D-COMMUTING SYK

BUILDING QUANTUM CHAOS FROM INTEGRABLE BRICKS

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Some formal aspects of Field theories and Holography

04/18/2025

Based on work with Ping Gao and Han Lin, **arxiv:2411.12806,** and related works

BACKGROUND

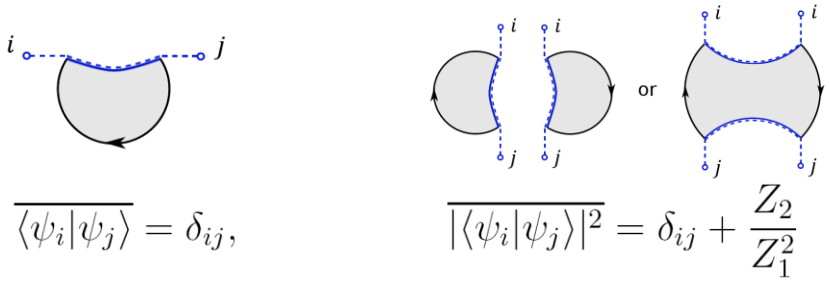
ENSEMBLE AVERAGE THEORIES

- Holographic model: Random Matrix Theories, Sachdev-Ye-Kitaev model, …

- Blackhole information paradox;
  connected amplitudes with disconnected boundaries;
  the factorization puzzle; …

(Penington, Shenker, Stanford, Yang, 22'; …

Witten and Yau 99'; Maldacena, Maoz 04'; ….)



$$\overline{\langle\psi_i|\psi_j\rangle} = \delta_{ij}, \qquad \overline{|\langle\psi_i|\psi_j\rangle|^2} = \delta_{ij} + \frac{Z_2}{Z_1^2}$$

$\Longleftrightarrow$

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + x_{ij}, \quad \overline{x_{ij}} = 0, \quad \overline{x_{ij}^2} = \frac{Z_2}{Z_1^2}$$

- Well-defineness of the large-N limit of "blackhole states"

(Schlenker, Witten 24'; …)

- Quantum Chaos V.S. Solvability and Integrability

CHAOS

VS

INTEGRABILITY

CHAOS VS INTEGRABILITY

- Many of the ensemble average theories are quantum chaotic, putting strong constraints on the holographic dual

- Many conventional theories are integrable, or have integrable sectors, which also have well-understood holographic description

  - Free/critical vector models
  - Symmetric orbifold CFTs
  - The plannar limit of N=4 SYM
  - …

- Can we observe chaotic and integrable behaviors in the same (sector of a) theory ?

INTEGRABILITY – CHAOS TRANSITION

- A transition between chaos and integrability ?

- There are mainly two types of such transitions in the literature

1. Change coupling constants directly
   - Turn on some coupling constants from zero
   - Couple an integrable theory with a chaotic theory, and make them compete as we vary some coupling constants

2. Do not change the couplings constants

   change other parameters instead

A 2D $\mathcal{N}$=(0,2) MODEL

CP, *JHEP* 12 (2018) 065, Ahn CP, JHEP 07 (2019) 092

- $$S = \int d^2z d\theta d\bar{\theta} \left(-\bar{\Phi}^a \partial_{\bar{z}} \Phi^a + \frac{1}{2} \bar{\Lambda}^i \Lambda^i \right) + \int d^2z d\theta \frac{J_{i a_1 \ldots a_q}}{q!} \Lambda^i \Phi^{a_1} \ldots \Phi^{a_q}$$

  Chiral: $\Phi^a = \phi^a + \sqrt{2}\theta\psi^a + 2\theta\bar{\theta}\partial_z \phi^a \,, \qquad a = 1 \ldots N$

  Fermi: $\Lambda^i = \lambda^i - \sqrt{2}\theta G^i + 2\theta\bar{\theta}\partial_z \lambda^i \,, \qquad i = 1 \ldots M$

- $N, M \gg 1$, with $\mu = \frac{M}{N}$ fixed (but tunable)

- IR solution $G_c^I(z_1, z_2) = \dfrac{n_I}{(z_1 - z_2)^{2h_I} (\bar{z}_1 - \bar{z}_2)^{2\tilde{h}_I}} \qquad I = \phi, \psi, \lambda, G$

