“2024引力与宇宙学”专题研讨会

Universal topological classifications of black hole thermodynamics

Mainly based on:

SWW, Yu-Xiao Liu, R.B. Mann,
PRD 110, L081501 (2024)

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2024. 11. 16

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- □ **Introduction**
- □ **BH thermodynamics topology**
- □ **Universal classifications**
- □ **Summary and outlook**

- **Topological study has been widely applied in various fields of physics. New concepts and phenomena are discovered.**
- **In 2017, a topological treatment has been used to investigate the light ring stability for ultracompact objects. They showed that the light rings always come in pairs, one being a saddle point and the other a local extremum of an effective potential.** [Cunha, Berti, Herdeiro, PRL 119, 251102 (2017)]
- **In 2020, they applied the similar approach to the Kerr-like black holes and observe that there exists at least one standard LR outside the horizon for each rotation sense.** [Cunha, Herdeiro, PRL 124, 181101 (2020)]
- **We extended the study to the static and spherically symmetric black hole, and found that the result holds for the asymptotic flat, AdS, and dS cases.** [Wei, PRD 102, 064039 (2020)]

- **The stability along the angular motion is found to be stable.** [Guo and Gao, PRD 103, 104031 (2021)]
- **Topological properties of equatorial timelike circular orbits around stationary black holes were firstly constructed. The main differences come from the fact that the light ring/photon sphere is independent of the black hole or particle parameters, while the equatorial timelike circular orbits shows a close relation with these parameters.** [Wei and Liu, PRD 107, 064006 (2023)]
- **From a thermodynamical perspective, the critical point, a second-order phase transition point, can be viewed as topological defect in a generalized parameter space. The critical points are classified into two classes with different properties.** [Wei and Liu, PRD 105, 104003 (2022)]

- **Employing with the generalized free energy, we showed that the actual black hole solution can be treated as the topological defect in the thermodynamical parameter space. Black hole solutions belong to different topological classes. The local stability from the heat capacity is also found to be one of the topological properties.** [Wei, Liu, Mann, PRL 129, 191101 (2022)]
- **Hawking-Page phase transition can also be studied via such perspective of topology.** [Yerra, Bhamidipati, Mukherji, PRD 106, 064059 (2022)]
- **The small-large black hole phase transition of the first-order can also be tested.** [Fan, PRD 107, 044026 (2023)]
- **Up to now, there are hundreds of articles that consider the topology of black hole thermodynamics, light ring/photon sphere, or the equatorial timelike circular orbits.**

arXiv:2410.14414v1 [gr-qc] 18 Oct 2024

Black Holes Inside and Out 2024

Visions for the future of black hole physics

August 26 to August 30, 2024

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Same as Ever: Looking for (In)variants in the Black Holes Landscape

Carlos A. R. Herdeiro

Studying this topological charge is an informative technique. For instance, in different asymptotics it could be shown that the topological charge is still $w = -1$ [13, 14], also implying the existence of standard LRs. This also holds for some topologically non-trivial

pling, with the discontinuity at the KK value [12]. There are also connections between the existence of ergo-regions and LRs [18] and various generalizations [19, 20], besides the technique inspiring some spin-offs, for instance to understand special timelike [21, 22] (rather than null) orbits, or the topology of BH thermodynamics [23].

[13] S.-W. Wei, "Topological Charge and Black Hole Photon Spheres," *Phys. Rev. D* **102** no. 6, (2020) 064039, arXiv:2006.02112 [gr-qc].

[21] S.-W. Wei and Y.-X. Liu, "Topology of equatorial timelike circular orbits around stationary black holes," *Phys. Rev. D* **107** no. 6, (2023) 064006, arXiv:2207.08397 [gr-qc].

[23] S.-W. Wei and Y.-X. Liu, "Topology of black hole thermodynamics," *Phys. Rev. D* **105** no. 10, (2022) 104003, arXiv:2112.01706 [gr-qc].

We usually wish to determine the exactly black hole with all the parameters are fixed. But thermodynamical topology concerns the properties of black hole solutions, especially at given temperature.

