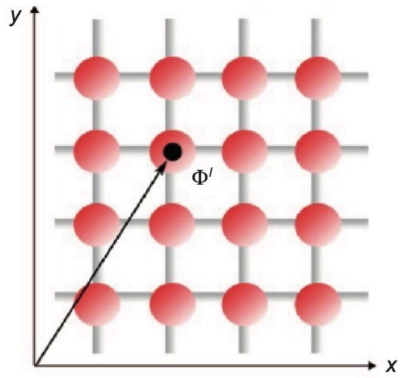
Alberto Nicolis (ISCAP, New York and Columbia U.), Riccardo Penco (ISCAP, New York and Columbia U.), Rosen (ISCAP, New York and Columbia U.) (Jul 1, 2013)

Published in: *Phys.Rev.D* 89 (2014) 4, 045002 • e-Print: 1307.0517 [hep-th] e.g. $d = 2 + 1$

Nicolis et al, PRD 89 (2014) 4, 045002

Dynamical fields: $\Phi^I = \boxed{\delta^I_i x^i} + \delta\Phi^I, \quad i = 1, 2$

Internal shift symmetry: $\Phi^I \to \Phi^I + c^I$ ➡ building block: $\mathcal{I}^{IJ} \equiv \partial_\mu \Phi^I \partial^\mu \Phi^J$



At leading order in derivatives:

$$S_{\rm eff}[\Phi] = -\int d^3x \sqrt{-g}\, V(X, Z), \quad X \equiv {\rm Tr}\left(\mathcal{I}^{IJ}\right), \quad Z \equiv \det\left(\mathcal{I}^{IJ}\right).$$

Stress tensor: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\rm eff}}{\delta g^{\mu\nu}}\Big|_{g=\eta} = -\eta_{\mu\nu} V + 2\partial_\mu \Phi^I \partial_\nu \Phi_I V_X + 2\left(\partial_\mu \Phi^I \partial_\nu \Phi_I X - \partial_\mu \Phi^I \partial_\nu \Phi^J \mathcal{I}_{IJ}\right) V_Z \ .$

$S^{(2)}_{\rm eff}\,[\delta\Phi]$ (Goldstone) ➡ $\boxed{\omega^2 = v^2_{\rm L,T}\, k^2 + \ldots, \quad v^2_{\rm T} = \frac{G}{\varepsilon + p}, \quad v^2_{\rm L} = \frac{B + G}{\varepsilon + p}}$

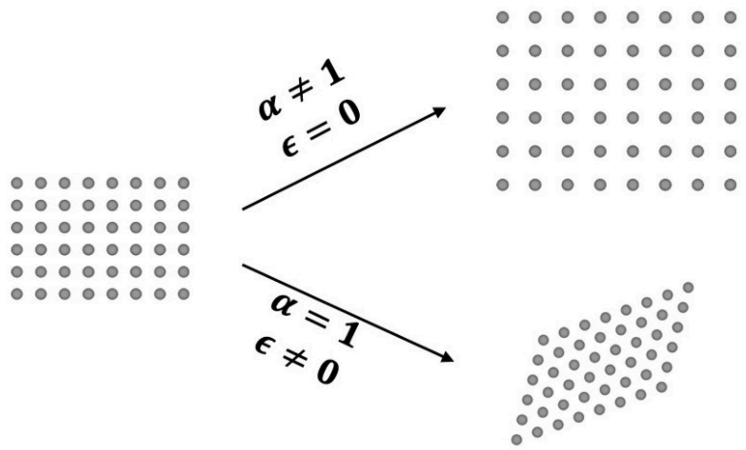
Sound waves in elastic medium

Generalization: $\Phi^I = \boxed{M_i^I x^i} + \delta\Phi^I, \;\; i = 1, 2$

Alberte *et al*, PRD 100 (2019) 6, 065015

broken rotations

$$M_i^I = \alpha \begin{pmatrix} \sqrt{1+\epsilon^2/4} & \epsilon/2 \\ \epsilon/2 & \sqrt{1+\epsilon^2/4} \end{pmatrix}$$



Sheared material ($\alpha = 1, \epsilon \neq 0$):

$$\omega^2 = v_{1,2}^2(\epsilon, \theta)k^2 + \cdots, \quad v_2^2 < v_1^2.$$

The EFT method predicts a phonon instability signaled by $v_2^2 < 0$ when $\epsilon > \epsilon_c$.

