Taub-NUT Blackholes with Additional Hair

刘海山

2024引力与宇宙学专题研讨会

This talk is based on work:

Yu-Qi Chen and Hai-Shan Liu, arxiv: 2406.03692, to appear in SCPMA

Hai-Shan Liu and Lei Zhang, arxiv: 2407.08208, JHEP 10 (2024) 067

Outline

- Taub-NUT spacetimes

- New Taub-NUT black holes in higher derivative gravity theories

Taub-NUT spacetime

- Taub-NUT spacetime is a solution to Einstein's general relativity, first discovered by Taub in 1951, subsequently rediscovered by Newman, Tamburino and Unti in 1963.

- The solution is a simple generalization of the Schwarzschild spacetime. In addition to a Schwarzschild-like mass parameter m, it contains one more parameter which is called NUT parameter n.

- The nature of this NUT parameter n was viewed as a "gravitational magnetic charge" by some people, thus the solution is a sort of gravitational dyon and its Euclidean continuation is the solution known as Kaluza-Klein monopole.

Taub-NUT spacetime

- People pointed out that there may exist NUT charge in the real world.

- Recent works shows that CRO J165540 and M87 might contain NUT charge.

  C.Chakraborty and S.Bhattacharyya, Phys.Rev.D98,043021(2018);
  C.Chakraborty and S.Bhattacharyya, JCAP 05,034(2019);
  M.Ghasemi-Nodehi and C.Chakraborty, EPJC 81,939(2021).

- Primordial black holes could contain NUT charge, too.

  C.Chakraborty and S.Bhattacharyya, Phys.Rev.D106,103028(2022);
  C.Chakraborty and B.Mukhopadhyay, EPJC 83,937(2023).

Taub-NUT spacetime

- The theory is just Einstein's general relativity

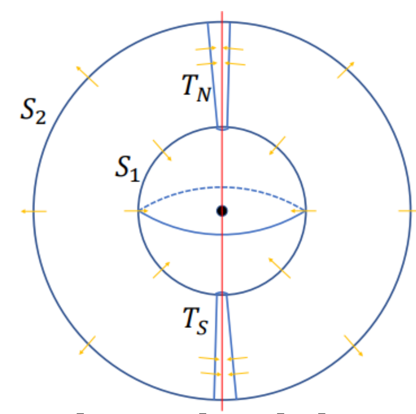
$$\mathcal{L} = \sqrt{-g}\mathrm{R}$$

- Taub-NUT metric is

$$\mathbf{ds^2 = -f(dt + 2n\cos\theta d\varphi)^2 + \frac{dr^2}{f} + (r^2 + n^2)(d\theta^2 + \sin^2\theta d\phi^2)}$$

$$\mathbf{f = \frac{r^2 - 2mr - n^2}{r^2 + n^2}}$$

with two parameters m and n.



- This metric does not have curvature singularities, but has the so-called Minser string singularities at north and south pole ($\theta = 0, \pi$), where the metric fails to be invertible.

I: Black holes with spin-2 hair

Higher order curvature gravity

$$L_4 = \sqrt{-g}\,(R + \beta\, R^2 + \alpha\, C^2 + \gamma\, GB)$$

- Linear spectrum: one massive scalar mode, one massless gravity mode and one massive gravity mode.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu};\ \ h = \eta^{\mu\nu} h_{\mu\nu}\,,\ h^T_{\mu\nu} = h_{\mu\nu} - \frac{1}{4} h \eta_{\mu\nu}$$

$$(\Box - \mu_0^2) h = 0, \qquad \Box(\Box - \mu_2^2) h^T_{\mu\nu} = 0\ , \qquad \mu_0^2 = \frac{1}{6\beta}\,, \quad \mu_2^2 = -\frac{1}{2\alpha}$$

- Asymptotic to Minkowski spacetime, the expansion of static spherical metric to linear order in large r is

$$ds_4^2 = -h dt^2 + \frac{dr^2}{f} + r^2(d\theta^2 + Sin^2\theta\, d\phi^2),$$

$$h = 1 - \frac{m}{r} - \frac{c_0}{r} e^{-\mu_0 r} - \frac{c_2}{r} e^{-\mu_2 r} + \cdots\,, \qquad f = \ \ldots$$

The theory contains several modes. It doesn't guarantee the theory admits black holes that carry the corresponding hair.

I: Black holes with spin-2 hair in high order curvature gravity

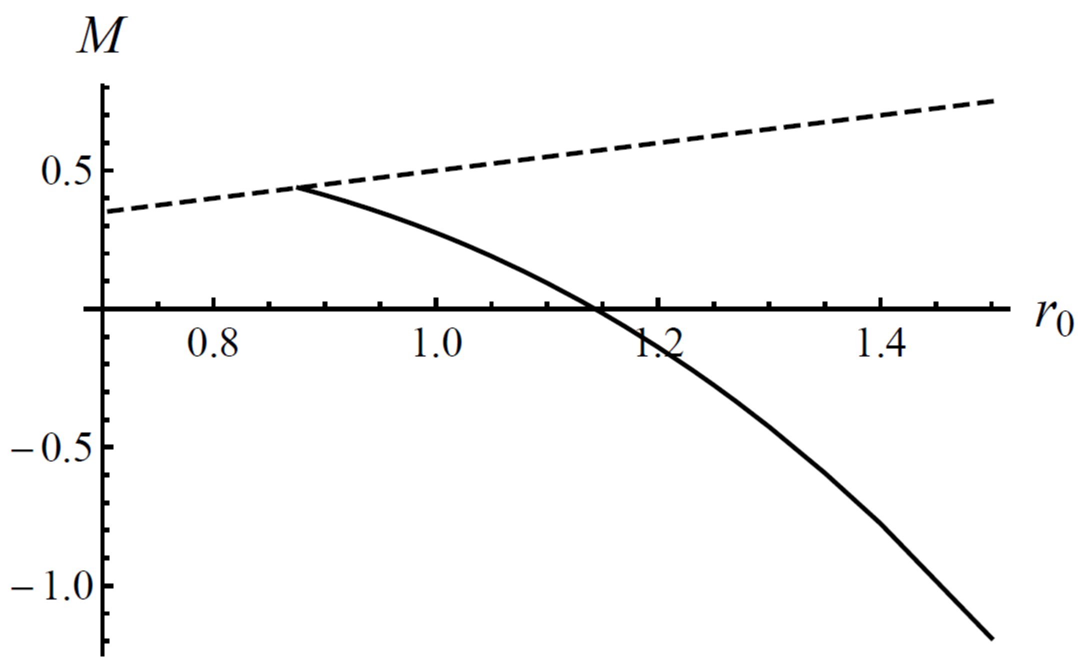
- In four dimensional spacetimes, any Ricci-flat solution (e.g. Schwarzschild black hole) is still a solution of quadratic curvature gravity.

- This raises a question: does there exist new black hole solution beyond Schwarzschild black hole in the high curvature gravity theory ?

I: Black holes with spin-2 hair in high order curvature gravity

Black hole solution with ghost massive spin-2 hair was constructed in Einstein-Weyl gravity($\beta = 0$).