➢ **Outlook** ***of PPT last year***

topological classes

-1 **0^+, 0^-** **+1**

How many thermodynamic universality classes do we have?

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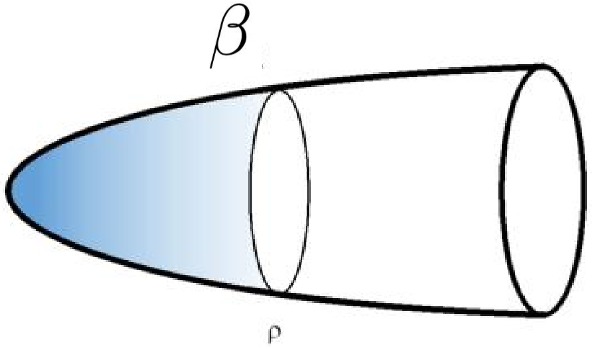
- **Partition function**

$$\mathcal{Z} = e^{-\beta F} = \int D[g] e^{-\frac{\mathcal{I}}{\hbar}} \sim e^{-\frac{\mathcal{I}}{\hbar}}$$



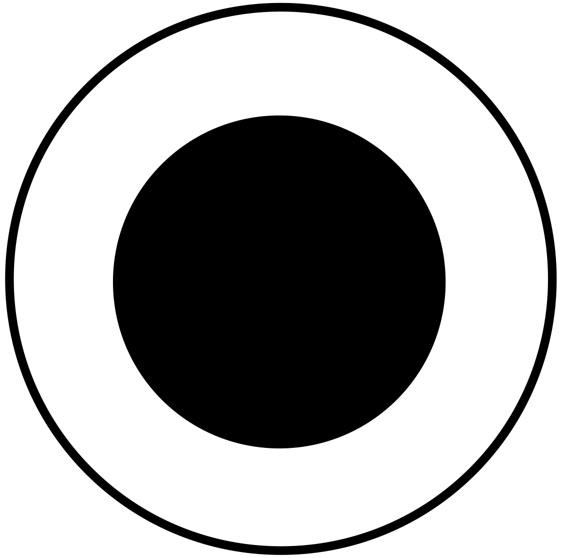
F **: Free energy**

$\mathcal{I}$ **: Euclidean action**

β **: Euclidean time**

G. W. Gibbons and S. W. Hawking, PRD 15, 2752 (1977)

- **Schwarzschild BH**
  - **thermodynamically unstable**
  - **imaginary energy fluctuations**



- **Shortcomings can be solved by placing BH in a cavity for the following BHs**

$$F = M - TS$$

$$M > \sqrt{3}/8\pi T$$

J. W. York,
PRD 33, 2092 (1986)

***M* and *T* are independent**

- **Generalized off-shell free energy**

$$\mathcal{F} = E - \frac{S}{\tau}$$

- **On-shell condition**

$$\tau = T_{\mathbf{BH}}^{-1}$$

- **Differential of generalized off-shell free energy**

$$
\begin{aligned}
\left(\frac{\partial \mathcal{F}}{\partial r_{\mathrm{h}}}\right) &= \left(\frac{\partial E}{\partial r_{\mathrm{h}}}\right) - \mathcal{T}^{-1}\left(\frac{\partial S}{\partial r_{\mathrm{h}}}\right) \\
&= \left(\frac{\partial E}{\partial S}\right)\left(\frac{\partial S}{\partial r_{\mathrm{h}}}\right) - \mathcal{T}^{-1}\left(\frac{\partial S}{\partial r_{\mathrm{h}}}\right) \\
&= \left(T - \mathcal{T}^{-1}\right)\left(\frac{\partial S}{\partial r_{\mathrm{h}}}\right)
\end{aligned}
$$

- **On-shell condition turns to**

$$
\frac{\partial \mathcal{F}}{\partial r_{\mathrm{h}}} = 0
$$

- **Vector**

$$\phi = \left(\frac{\partial \mathcal{F}}{\partial r_{\mathrm{h}}}, -\cot\Theta\csc\Theta\right)$$

Wei, Liu, Mann,
PRL 129, 191101 (2022)