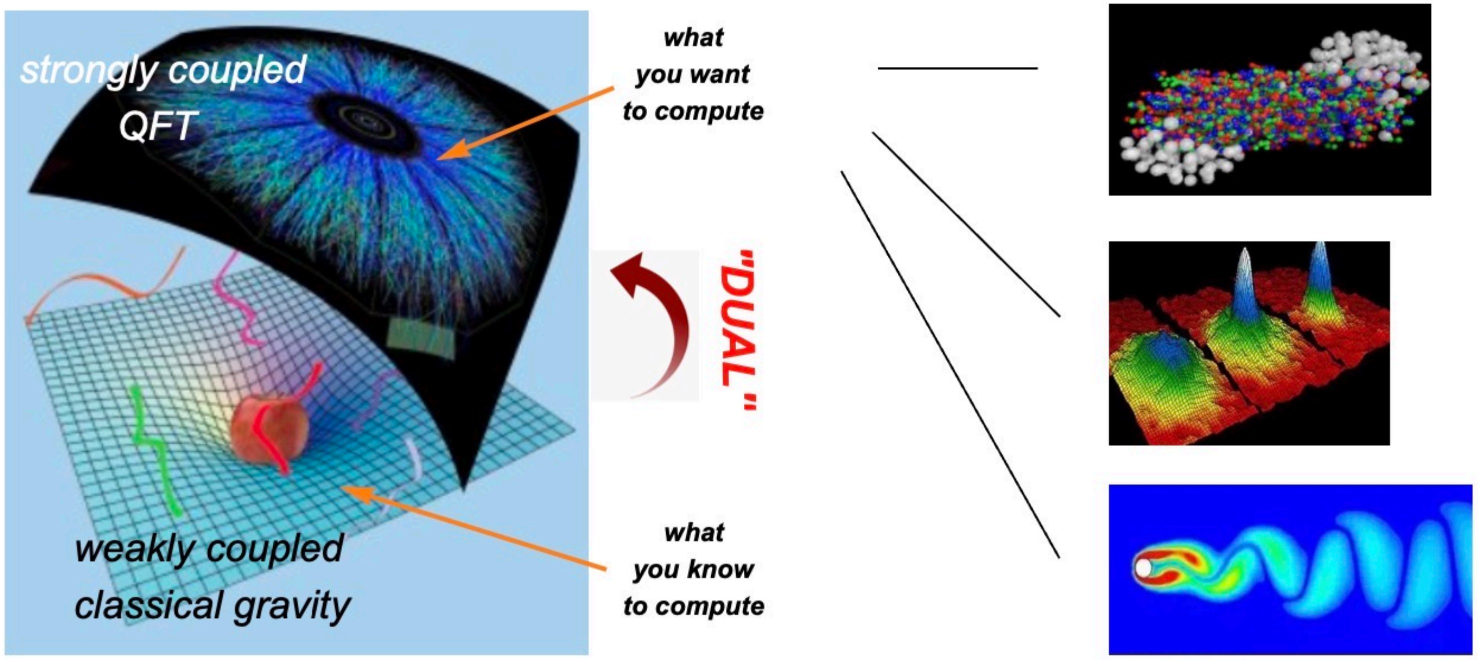
In the method above, the dissipative effects at finite temperatures are neglected.

However, dissipative effects are often crucial:

- Vibrational density of states
- Thermodynamic properties
- Transport properties
- Phase transitions (e.g. melting transition)
- Shear failure
- …

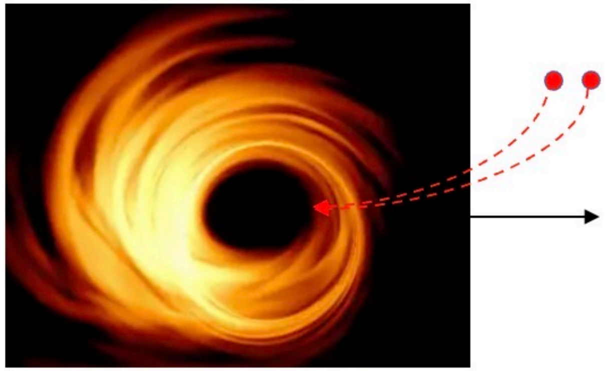
We need new theoretical methods.

AdS/CFT correspondence (holography) connects a weakly coupled gravity in an asymptotic AdS space with a strongly coupled quantum field theory living on boundary.



It provides a perturbative approach to understand strongly-coupled quantum many-body systems.

Moreover, it realizes dissipations economically : black hole solutions.



viscosity
conductivities
…

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

P. **K**ovtun, D. **S**on, A. **S**tarinets,
PRL 94 (2005) 111601

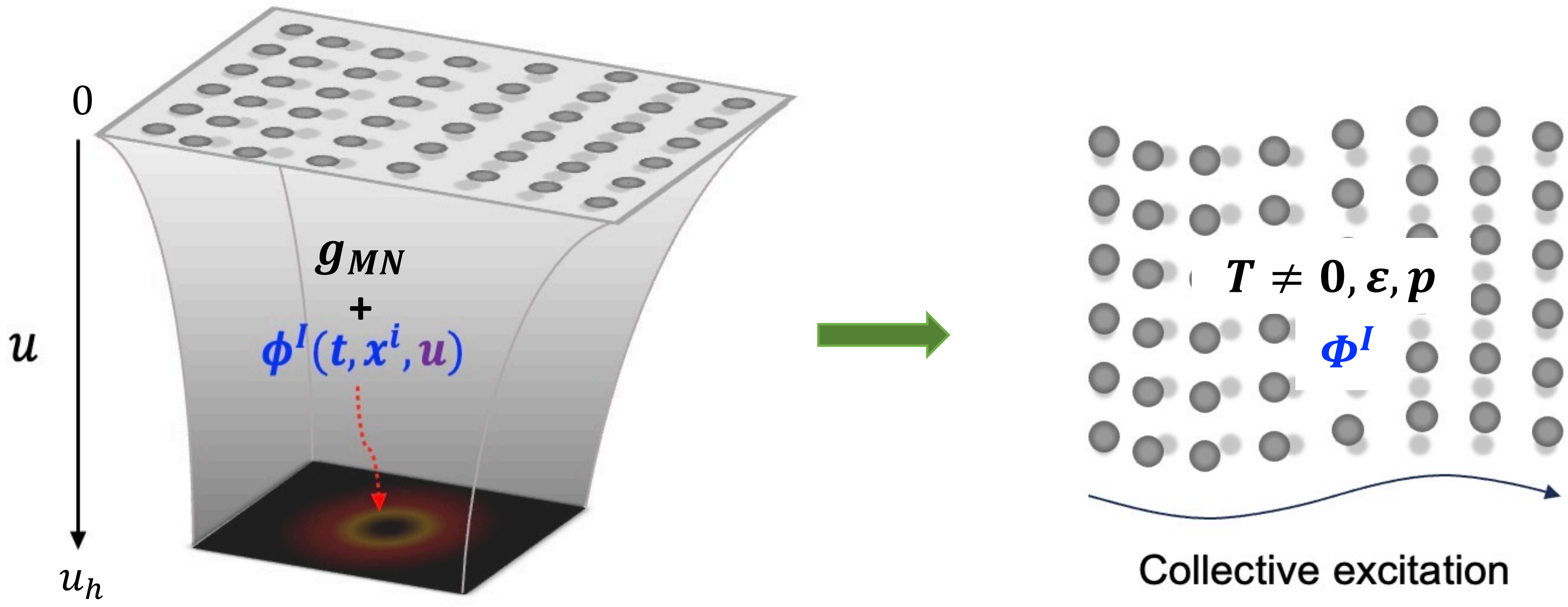
Broken translations have been realized in holography during last decade:

- Periodic lattice, Josephson junction, superfluid vortex, dark soliton, density waves, … (inhomogeneous)

A. Adams, T. Andrade, P. Chesler, S. Hartnoll, G. Horowitz, H. Liu, K. Schalm, A. Krikun, J. Santos, D. Tong, L. Yaffe, J. Zannen,…

R. G. Cai, M. Guo, S. Q. Lan, L. Li, Y. Ling, Y. X. Liu, Z. Y. Nie, Y. Tian, Y. Q. Wang, J. P. Wu, C. Y. Xia, P. Yang, H. Zeng, H. Zhang, H. Q. Zhang,…

- Q-lattice model, axion model,… (homogeneous)



Holographic axion model

Bulk action:

Baggioli, Kim, Li, **WJL**, SCPMA 64 (2021) 7, 270001.

$$S = \int d^4x\sqrt{-g}\left[R - 2\Lambda - m^2 V(X,Z)\right] + \dots,$$

$$X \equiv \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \phi^I \partial^\mu \phi^J\right], \quad Z \equiv \det\left[\partial_\mu \phi^I \partial^\mu \phi^J\right], \quad I = 1,2$$

Vegh, 1301.0537

Consider: $\bar{\phi}^I = M_i^I\, x^i$ → Broken **Diffeos** in bulk → Broken **Transls** on boundary

e.g. $V = X^N Z^M$

Near boundary $u \to 0$: $\phi^I(u,\vec{x},t) = \phi^I_{(0)}(\vec{x},t) \dots + \phi^I_{(1)}(\vec{x},t)\, u^{5-2N-4M} + \dots$

Leading term=source. Subleading term=expectation value

Explicit breaking → $5 - 2N - 4M > 0$

Spontaneous breaking → $5 - 2N - 4M < 0$

Emergent phonons on boundary

$\alpha = 1, \epsilon = 0$ $\quad (M_i^I = \delta_i^I)$

Background:

$$ds^2 = \frac{L^2}{u^2}\left[-f(u)dt^2 + \frac{1}{f(u)}du^2 + dx^2 + dy^2\right]$$

$$f(u) = u^3 \int_u^{u_h} ds \left(\frac{3}{s^4} - \frac{m^2 V(\bar{X}, \bar{Z})}{s}\right) \qquad \Lambda = -\frac{3}{L^2} \equiv -3$$

Linearized fluctuations ($k^i = (k, 0)$):

$$\delta\chi_{\mathbf{A}}(u, x^\mu) = \int_{-\infty}^{+\infty} \frac{d\omega d^2 k_i}{(2\pi)^4}\, e^{i\,(k_i x^i - \omega\, t)}\, \delta\chi_{\mathbf{A}}(u, \omega, k^i).$$

$$\delta\chi_{\mathbf{L}} = \{\delta g_{tt},\, \delta g_{xx},\, \delta g_{yy},\, \delta g_{tu},\, \delta g_{uu},\, \delta\phi^x\},$$

$$\delta\chi_{\mathbf{T}} = \{\delta g_{ty},\, \delta g_{xy},\, \delta g_{uy},\, \delta\phi^y\},$$

Quasi-normal modes(QNMs) ⟷ Poles of retarded Green functions ⟷ **Dispersion relations**

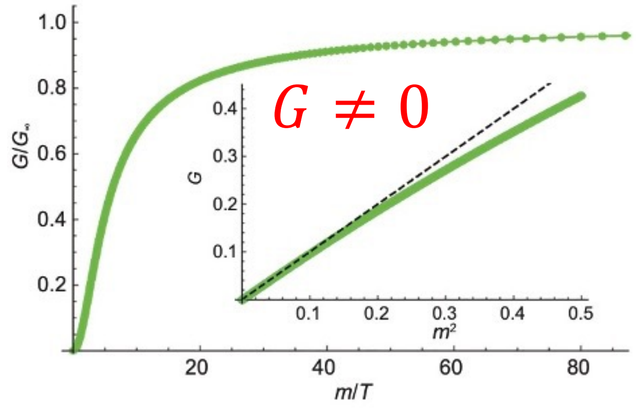
$V = X^3$

Gapless collective modes (for small k):

$$\omega_{\mathbf{T}} = \pm v_{\mathbf{T}}\, k - \frac{i}{2}\,\Gamma_{\mathbf{T}}\, k^2\,,$$

$$\omega_{\mathbf{L}} = \pm v_{\mathbf{L}}\, k - \frac{i}{2}\,\Gamma_{\mathbf{L}}\, k^2\,,$$