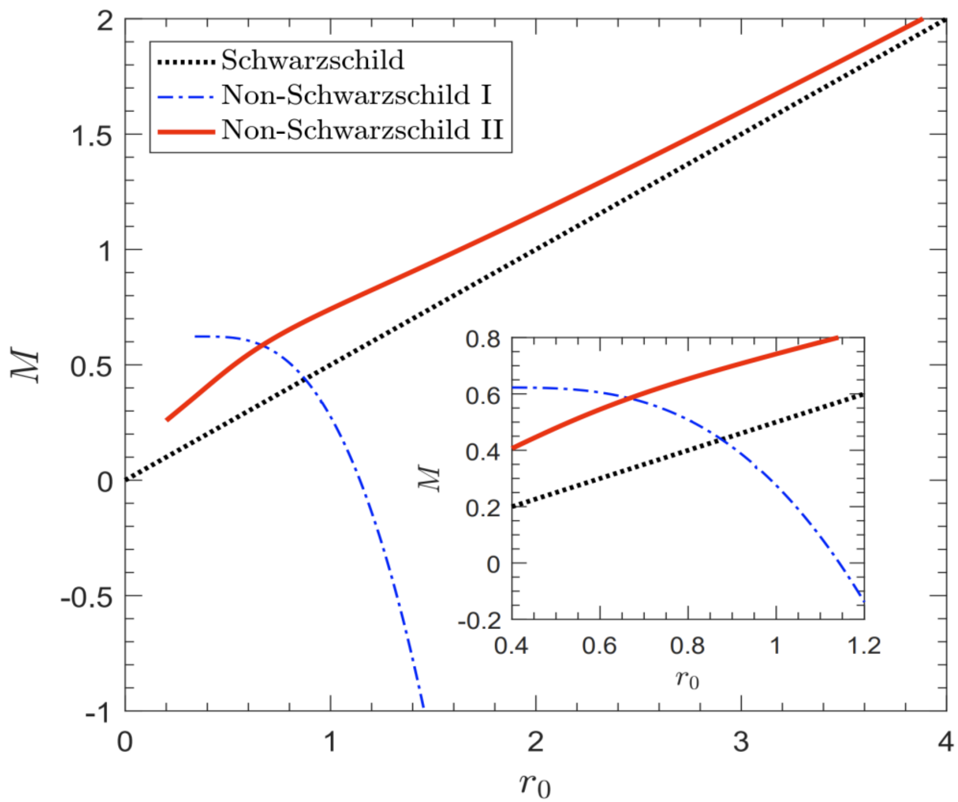
Lu, Perkins, Pope and Stelle, Phys.Rev.Lett.114, 171601, 2015



I: Black holes with spin-2 hair in high order curvature gravity

Later, a new branch solution was constructed in Einstein-Weyl gravity($\beta = 0$) by Huang, Liu and Zhang.

Huang, Liu and Zhang, JHEP 02 (2023) 057



I: Black holes with spin-2 hair in high order curvature gravity

- No new branch was discovered after these.

I: Black holes with spin-2 hair in high order curvature gravity

- No new branch was discovered after these.

- Are these all ?

I: Black holes with spin-2 hair in high order curvature gravity

- No new branch was discovered after these.

- Are these all ?

- As we mentioned earlier, Taub-NUT black hole is also Ricci-flat.

No scalar hair theorem

- Theory

$$L_4 = \sqrt{-g}\,(R + \beta\, R^2 + \alpha\, C^2)$$

- Taub-NUT ansatz

$$ds_4^2 = -h(r)(dt + 2n\cos\theta d\phi)^2 + \frac{dr^2}{f(r)} + (r^2+n^2)(d\theta^2 + \sin\theta^2 d\phi^2)$$

- The trace of the equations of motion is

$$\beta(\Box - m_0^2)R = 0$$

$m_0 = 1/\sqrt{6\beta}$ is the mass of scalar mode. The equation can be turned to

$$\int dx^4 \sqrt{g} R(\Box - m_0^2)R = \int dx^4 \Big[\partial_r(\sqrt{g} g^{rr} R \partial_r R) - \sqrt{g}(g^{rr}\partial_r R \partial_r R + m_0^2 R^2)\Big] = 0$$

It can be seen that R must vanish $R = 0$.

Lichnerowicz Mode

- Does new Taub-NUT black hole exist ? And if yes, where are they?

- The Lichnerowicz Mode analysis of the theory can give us an answer.

Lichnerowicz Mode

- Making a tensor perturbation under Taub-NUT background.

$$(\Delta_L + \frac{1}{2\alpha})\delta R_{\mu\nu} = 0$$

$$\Delta_L \delta R_{\mu\nu} = -\Box \delta R_{\mu\nu} - 2R_{\mu\nu\rho\sigma}\delta R^{\rho\sigma}$$

- And $\delta R_{\mu\nu}$ is a transverse and traceless tensor.

$$g^{\mu\nu}\delta R_{\mu\nu} = 0 \quad \nabla^{\mu}\delta R_{\mu\nu} = 0.$$

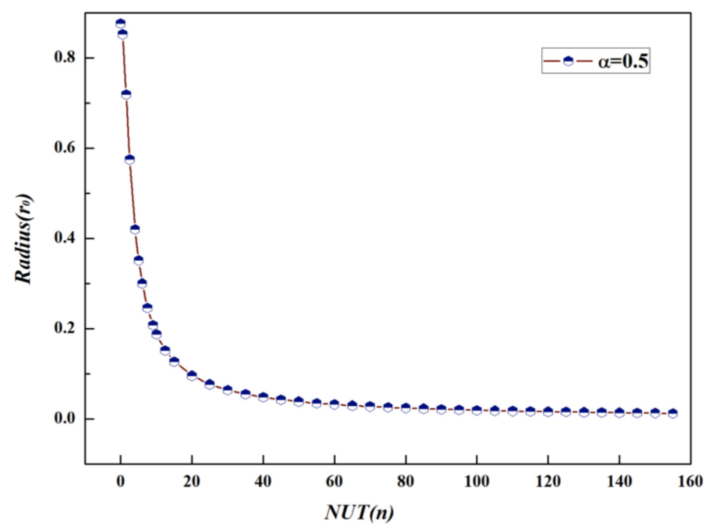
- It implies that once the Lichnerowicz operator has a transverse traceless eigenfunction with eigenvalue $\lambda = -\frac{1}{2\,\alpha}$, then there exists a linearized perturbation away from the Taub-NUT solution.

Lichnerowicz Mode

- The Lichnerowicz equation can not be solved analytically, but it can be solve in the large r, which gives the Yukawa like mode

$$e^{\pm\sqrt{-\lambda r}}$$

- The equation can be solved numerically, the relation between r_0 and NUT parameter n is shown for $\alpha = 1/2$.



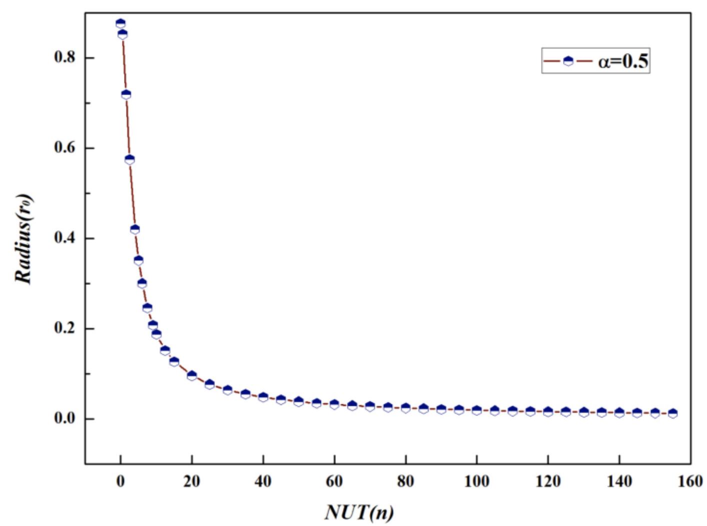
- It can be seen that when n=0, r_0 =0.876, which covers the Schwarzschild case

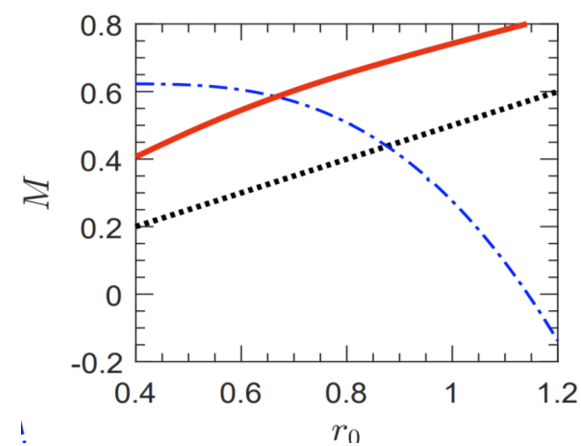
Lichnerowicz Mode

- The Lichnerowicz equation can not be solved analytically, but it can be solve in the large r, which gives the Yukawa like mode

$$e^{\pm\sqrt{-\lambda r}}$$

- The equation can be solved numerically, the relation between r_0 and NUT parameter n is shown for $\alpha = 1/2$.