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

$$\tilde{h}_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\psi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad \tilde{h}_\lambda = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}, \quad \tilde{h}_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2} \,.$$

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- IR solution $G_c^I(z_1, z_2) = \frac{\quad}{(\quad)^{2h_I}}$ $I = \phi, \psi, \lambda, G$

Tune this !

$$h_\phi = \frac{\mu q - 1}{2\mu q^2 - 2}, \quad h_\psi = \frac{\mu q^2 + \mu q - 2}{2\mu q^2 - 2}, \quad h_\lambda = \frac{q - 1}{2\mu q^2 - 2}, \quad h_G = \frac{\mu q^2 + q - 2}{2\mu q^2 - 2}$$

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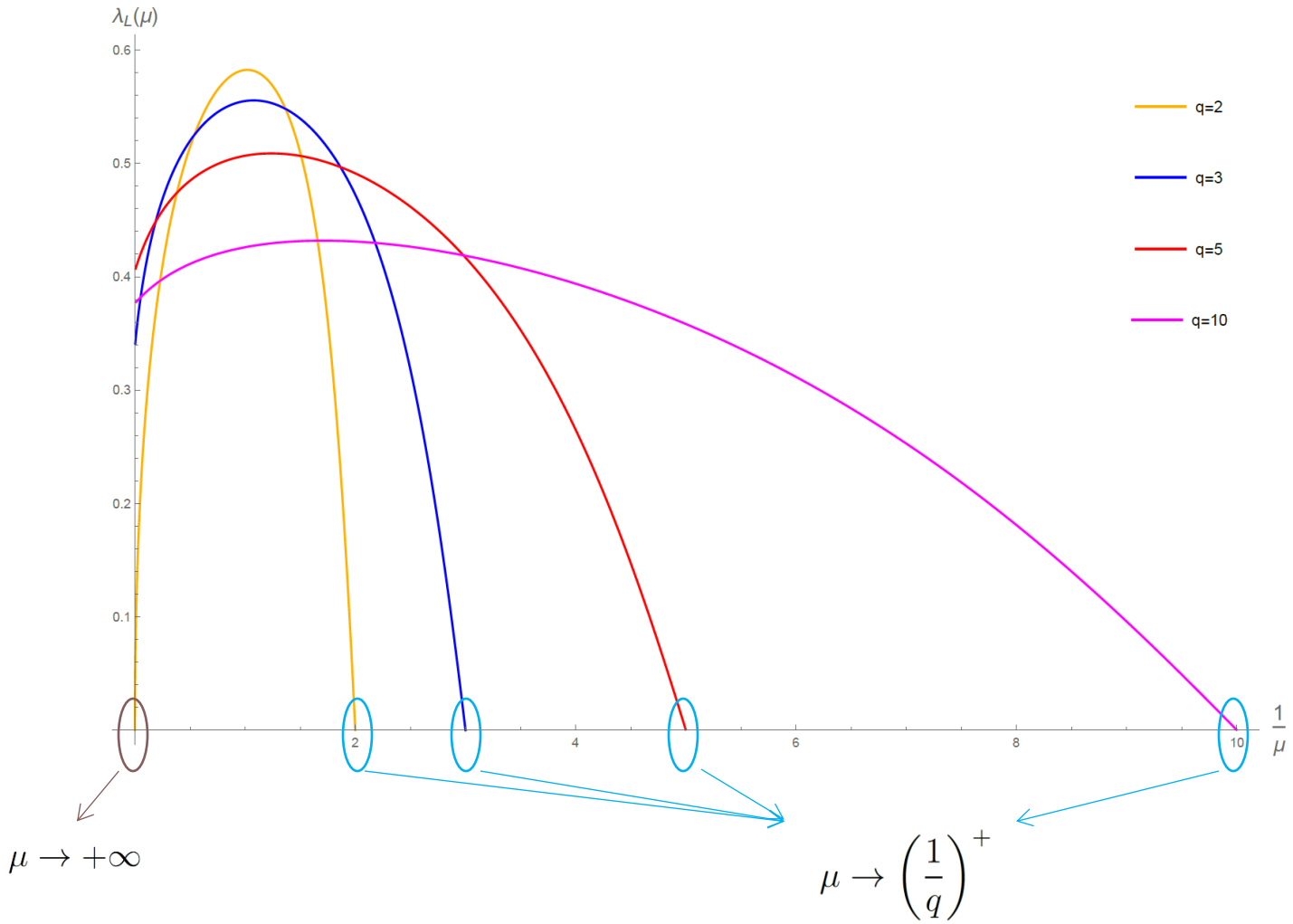
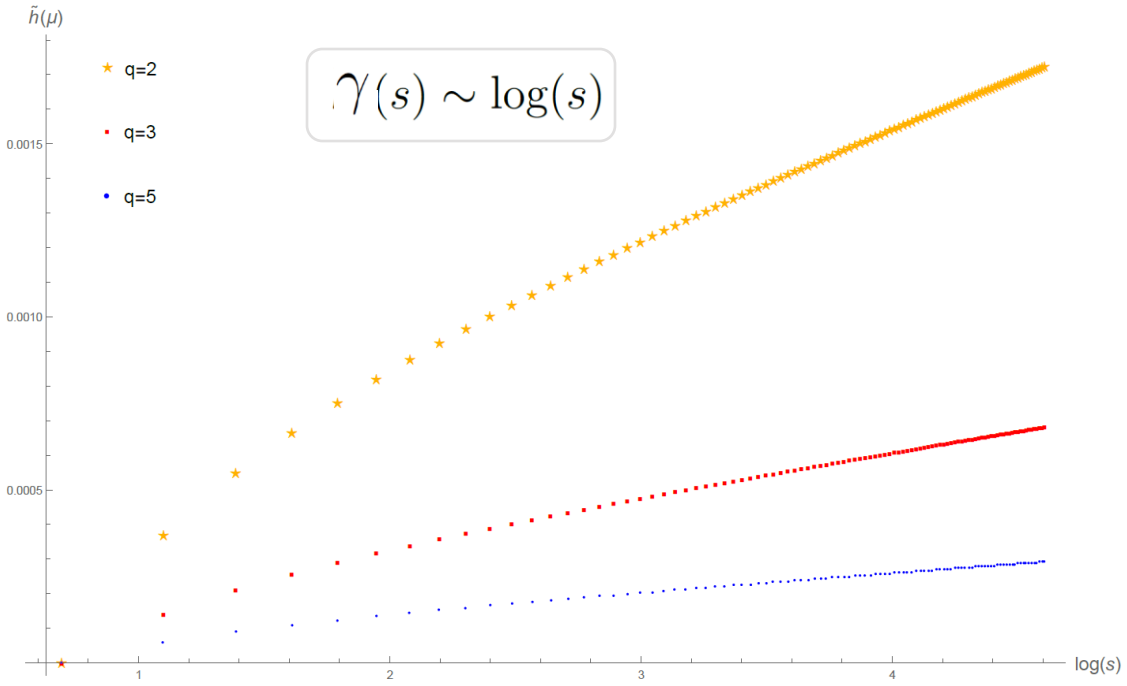
TWO INTERESTING LIMITS