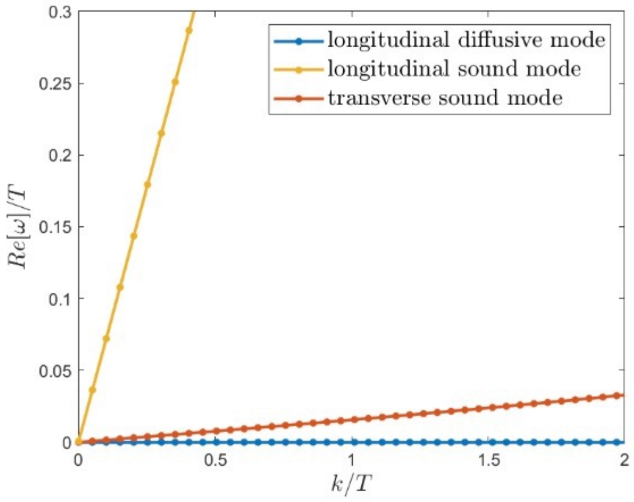
$$v_{\mathrm{T}}^2 = \frac{G}{\varepsilon + p}, \quad v_{\mathrm{L}}^2 = \frac{B+G}{\varepsilon + p}.$$

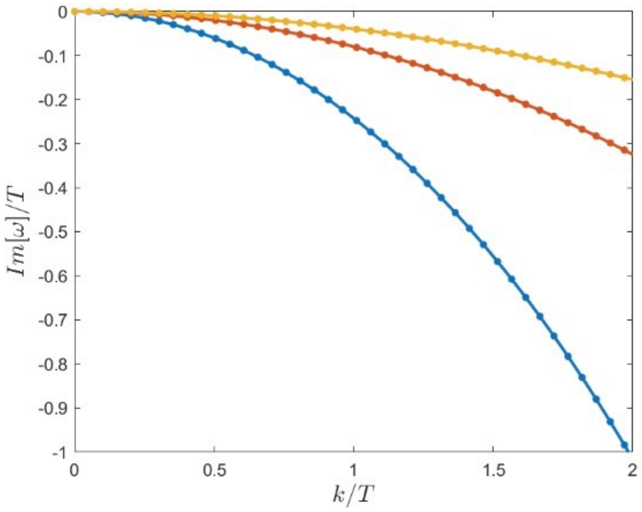


$$\omega_{\mathbf{L}} = -i\, D_{\phi}\, k^2\,.$$

Longitudinal diffusive mode in a solid.

Zippelius, Halperin, Nelson, PRB 22, 2514 (1980).





Mechanical instabilities under shear strain

$\alpha = 1, \epsilon \neq 0$

Baggioli, Castillo, Pujolas,
JHEP 09 (2020) 013.

$$ds^2 = \frac{1}{u^2}\left[-f(u)e^{-\chi(u)}dt^2 + \frac{1}{f(u)}du^2 + \gamma_{ij}(u)dx^i dx^j\right],$$

$$\gamma = \begin{pmatrix} \cosh h(u) & \sinh h(u) \\ \sinh h(u) & \cosh h(u) \end{pmatrix}, \quad M_i^I = \alpha \begin{pmatrix} \cosh(\Omega/2) & \sinh(\Omega/2) \\ \sinh(\Omega/2) & \cosh(\Omega/2) \end{pmatrix}$$

$$\epsilon = 2\sinh\left(\frac{\Omega}{2}\right)$$

Background equations,

$$2\chi' - u h'^2 = 0\,,$$
$$f\,(u\,\chi' + 6) + \left[V\,(\bar{X},\bar{Z}) - 2\,u\,f' - 6\right] = 0\,,$$
$$h'' + h'\left(\frac{f'}{f} - \frac{2}{u}\right) - \frac{1}{4}\,u\,h'^3 - \frac{m^2 \sinh(\Omega - h)\,V_X(\bar{X},\bar{Z})}{f} = 0,$$

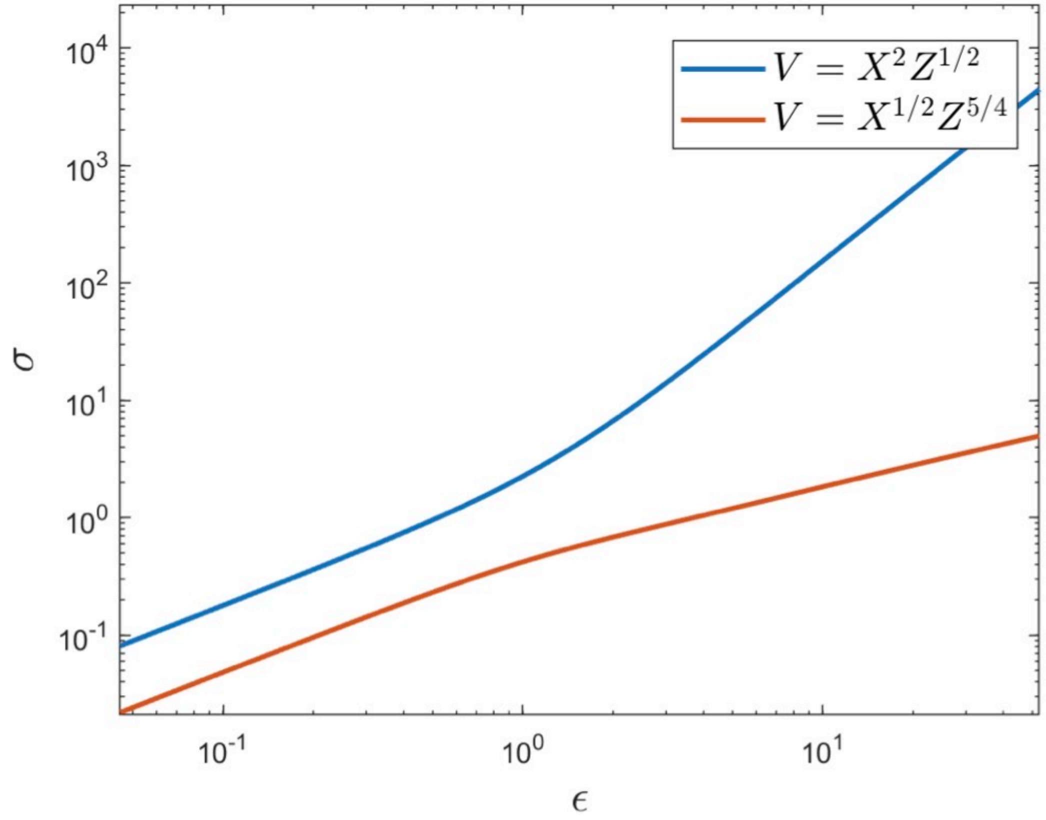
where $V_X \equiv \frac{\partial V}{\partial X}$. Near boundary expansion ($u \to 0$),