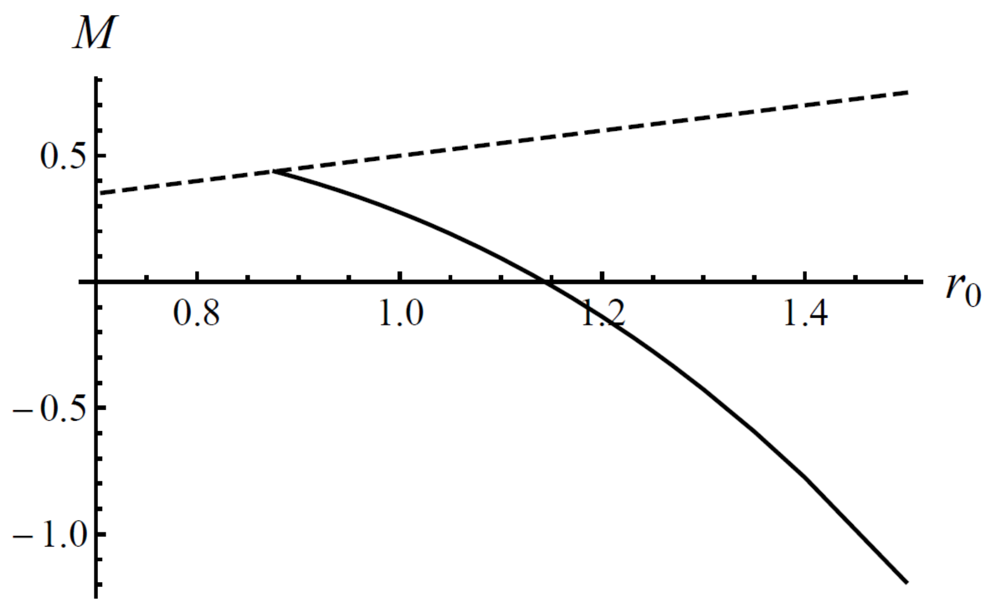
- It can be seen that when n=0, r_0 =0.876, which covers the Schwarzschild case

Lichnerowicz Mode

- The Lichnerowicz is very important for us to find new black hole numerically.

- First, it can tell us if a black hole exists.

- Second, it can tell us where the new black hole will emerge.



New Taub-NUT black holes in high order curvature gravity

- Theory

$$L_4 = \sqrt{-g}\,(R + \alpha\, C^2)$$

- Taub-NUT ansatz

$$ds_4^2 = -h(r)(dt + 2n\cos\theta d\phi)^2 + \frac{dr^2}{f(r)} + (r^2+n^2)(d\theta^2 + \sin\theta^2 d\phi^2)$$

- Near horizon r_0, the expansions are

$$f(r) = f_1(r-r_0) + f_2(r-r_0)^2 + f_3(r-r_0)^3 + \ldots$$

$$h(r) = h_1(r-r_0) + h_2(r-r_0)^2 + h_3(r-r_0)^3 + \ldots$$

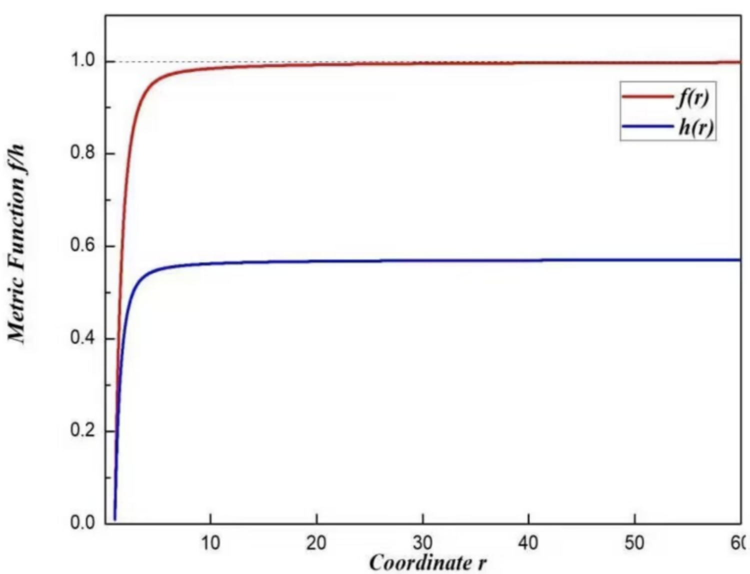
with

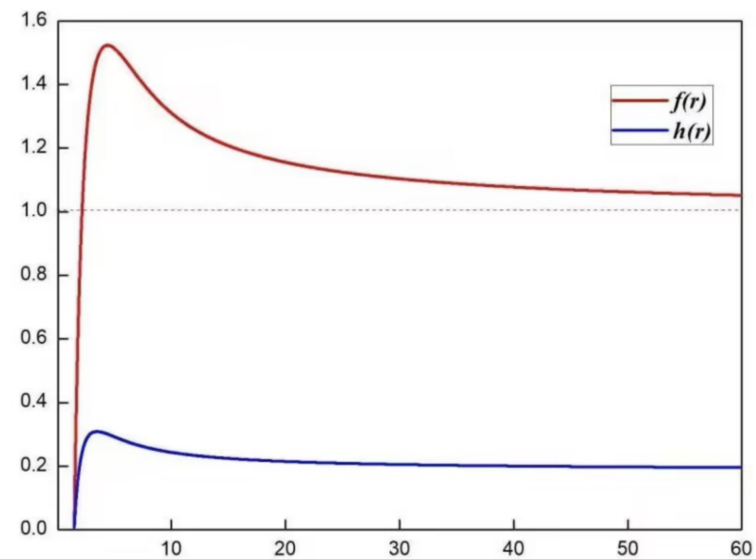
$$f_2 = \frac{-3n^2[1+4\alpha f_1(f_1-h_1) - f_1 r_0]}{8\alpha f_1 r_0(r_0^2+n^2)} + \frac{8\alpha f_1(1-2f_1r_0) + 3r_0(f_1r_0-1)}{8\alpha f_1(r_0^2+n^2)}$$

$$h_2 = \frac{h_1 n^2[1+4\alpha f_1(f_1-h_1) - f_1 r_0]}{8\alpha f_1^2 r_0(r_0^2+n^2)} + \frac{8\alpha f_1(-1+2f_1r_0) + r_0(f_1r_0-1)}{8\alpha f_1^2(r_0^2+n^2)}$$

New Taub-NUT black holes in high order curvature gravity

- Numerically, we shall integrate from horizon to infinity, using shooting method to get rid of the diverging mode $e^{+\sqrt{-\lambda r}}$ and keep the converging mode $e^{-\sqrt{-\lambda r}}$.