- **Zero point**

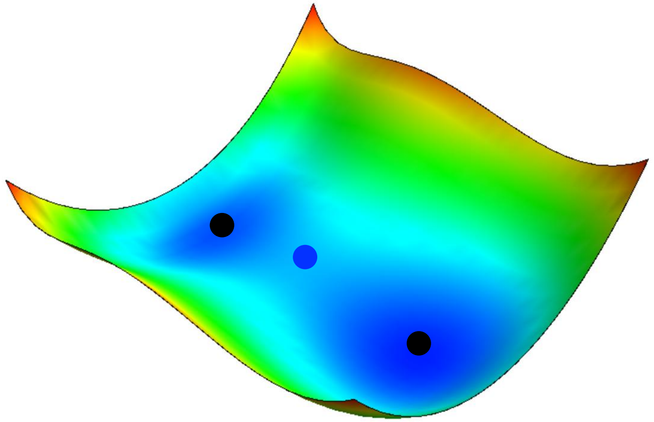
$$\Theta = \pi/2 \text{ and } \tau = T^{-1}$$

BHs exactly locate at the zero points of vector

- Θ**: auxiliary parameter**

Do not introduce new zero points
Well behaved in the parameter space
Convenient calculation of topological charge
Independent of the sign of auxiliary term

- **Generalized free energy**



$$\frac{\partial \mathcal{F}}{\partial r_h} \propto (T - 1/\tau) \qquad \frac{\partial^2 \mathcal{F}}{\partial r_h^2}\Big|_{on-shell} \propto \frac{T}{C_P}$$