$$h\,(u) = \mathcal{H}_0 + \ldots + \mathcal{H}_3\,u^3 + \ldots$$

$\langle T_{xy} \rangle \equiv \sigma$

Non-linear elastic response can be studied by looking at $\sigma = \frac{3}{2}\mathcal{H}_3$ as a function of $\epsilon = 2\sinh(\Omega/2)$.

Baggioli, Li, **WJL**, Sun, JHEP 05 (2024) 198.

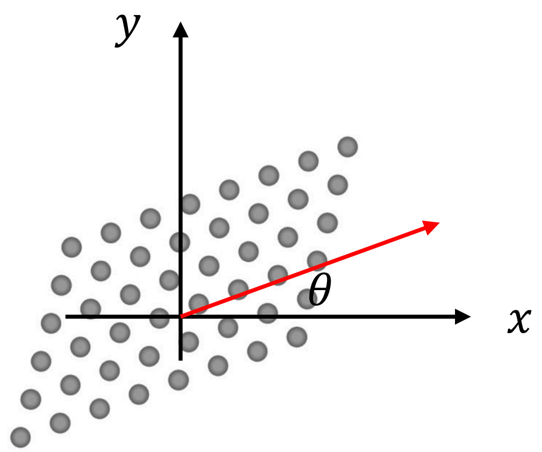


Stress-strain curve

Turn on fluctuations (modes are all coupled)

$$\delta\chi_A \sim \exp(-i\omega t + ikx\cos\theta + iky\sin\theta)$$

around the strained background and calculate QNMs.