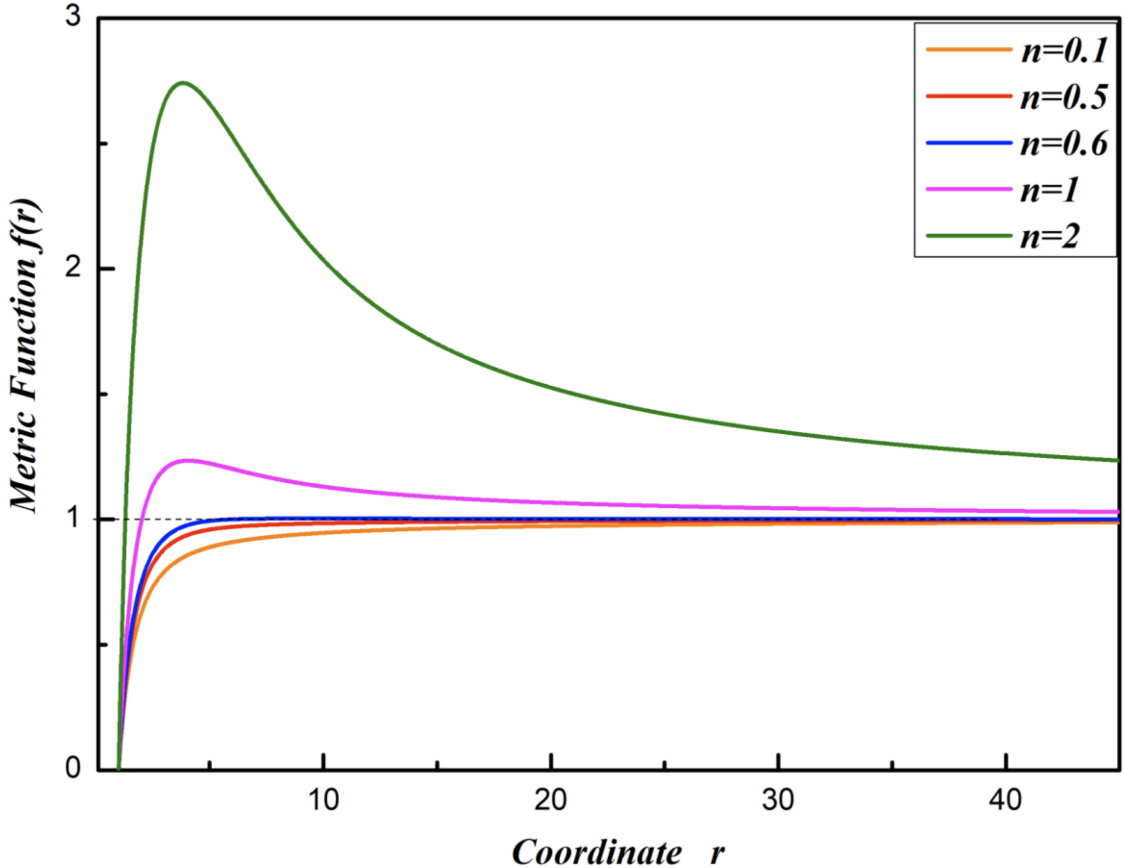




- $\alpha = 0.5, n = 0.5;$ ($r_0 = 1$, $m > 0$; $r_0 = 1.5$, $m < 0$)

New Taub-NUT black holes in high order curvature gravity

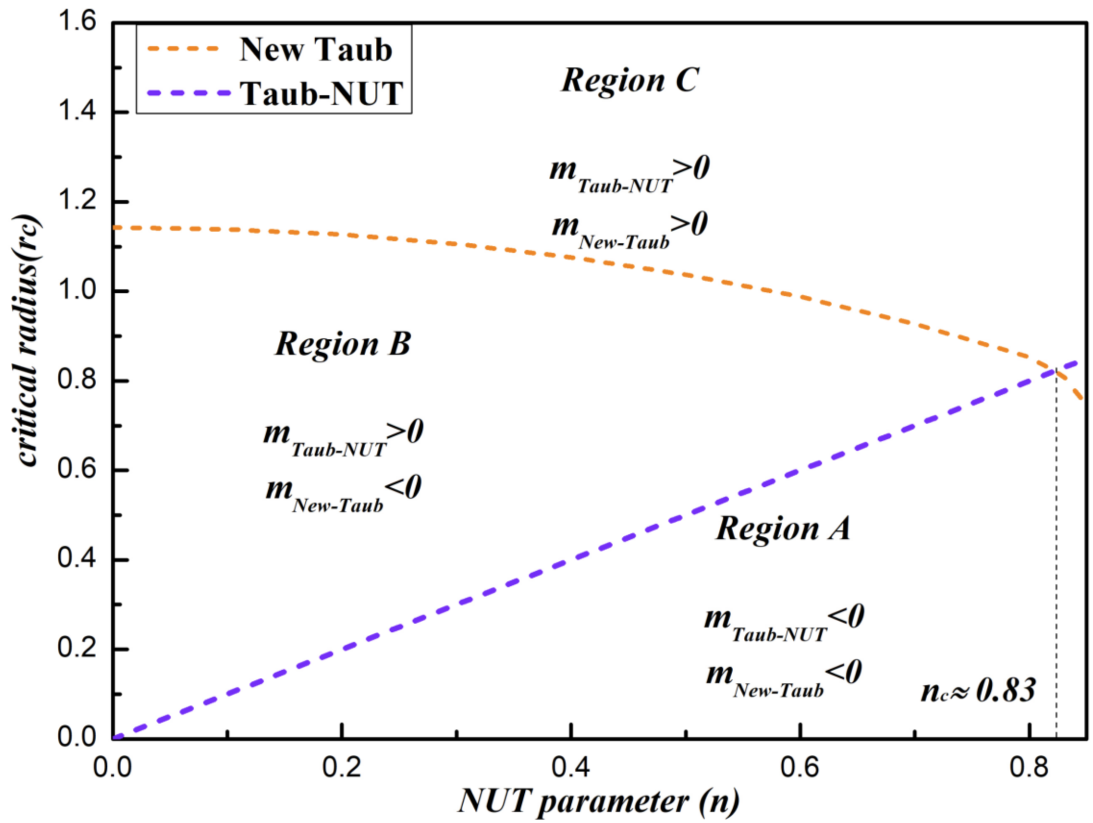
- Metric profile with different NUT parameter n