- Lyapunov exponent drops to zero
- "Integrability" takes over ?
- Large symmetries ?
- This happens at fixed J, the transition is due to the screening effect of the interactions

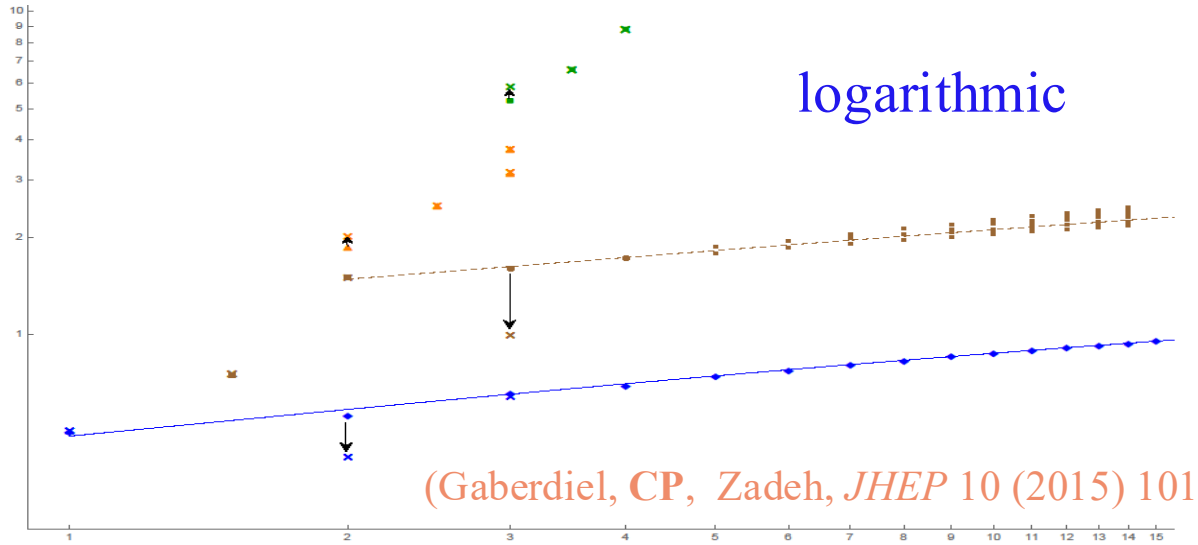
CONNECTIONS WITH CONVENTIONAL THEORIES

- Dispersion relation of this **SYK** model: the anomalous dimension γ logarithmically depends on the spin s



(**CP**, *JHEP* 12 (2018) 065)

- **Higher-spin** perturbation computation

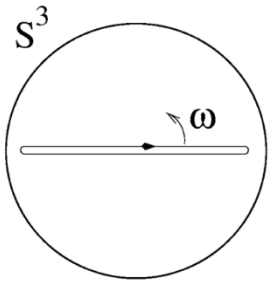


logarithmic

(Gaberdiel, **CP**, Zadeh, *JHEP* 10 (2015) 101)

- Rotating folded closed long **string** in AdS

$$E - S = \frac{\sqrt{\lambda}}{\pi} \ln\left(S/\sqrt{\lambda}\right) + \cdots \qquad \lambda = g_{\mathrm{YM}}^2 N$$



logarithmic due to the AdS geometry

(Gubser, Klebanov, Polyakov, *Nucl.Phys.B* 636 (2002) 99-114)

A 3D DISORDERED MODELS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129, 011603

- We can in fact consider more general 2+1d disordered models
  - ✓ with supersymmetry

$$L_{susy} = -\int d^2\theta d^2\bar{\theta}\left(\bar{p}_i(y^+)\,p^i(y) + \bar{s}_a(y^+)s^a(y)\right) - \left[\int d^2\theta \frac{1}{2} g_{aij}\, s^a(y)\, p^i(y) p^j(y) + \text{c.c}\right]$$

where p^i and s^a are chiral $\mathcal{N}$=2 multiplets with $i = 1 \ldots N$ and $a = 1 \ldots M$

  - ✓ or without supersymmetry

$$L_{bos} = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi^i + \frac{1}{2} g_{aij}\sigma^a \phi^i \phi^j - \frac{1}{4}(\sigma^a)^2$$