w= -1 *local unstable*

w= +1 *local stable*

Local thermodynamical stability is a topological property

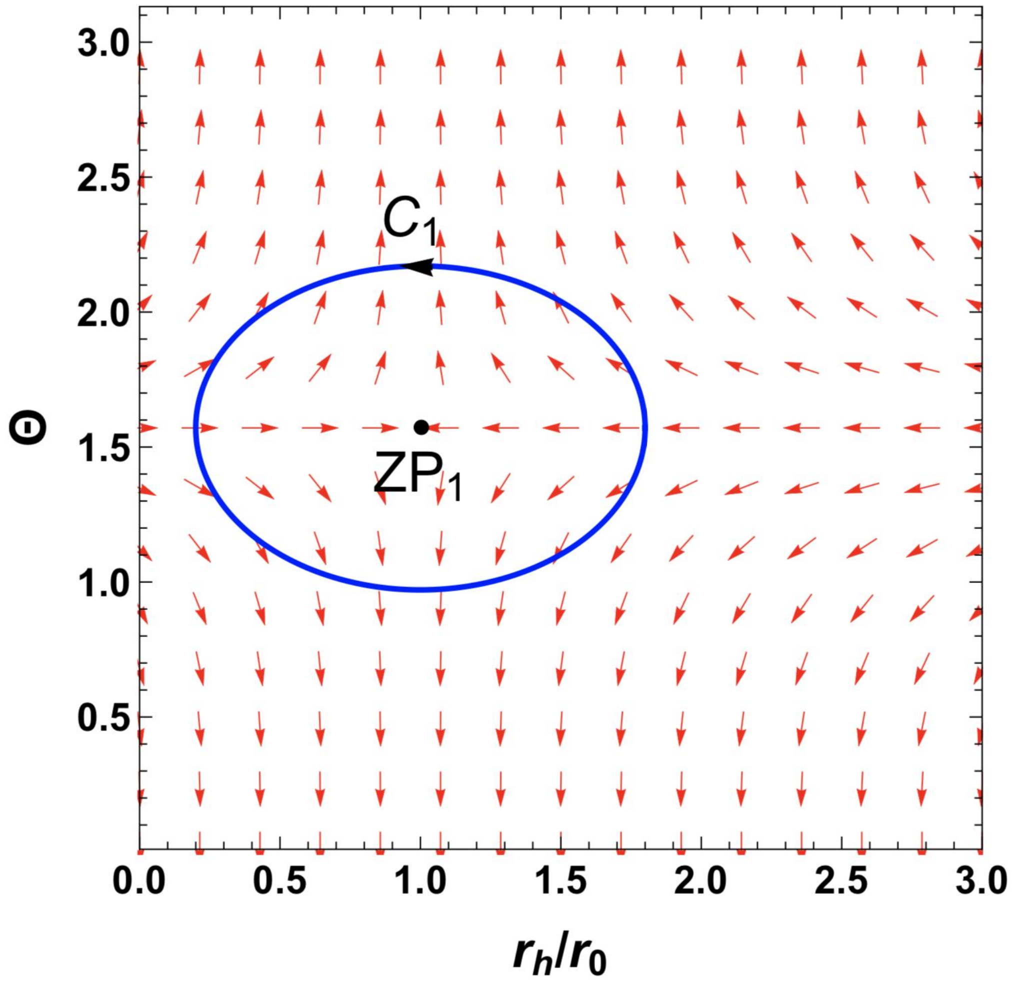
- **Schwarzschild BH**

$$\tau/r_0 = 4\pi$$

$$\phi^{r_\mathrm{h}} = \frac{1}{2} - \frac{2\pi r_\mathrm{h}}{\tau},$$
$$\phi^{\Theta} = -\cot\Theta\csc\Theta.$$

w_{ZP1}= -1 *local unstable*

W= -1



	Sch BH	RN BH	RN-AdS BH
W	-1	0	1
generation point	0	1	1 or 0
annihilation point	0	0	1 or 0

three topological classes !?

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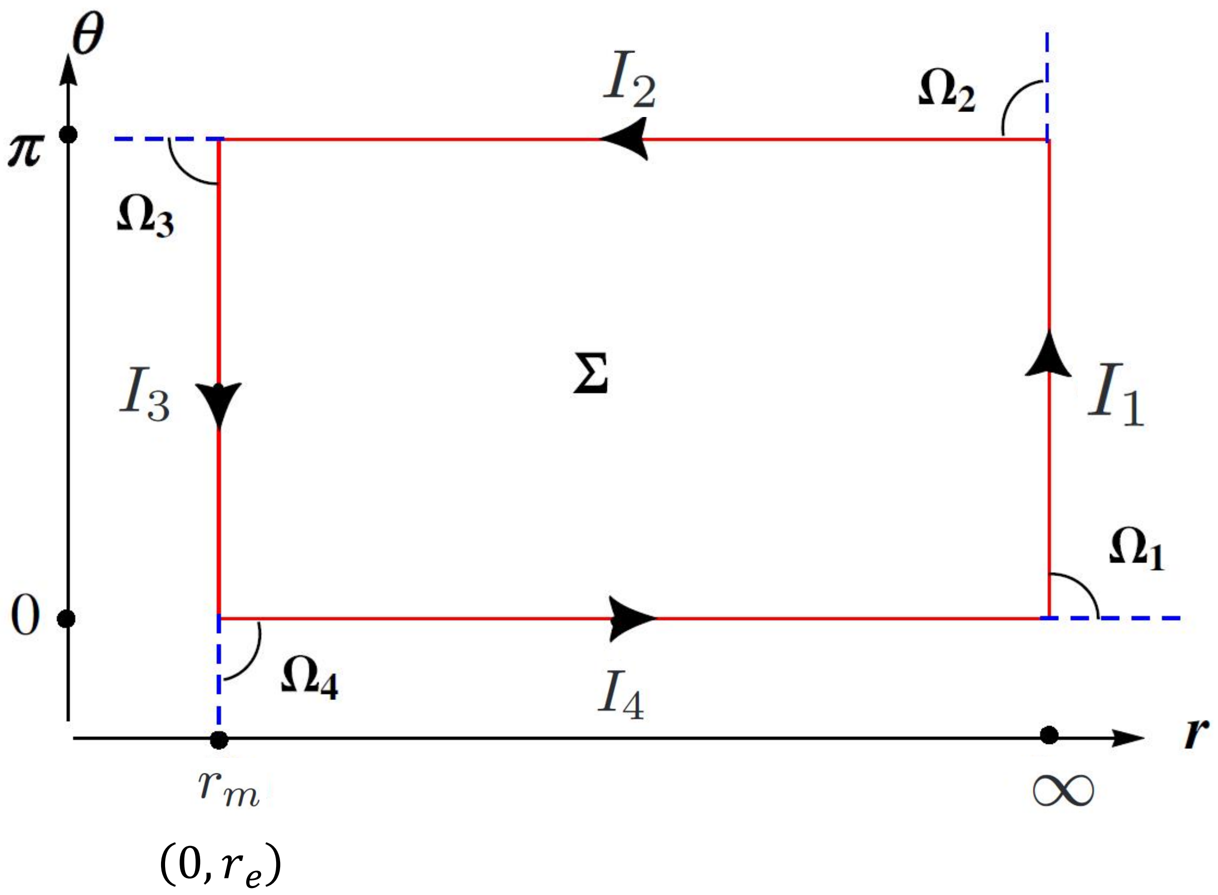
➢ Whole parameter space