- $\alpha = 0.5, \quad r_0 = 1$

New Taub-NUT black holes in high order curvature gravity

- Critical line of m=0



New Taub-NUT black holes in high order curvature gravity

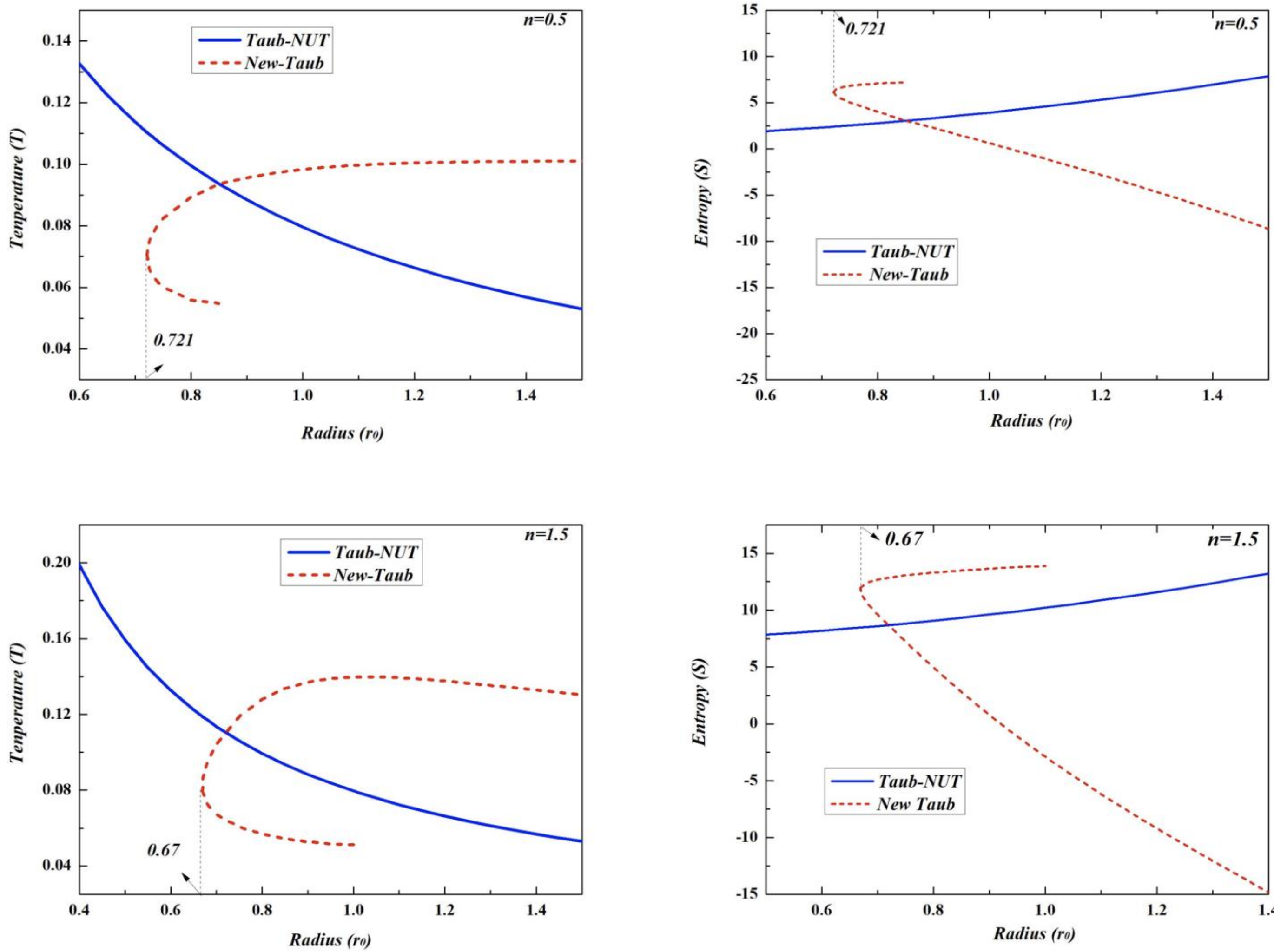
- Temperature and entropy

$$T = \frac{\sqrt{f'(r_0)h'(r_0)}}{4\pi} = \frac{\sqrt{1 + r_0\delta}}{4\pi r_0}$$

$$S = -\frac{1}{8}\int \sqrt{h}d\Omega\epsilon_{ab}\epsilon_{cd}\frac{\partial L}{\partial R_{abcd}} = \pi(r_0^2 + n^2) - 4\pi\alpha r_0\delta$$

New Taub-NUT black holes in high order curvature gravity

- Temperature and entropy



II scalarization: Black holes with scalar hair

- No hair theorem: it is well-know that Einstein gravity minimally coupled to massless scalar can not supports a black hole with non trivial scalar hair.

$$L_4 = \sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 \right)$$

- The integration

$$\int dx^4 \sqrt{g}\phi \nabla_\mu \nabla^\mu \phi = -\int dx^4 \sqrt{g}(\nabla^\mu \phi)^2$$

- EOM for scalar is

$$\nabla_\mu \nabla^\mu \phi = 0$$

so

$$\phi = 0$$

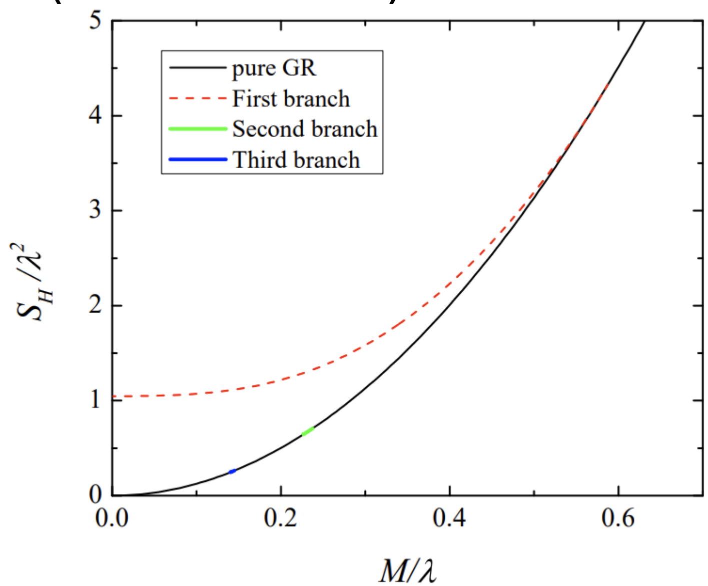
Scalarization in EsGB

- Recently, a way of scalariztion of black holes by using Gauss-Bonnet terms was carried out.

Antoniou, Bakopoulos, Kanti, Phys.Rev.Lett.120,131102,2018;
Doneva and Yazadjiev, Phys.Rev.Lett.120,131103,2018;
Silva, Sakstein, Gualtieri, Sotiriou, Berti, Phys.Rev.Lett.120,131104,2018.

- It was shown that Einstein-scalar theory can excite new scalar hairy black hole by adding non-minimally coupled term

$$L_4 = \sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2 \right) + \frac{\lambda^2}{12}(1 - e^{-6\phi^2})GB$$



- How about Taub-NUT background ?

Scalarization of Taub-NUT

- Theory

$$L_4 = \sqrt{g}\left(R - \frac{1}{2}(\partial\phi)^2\right) + \frac{\lambda^2}{12}(1 - e^{-6\phi^2})GB$$

- Test scalar under Taub-NUT background

$$\nabla_\alpha \nabla^\alpha \delta\varphi + \frac{\lambda^2}{4} R_{GB}^2 \delta\varphi = 0$$