where ϕ_i and σ^a are bosonic fields with $i = 1 \ldots N$ and $a = 1 \ldots M$

- Can solve the model in the large-N limit in the IR analytical

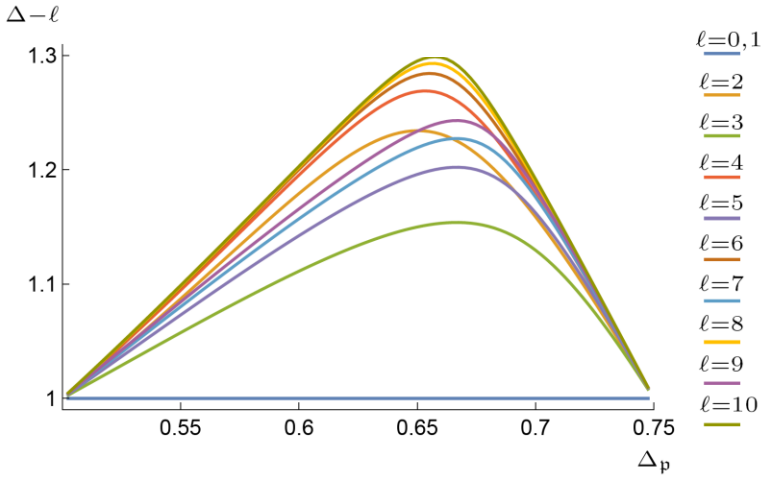
$$N \to \infty, \quad \lambda \equiv M/N, \quad \text{fixed}$$

Tune this !

3D DISORDERED MODELS: HIGHER-SPIN LIMITS

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- Most of the details of the model are quite different from the 0+1d and 1+1d models
- Nevertheless there is again a clear connection to higher-spin theories
- There are special limits
  - Free p $\lambda \to 0,$ $\Delta_p = \frac{1}{2},$ $\Delta_s = 1$ higher-spin
  - Free s $\lambda \to \infty,$ $\Delta_p = \frac{3}{4},$ $\Delta_s = \frac{1}{2}$ higher-spin
  - Diagonal $\lambda \to \frac{1}{2},$ $\Delta_p = \frac{2}{3},$ $\Delta_s = \frac{2}{3}$ the 2+1D SYK model



- This indicates the connection to higher-spin theories is probably universal

A RECENT ATTEMPT

BACKGROUND

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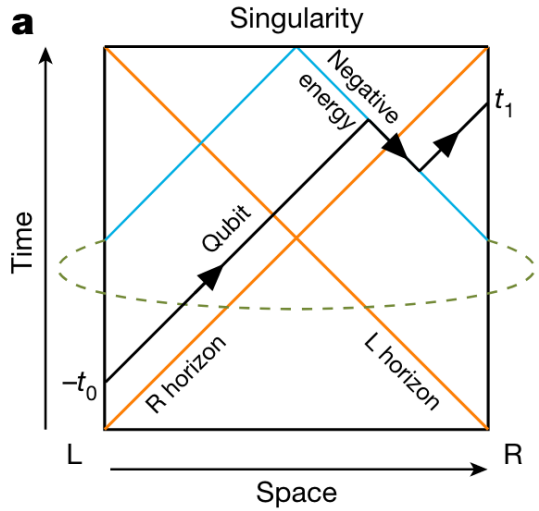
Traversable wormhole dynamics on a quantum processor

Daniel Jafferis, Alexander Zlokapa, Joseph D. Lykken, David K. Kolchmeyer, Samantha I. Davis, Nikolai Lauk, Hartmut Neven & Maria Spiropulu ✉

Nature **612**, 51–55 (2022) | Cite this article



Gao-Jafferis-Wall JHEP 12 (2017)