$$C = I_1 \cup I_2 \cup I_3 \cup I_4$$



$$w = \frac{1}{2\pi} \oint_C d\Omega$$

➢ Asymptotic limits of Hawking temperature

$$case\ I\ :\quad \beta(r_m)=0, \qquad \beta(\infty)=\infty,$$
$$case\ II\ :\quad \beta(r_m)=\infty, \qquad \beta(\infty)=\infty,$$
$$case\ III\ :\quad \beta(r_m)=\infty, \qquad \beta(\infty)=0,$$
$$case\ IV\ :\quad \beta(r_m)=0, \qquad \beta(\infty)=0$$

	I_1	I_2	I_3	I_4	W
case I	←	↑	→	↓	-1
case II	←	↑	←	↓	0
case IIII	→	↑	←	↓	+1
case IV	→	↑	→	↓	0

$$\phi^{r_h}=\frac{\partial\tilde{\mathcal{F}}}{\partial r_h}=\frac{\partial M}{\partial S}\frac{\partial S}{\partial r_h}-\frac{1}{\tau}\frac{\partial S}{\partial r_h}=\frac{\partial S}{\partial r_h}\left(\frac{1}{\beta}-\frac{1}{\tau}\right)$$

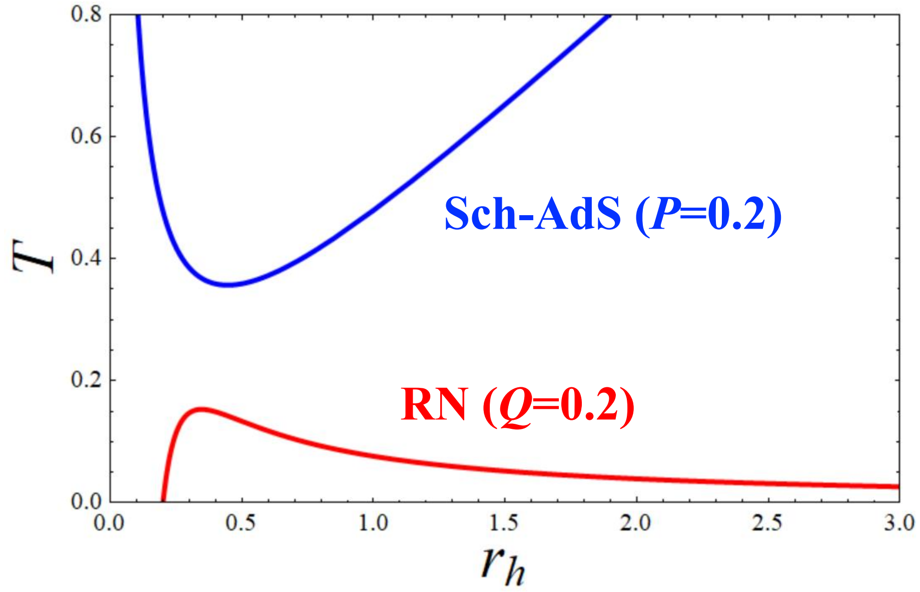
➢ Representative examples

Case II: RN BH

- **Upper bound of temperature**
- **Extremal case**
- **Minimum horizon radius**