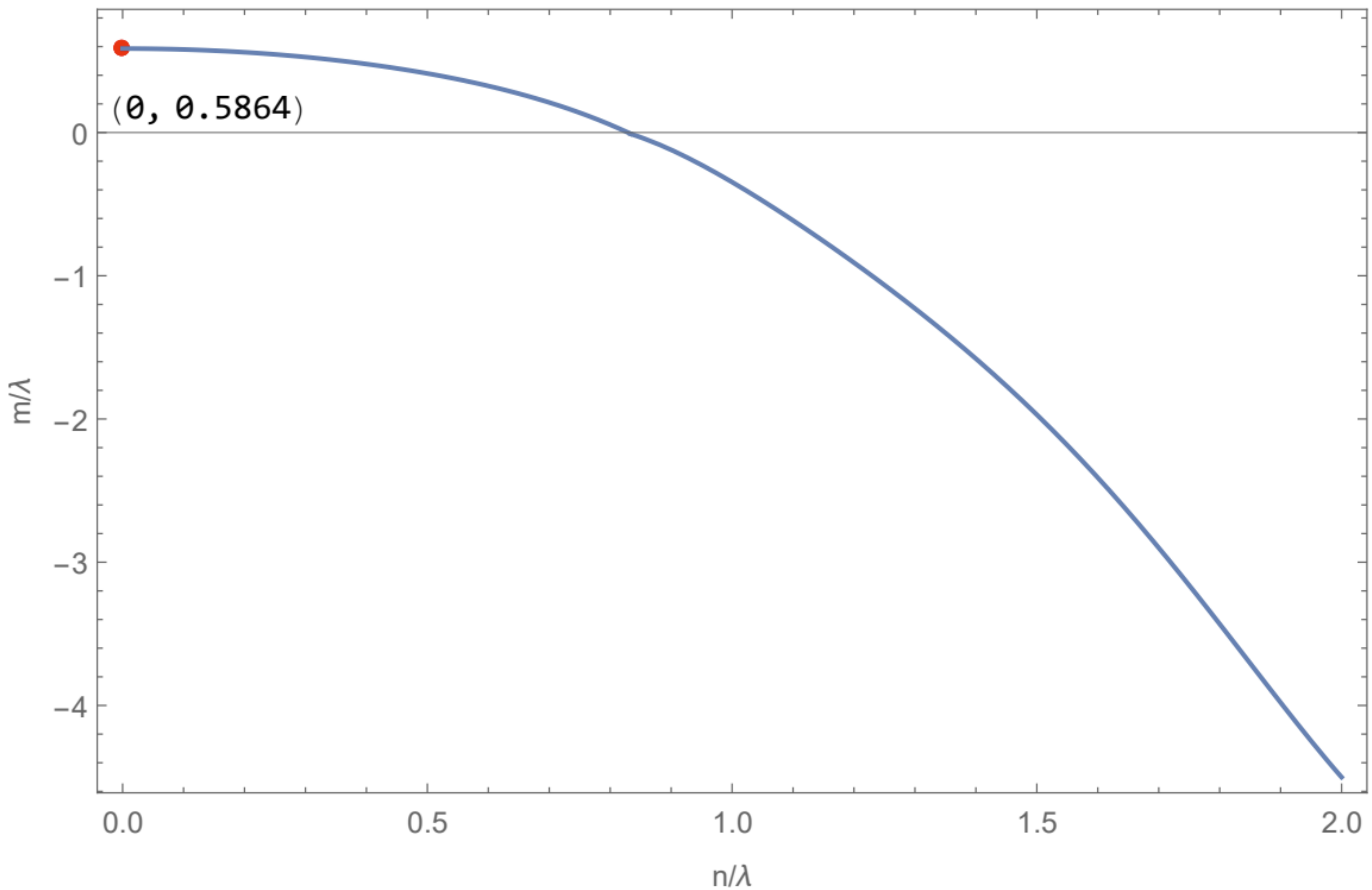
- Performing a spherical harmonics of the scalar, the radial equation is

$$\frac{1}{(r^2+n^2)}\frac{d}{dr}\left[(r^2 - 2mr - n^2)\frac{dU_l}{dr}\right] - \left[\frac{l(l+1)}{n^2+r^2} - \frac{\lambda^2}{4}R_{GB}^2\right]U_l = 0 .$$

It becomes an eigenvalue problem, fixing (n, λ), the equation can admit solutions for a discrete parameter m.

Scalarization of Taub-NUT

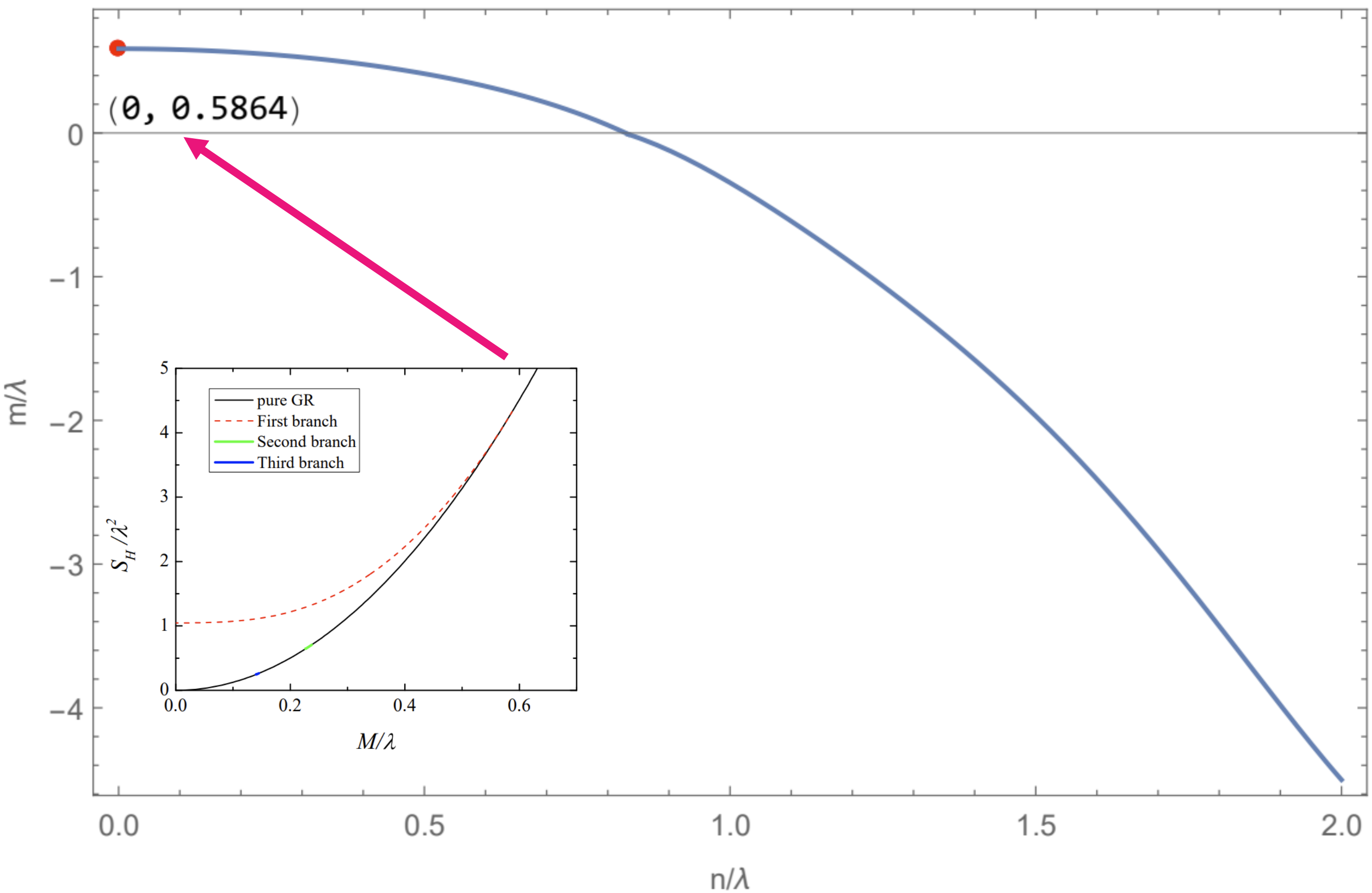
- The equation can be solved numerically, which gives



- This analysis shows the Taub-NUT black hole is not stable in these points, where new black holes with scalar hair may emerge.

Scalarization of Taub-NUT

- The equation can be solved numerically, which gives



- This analysis shows the Taub-NUT black hole is not stable in these points, where new black holes with scalar hair may emerge.