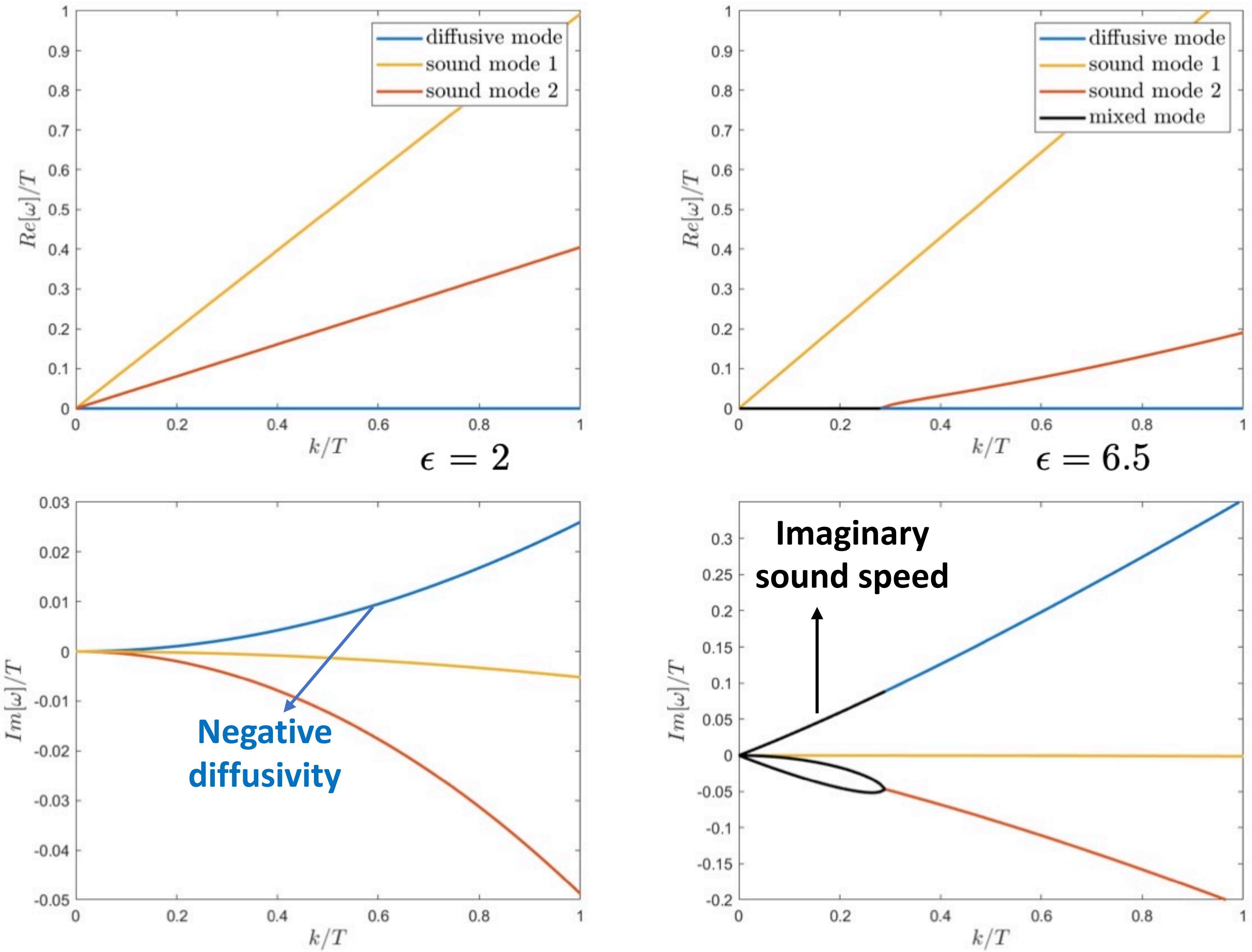


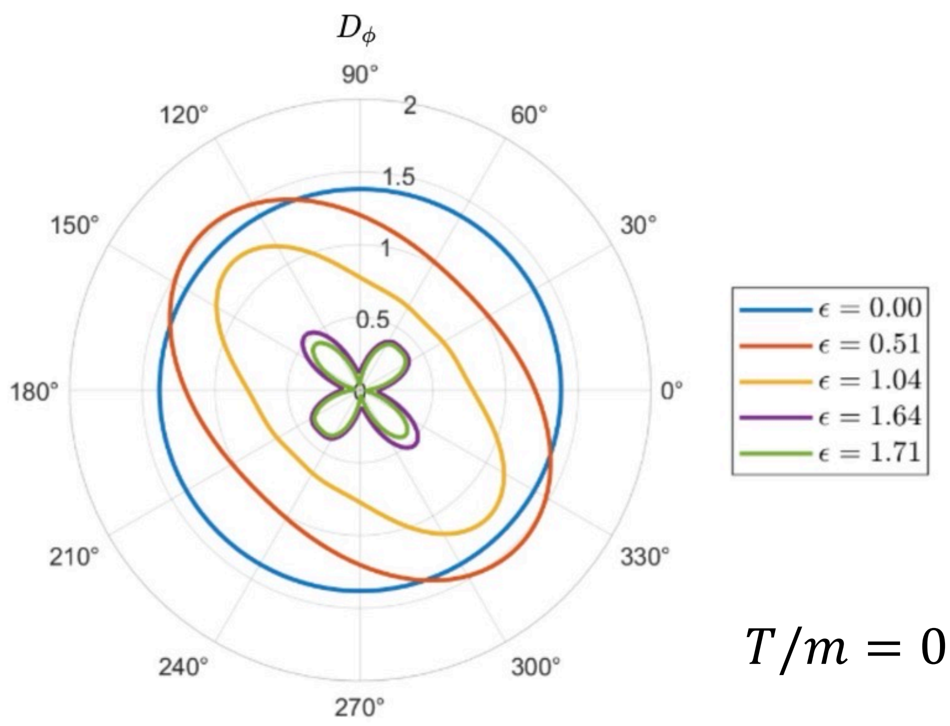
Dispersion relations:

$$\omega = \pm v_{1,2}(\theta,\epsilon)\,k - \frac{i}{2}\Gamma_{1,2}(\theta,\epsilon)\,k^2$$

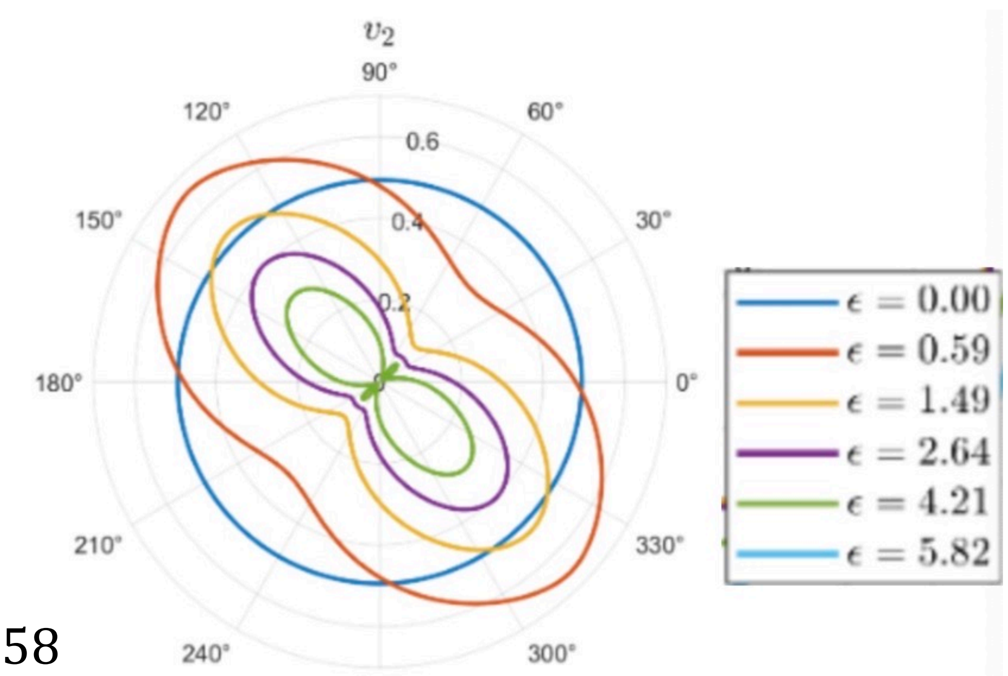
$$\omega = -i\,D_\phi(\theta,\epsilon)\,k^2\,.$$