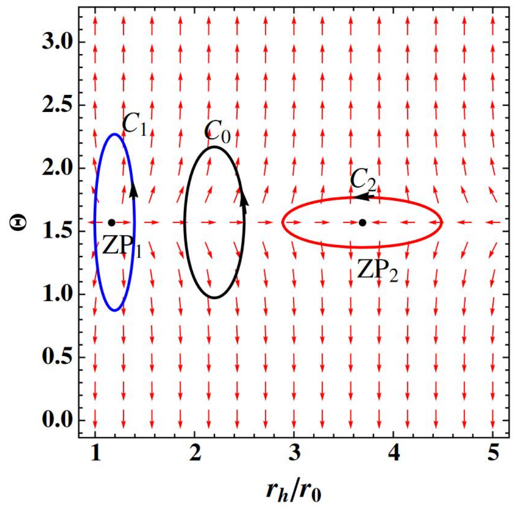
Case IV: Sch-AdS BH

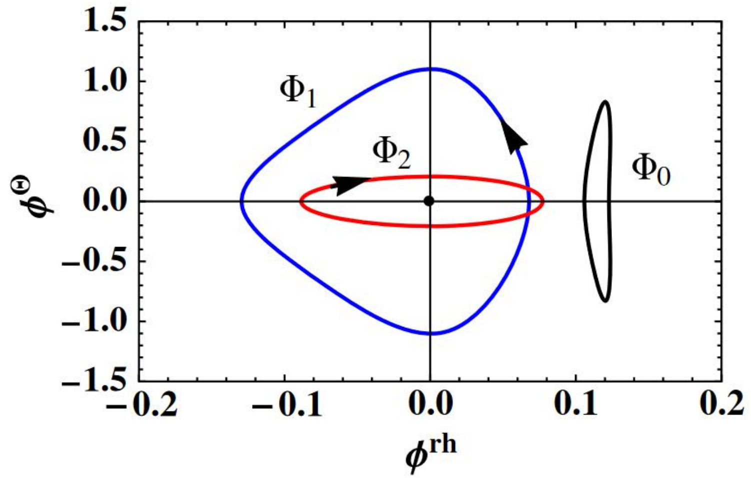
- **Lower bound of temperature**
- **No extremal case**
- **No horizon radius constrain**