New Taub-NUT with scalar hair

- For Taub-NUT ansatz, the equations of motion are very complicated.

$$4fh^2r\left[2n^2\Psi_r + r(n^2+r^2)\Psi_r'\right] - 3n^2f^2(n^2+r^2)\left(-1+2\Psi_r h' + 4h\Psi_r'\right) -$$
$$fh(n^2+r^2)^3\varphi'^2 - fh(n^2+r^2)\left[2n^2+r^2-6r^2\Psi_r h' + 4(n^2+r^2)\Psi_r'\right] +$$
$$f(n^2+r^2)^2\left[1-(r+2\Psi_r)\,h'\right] = 0,$$

$$4f^2h\left[n^2+(n^2+r^2)^2\varphi'^2\right] - h(n^2+r^2)(n^2+r^2-4hr\Psi_r)f'^2 + 2f^2r(n^2+r^2)h' -$$
$$4n^2f^3\left(2\Psi_r h' + 4h\Psi_r' - 1\right) + f(n^2+r^2)^2 f'h' + 2fhr(n^2+r^2)\left(1-6\Psi_r' h'\right)f' +$$
$$2fh(n^2+r^2)^2f'' - 8fh^2r(n^2+r^2)\Psi_r f'' - 8fh^2\left[n^2\Psi_r + r(n^2+r^2)\Psi_r'\right]f' = 0,$$

$$fh(n^2+r^2)^4\varphi' f' + f^2\left(n^2+r^2\right)^4\varphi' h' + 2f^2h\left(n^2+r^2\right)^2\left[2r(n^2+r^2)\varphi' + (n^2+r^2)^2\varphi''\right] +$$
$$\Big\{fh(n^2+r^2)^2\left[3r^2\lambda^2h'f' - 2\lambda^2(n^2+r^2)f''\right] - 3n^2\lambda^2ff'\left(n^2+r^2\right)^2\left(f'h+fh'\right) +$$
$$4n^2\lambda^2f^3\left[2h\left(n^2-3r^2\right) + r\left(n^2+r^2\right)h'\right] + \lambda^2h(n^2+r^2)(n^2+r^2-r^2h)f' +$$
$$2\lambda^2fh^2r(n^2+r^2)\left[2n^2f' + r(n^2+r^2)f''\right] - \lambda^2f\left(n^2+r^2\right)^3f'h' +$$
$$24n^2rf^2h(n^2+r^2)^2f' - 6n^2\lambda^2f^2h\left(n^2+r^2\right)^2f''\Big\}\frac{dF(\varphi)}{d\varphi} = 0,$$

$$(n^2+r^2)h\left[r(n^2+r^2) + 2(n^2+r^2-3r^2h)\Psi_r\right]f' - n^2f^2(n^2+r^2+8rh\Psi_r) - (n^2+r^2)^2f -$$
$$(n^2+r^2)fh\left[-r^2-6n^2f'\Psi_r + (n^2+r^2)^2\varphi'^2\right] = 0,$$

New Taub-NUT with scalar hair

- Asymptotic form in the horizon

$$h(r) = \frac{1+\delta}{r_h}(r-r_h) + \mathcal{O}(r-r_h)^2\,,$$

$$f(r) = \frac{r_h\left[e^{\left(-12\varphi_h^2\right)}\left(n^2+r_h^2\right)\delta + 6\varphi_h^2(1+\delta)^2\lambda^4\right]}{6n^2\varphi_h^2(1+\delta)\lambda^4}(r-r_h) + \mathcal{O}(r-r_h)^2\,,$$

$$\varphi(r) = \varphi_h - \frac{e^{6\varphi_h^2}r_h\delta}{2\varphi_h(1+\delta)\lambda^2}(r-r_h) + \mathcal{O}(r-r_h)^2\,.$$

- Asymptotic form in the infinity

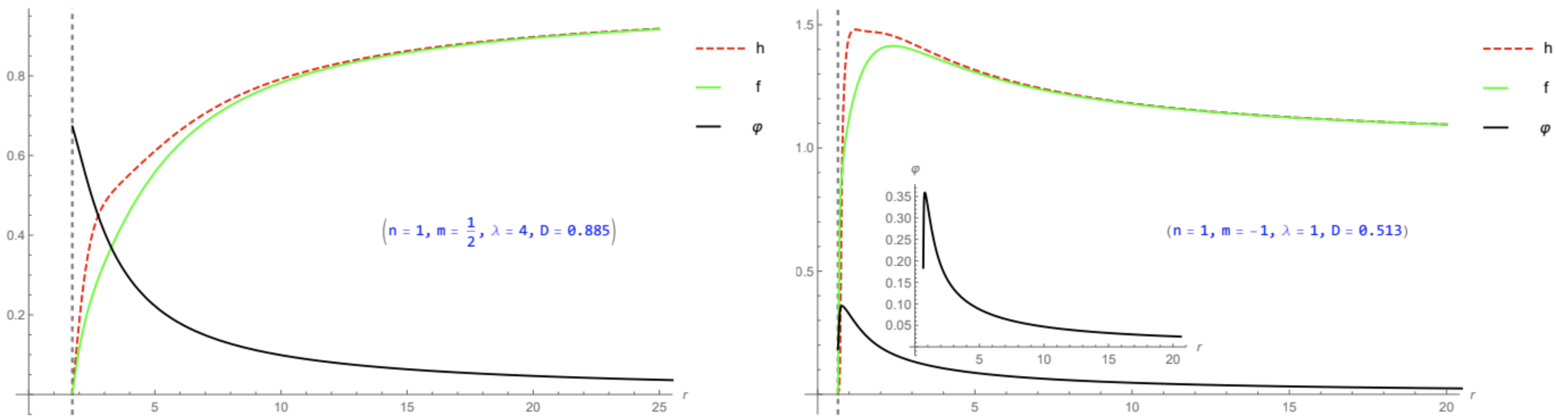
$$h(r) = 1 - \frac{2M}{r} + \frac{D^2-2n^2}{r^2} + \frac{M\left(D^2+2n^2\right)}{r^3} + \frac{\left(4D^2M^2+6n^4+2n^2D^2\right)}{3r^4}$$
$$+\frac{M}{12r^5}\left\{63D^4 - 4D^2\left(142M^2+65n^2-4\lambda^2\right) + 8\left(80M^4+12n^2-51n^2\lambda^2\right)\right.$$
$$\left.+8M^2\left(80n^2-21\lambda^2\right)\right\} + \ldots,$$

$$f(r) = 1 - \frac{2M}{r} - \frac{2n^2}{r^2} + \frac{M\left(D^2+6n^2\right)}{3r^3} + \frac{2D^2M^2+2n^2\left(D^2+3n^2\right)}{3r^4}$$
$$+\frac{M}{60r^5}\left\{55D^4 - 4D^2\left(130M^2+50n^2-28\lambda^2\right) + 8\left(80M^4-51n^2\lambda^2\right)\right.$$
$$\left.+8M^2\left(80n^2-2\lambda^2\right)\right\} + \ldots,$$