$V(X, Z) = X^2 Z^{1/2}$ $(\theta = 0,\ T/m = 0.158)$



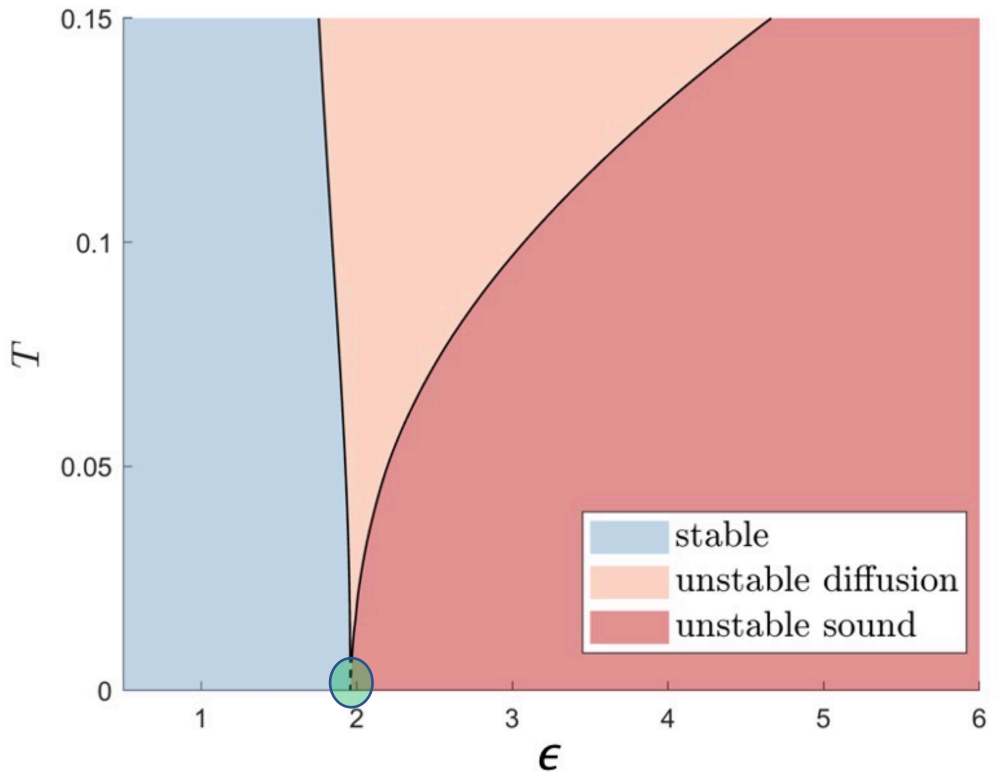


$T/m = 0.158$



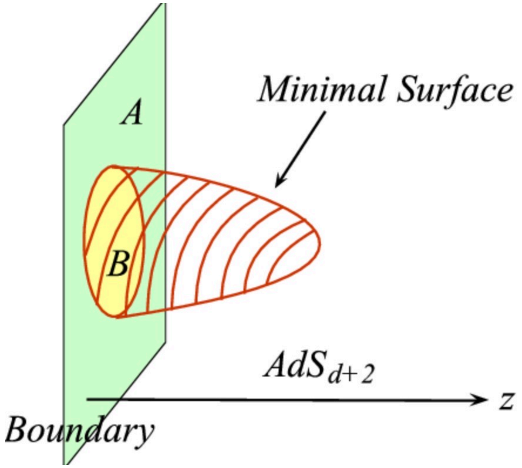
$\epsilon_0^{(\mathrm{d,s})}(\theta)$: The value of strain to make the diffusion constant/ sound speed vanishing along certain directions. The minimal $\epsilon_0^{(\mathrm{d,s})}(\theta)$ can be identified as the maximal strain the system can sustain.

$$\epsilon_c^{(\mathrm{d})} = \epsilon_{0,\mathrm{min}}^{(\mathrm{d})} \approx 1.74 \qquad \epsilon_c^{(\mathrm{s})} = \epsilon_{0,\mathrm{min}}^{(\mathrm{s})} \approx 4.91$$



For a wide class of solid models ($V(X) = X^N Z^M$), it is found that $\epsilon_c^{(d)} < \epsilon_c^{(s)}$ which implies the shear failure is always initiated by the diffusion instability.

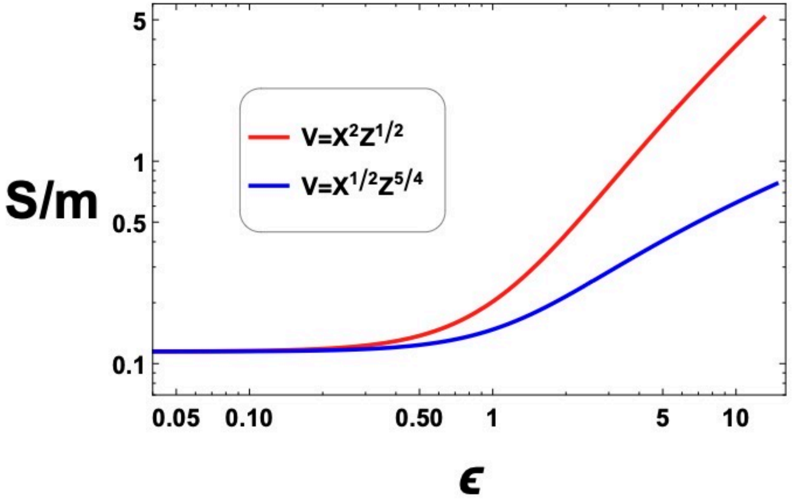
Holographic entanglement entropy

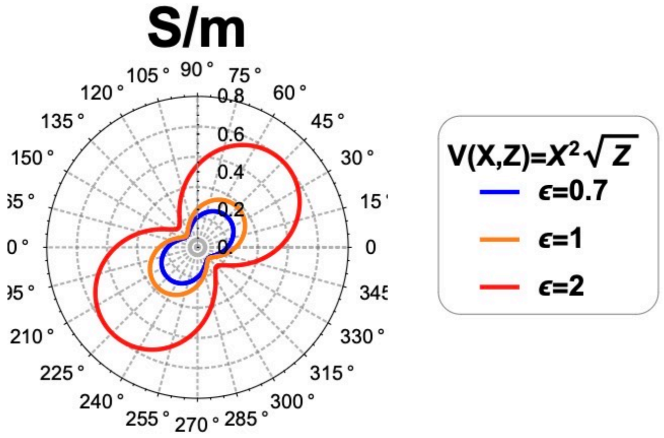


Xu, **WJL**, in preparation.