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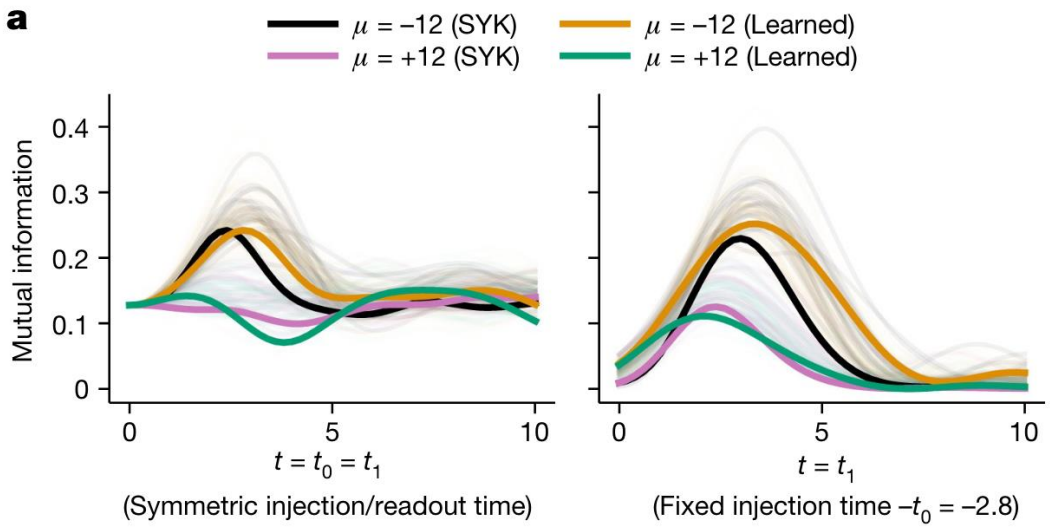
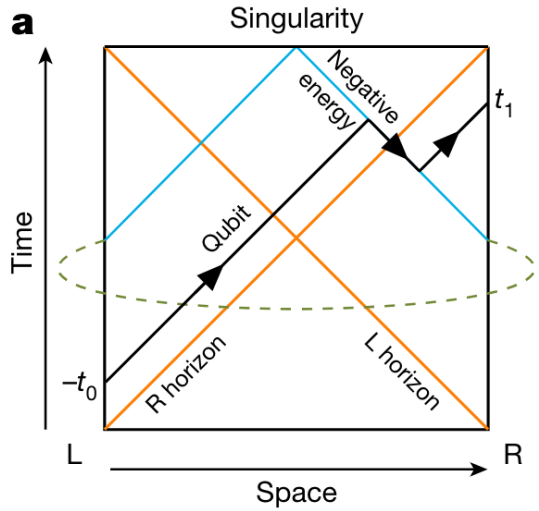
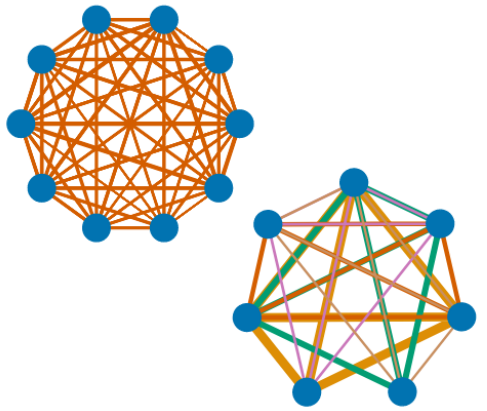