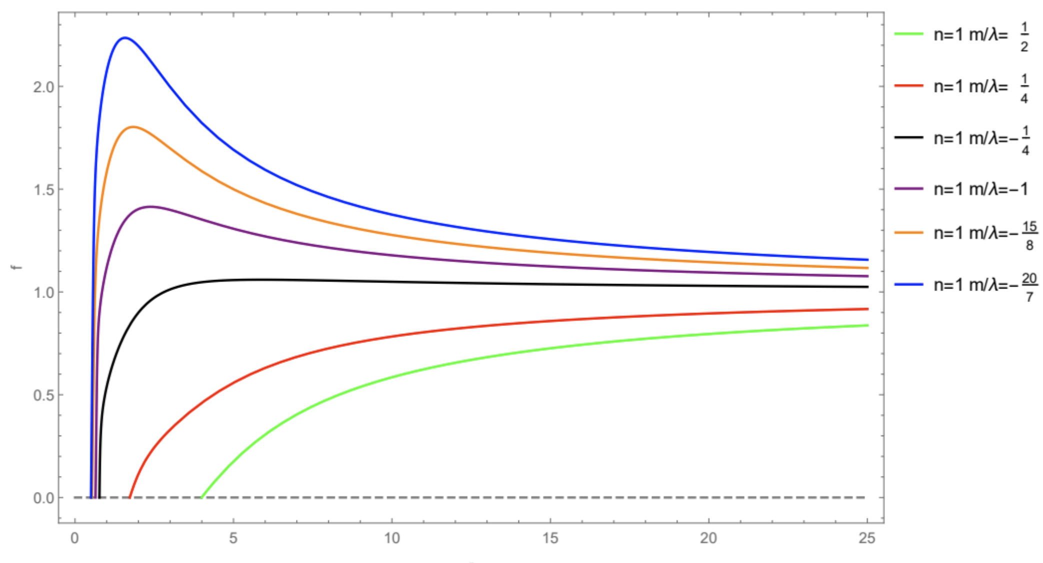
$$\varphi(r) = \frac{D}{r} + \frac{DM}{r^2} - \frac{D\left[D^2-2\left(4M^2+n^2\right)\right]}{6r^3} + \frac{DM\left(6M^2-2D^2+3n^2\right)}{3r^4}$$
$$+\frac{D}{120r^5}\left\{9D^4 + 24\left[16M^4+n^4+3n^2\lambda^2+3M^2\left(4n^2-\lambda^2\right)\right]\right.$$
$$\left.-4D^2\left(58M^2+13n^2\right)\right\} + \ldots$$

New Taub-NUT with scalar hair

- The equations can be solved numerically

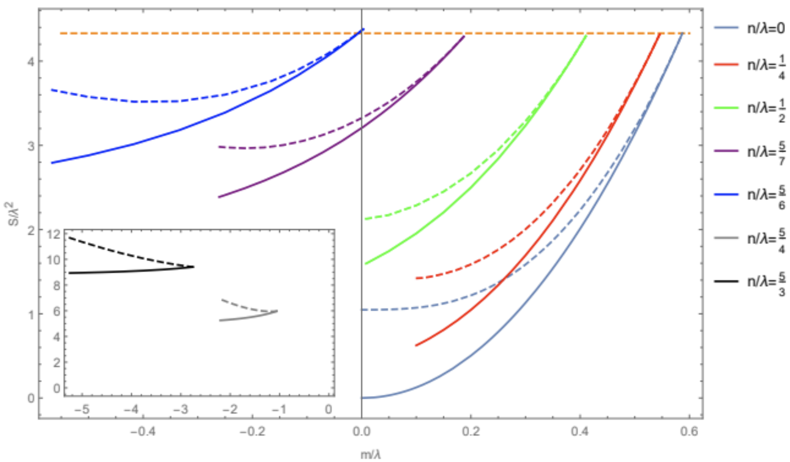


(a) $m > 0$ (b) $m < 0$

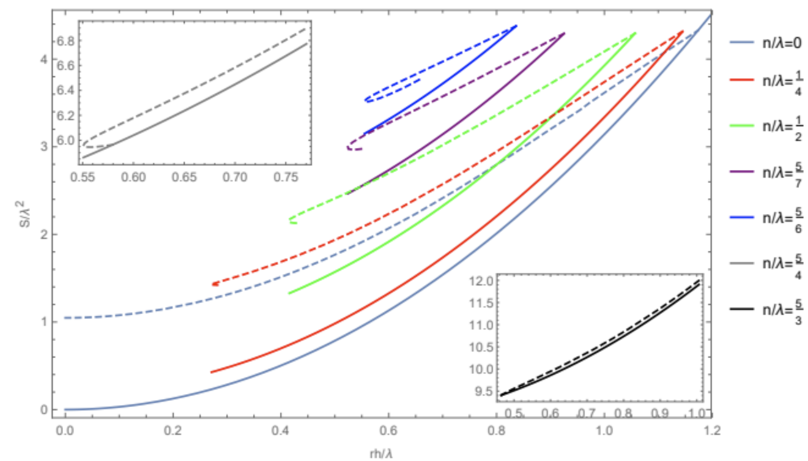


New Taub-NUT with scalar hair

- The entropy of the black hole



(a) S/λ^2 as a function of m/λ



(b) S/λ^2 as a function of r_h/λ

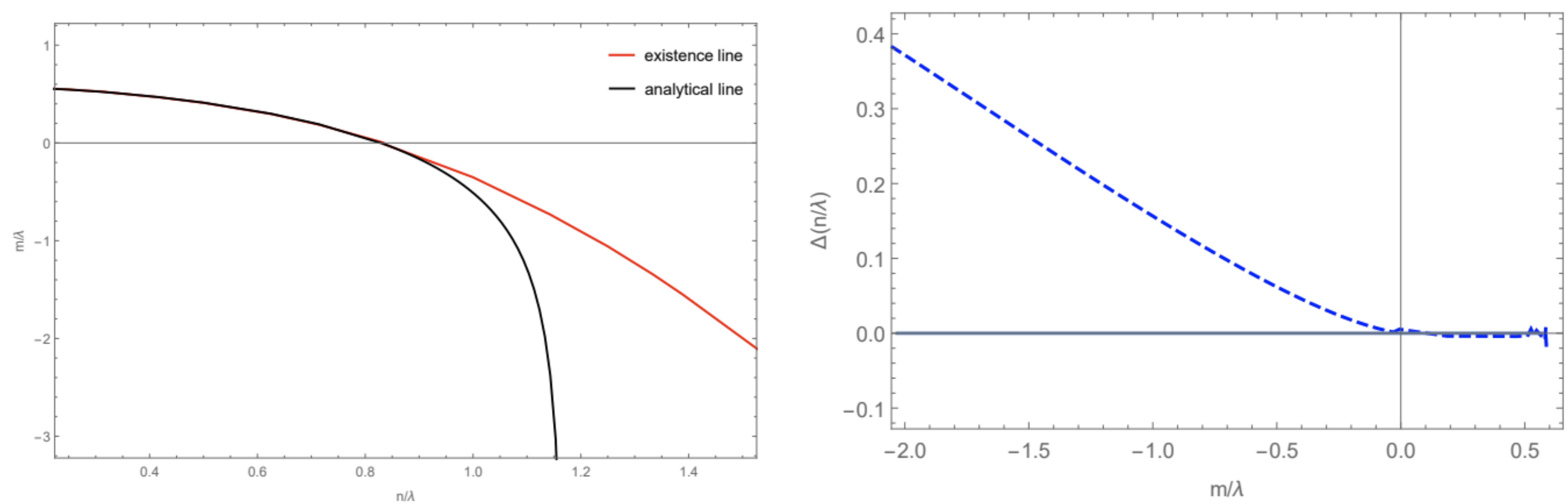
- The entropy of scalarized black hole is universal at the bifurcation point for different NUT parameter, and it is the highest value of the scalarized black hole.

$$S_{\text{scalar}-\text{free}}(m,n) < S_{\text{scalarized}}(m,n) \leq S_{\text{scalarized}}(m_{\max}(n)) = S_{\text{scalar}-\text{free}}(m_{\max}(n))$$

New Taub-NUT with scalar hair

- The universal value at bifurcation points shows a relation between m and n

$$\left(\frac{m}{\lambda}\right)^2+\left(\frac{n}{\lambda}\right)^2+\frac{m}{\lambda}\sqrt{\left(\frac{m}{\lambda}\right)^2+\left(\frac{n}{\lambda}\right)^2}\approx\frac{2.16}{\pi}$$



(a) (b)

- The universal value only holds for positive mass parameter.

Conclusion and outlook

- We constructed new Taub-NUT black holes with scalar hair or spin-2 hair in higher derivative gravity theories.

- The thermodynamics, especially the definition of mass and NUT charge for new Taub-NUT black hole in higher derivative gravity is worth further investigation.

Thank you!