Ryu-Takayanagi formula ：

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$





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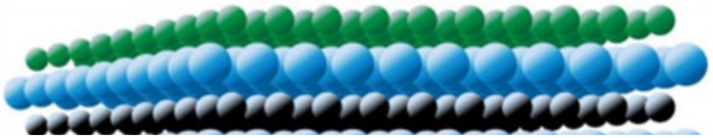
Entanglement of an **infinite strip** grows rapidly only at non-linear elastic regime.

Multilayer materials



Gravity dual???

$$\phi^I \longrightarrow \{\phi^I_a, a = 1,2, \dots\}$$

Multiple-axion model:

Xia, **WJL**, 2405.17092

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \Lambda - \mathcal{W}(X_1, X_2, \cdots, X_{\mathcal{N}})\right]$$

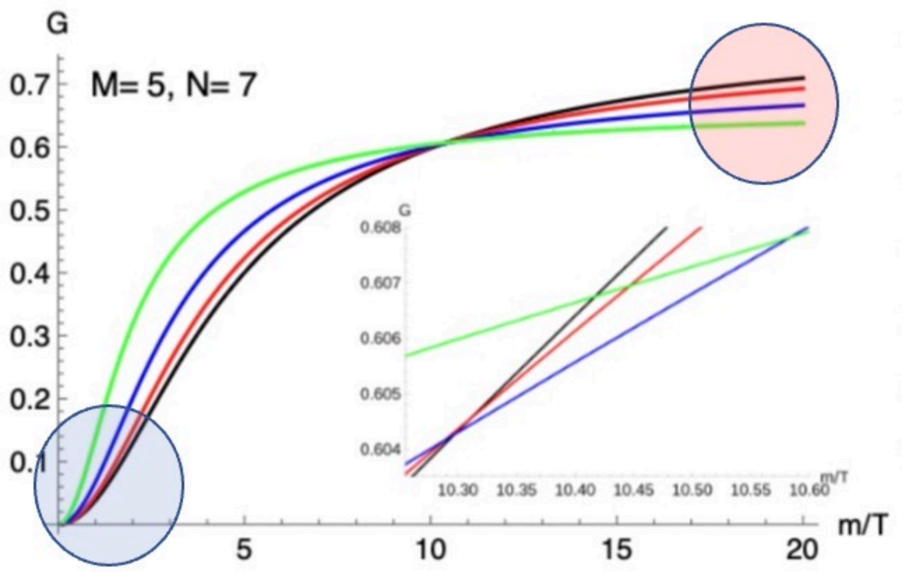
$$X_a \equiv \frac{1}{2}\partial_\mu\phi_a^I\partial^\mu\phi_a^I, \quad \bar{\phi}_a^I = \delta_i^I x^i, \quad a = 1, 2, \ldots$$

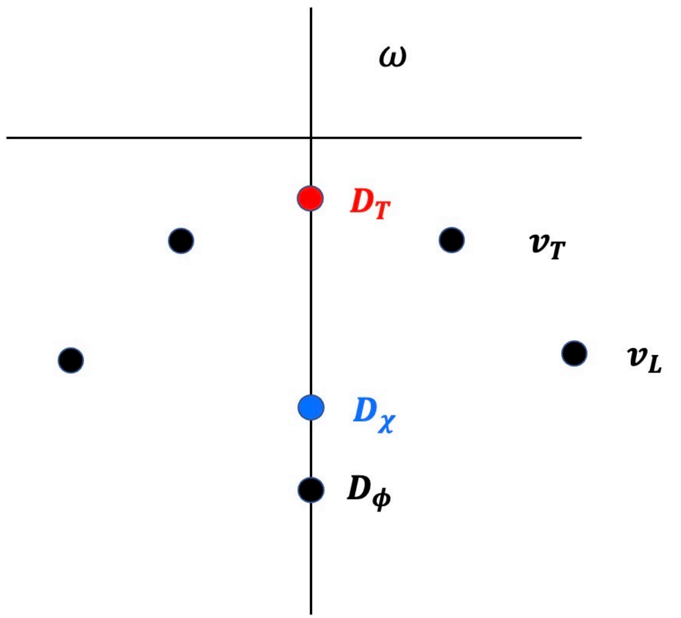
Double axion:

$$\mathcal{W}(X, Y) = m^2 V(X) + n^2 W(Y) + l^2 Z(X, Y)$$

$$V(X) = X^M, W(Y) = Y^N, Z(X, Y) = X^P Y^Q$$

$$X \equiv X_1 \equiv \frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^I \qquad Y \equiv X_2 \equiv \frac{1}{2}\partial_\mu\chi^I\partial^\mu\chi^I$$





Adding more sets of axions in the bulk just increases the amount of diffusive modes on boundary.

Outlook

- Hydrodynamic description of the holographic models (physical nature of the diffusive mode).
- Dynamical strains (ductile-brittle transition).
- More realistic models (e.g. including dilaton).

Thank you for listening!