SYK model

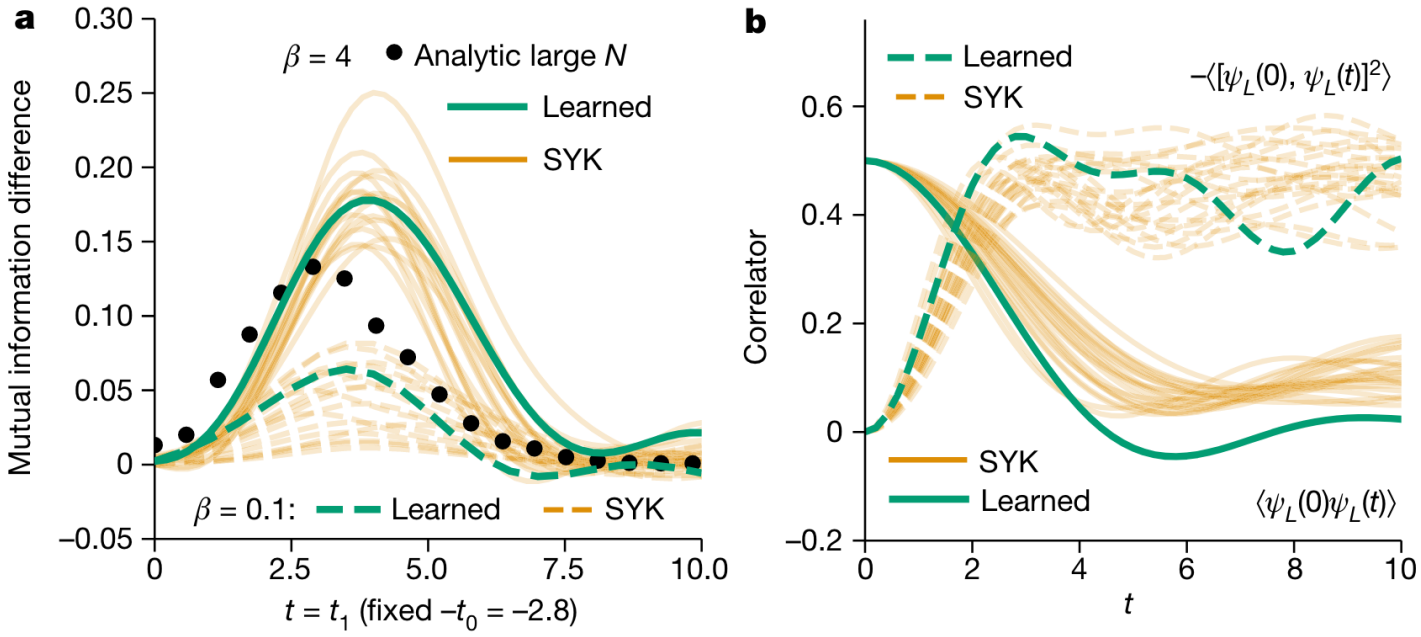


$$|\mathrm{TFD}\rangle = \frac{1}{Z}\sum_n e^{-\beta E_n/2}\,|\mathrm{n}\rangle_L\,|\mathrm{n}\rangle_R$$

$$H = H_L + H_R$$

TRAVERSABLE WORMHOLES ?

$$H_{L,R} = -0.36\psi^1\psi^2\psi^4\psi^5 + 0.19\psi^1\psi^3\psi^4\psi^7 - 0.71\psi^1\psi^3\psi^5\psi^6 + 0.22\psi^2\psi^3\psi^4\psi^6 + 0.49\psi^2\psi^3\psi^5\psi^7,$$



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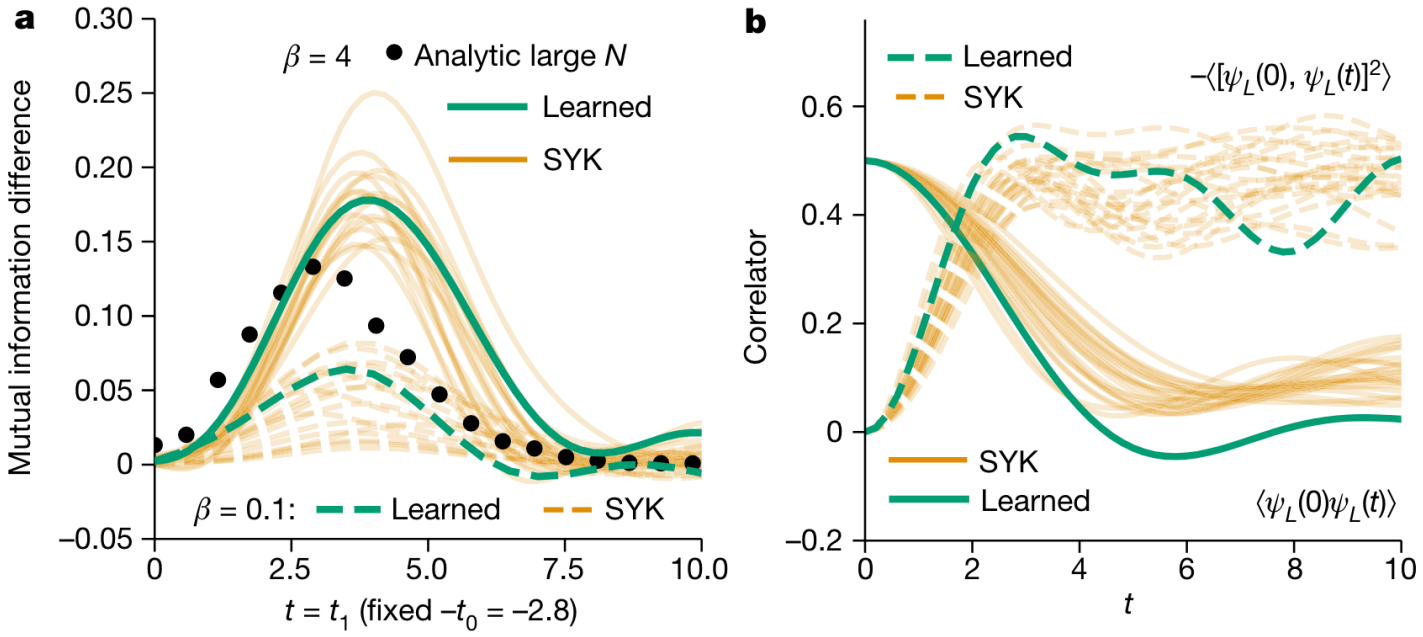