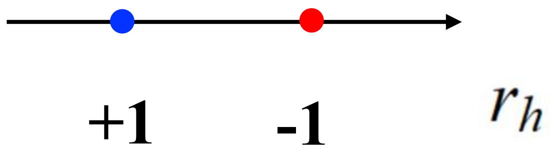


significant differences

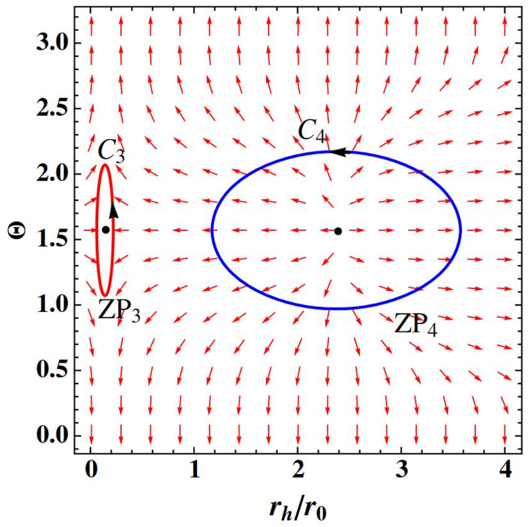
Case II: RN BH

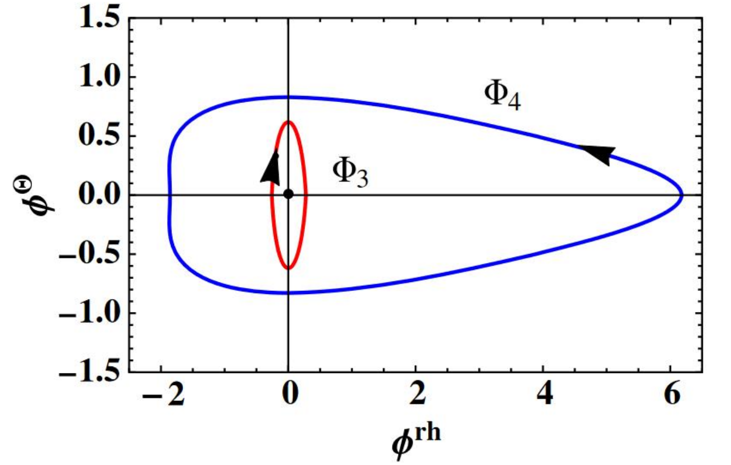


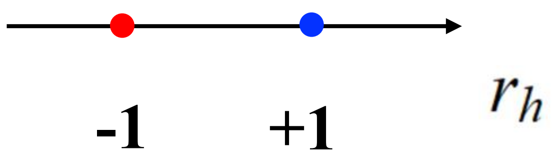




Case IV: Sch-AdS BH







- **Four topological classifications**

W^{1-},	W^{0+},	W^{0-},	W^{1+}
Sch	**RN**	**Sch-AdS**	**RN-AdS**

➢ State systematic orderings

$$W^{1-} \quad [-,(+,-),\ldots,(+,-)]$$
$$W^{0+} \quad [+,(-,+),\ldots,-]$$
$$W^{0-} \quad [-,(+,-),\ldots,+]$$
$$W^{1+} \quad [+,(-,+),\ldots,(-,+)]$$

$$W^{1-}=[-,-], \qquad W^{0+}=[+,-],$$
$$W^{0-}=[-,+], \qquad W^{1+}=[+,+].$$

- **Innermost states**

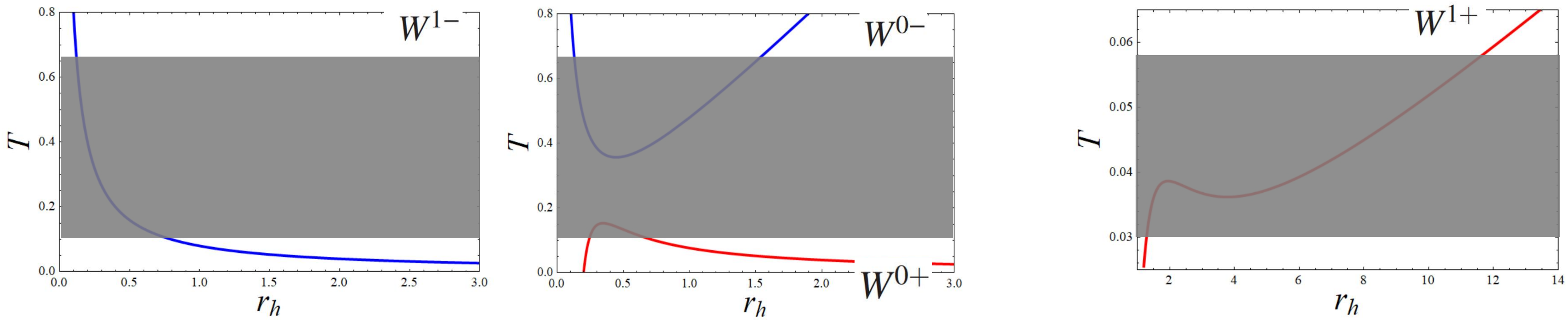
  unstable, stable, unstable, stable

- **Outermost states**

  unstable, unstable, stable, and stable

➢ Universal thermodynamical properties

	Innermost	Outermost	Low T	High T
W^{1-}	Unstable	Unstable	Unstable large	Unstable small
W^{0+}	Stable	Unstable	Unstable large + stable small	No
W^{1+}	Stable	Stable	Stable small	Stable large
W^{0-}	Unstable	Stable	No	Unstable small + stable large



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- **Topological approach provides a new perspective to the black hole thermodynamics.**
- **Different black hole systems are classified into four possible topological classes. It provides a strategy for one to select expected black hole solution for their own purposes.**
- **Although it seems that a lot of work has been done, further study on the black hole thermodynamical topology is still worthwhile.**

Thanks for your attention!