H is commuting

lots of conserved quantities?

The theory is likely to be integrable?

But gravity is chaotic …

This is related to the fact that one need to select a very specialset of couplings to observe these properties

THE COMMUTING SYK IS EASIER TO REALIZE ON QUANTUM COMPUTERS.

CAN WE DO SOMETHING BETTER ?

MORE IS DIFFERENT ?

Gao, Lin, CP, 2411.12806

- We can consider the following alternative model

$$\tilde{H} = \frac{1}{\sqrt{d}} \sum_{a=1}^{d} \tilde{H}_a, \quad \tilde{H}_a = \sum_{i_1 < \cdots < i_p} J^a_{i_1 \cdots i_p} \mathcal{X}^a_{i_1} \cdots \mathcal{X}^a_{i_p} ,$$

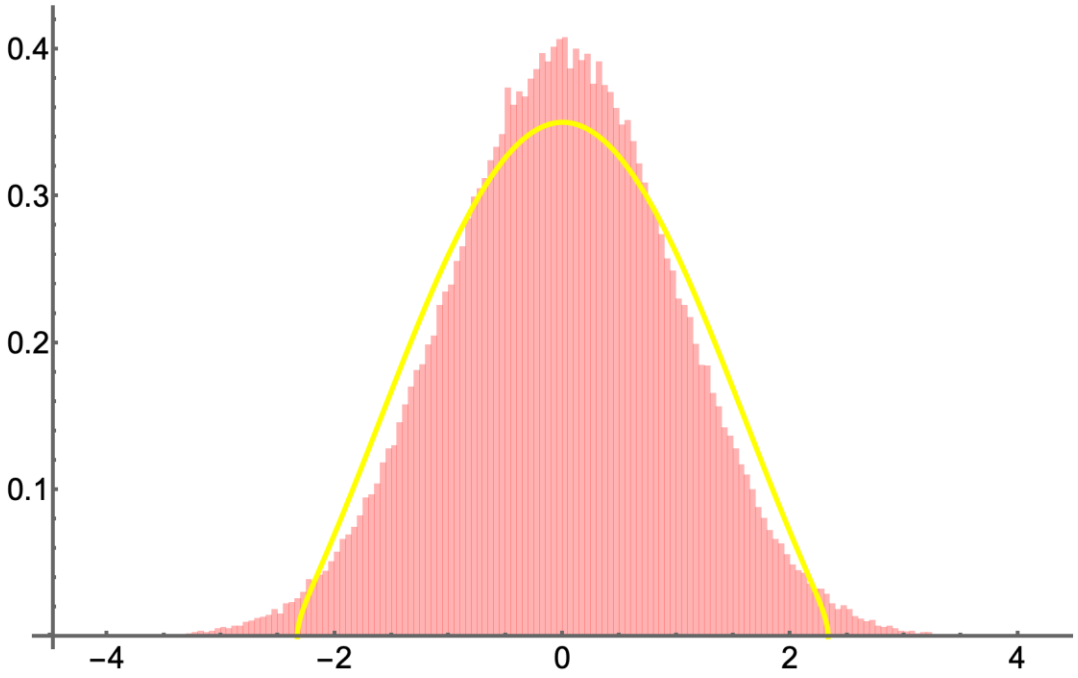
$$\mathcal{X}^a_j \equiv i \psi_{2j-1} \psi_{(2j-4+2a)_{2N}+2}$$

- $d = 1$, a commuting SYK
- $d > 1$, the model is not commuting, but similar enough
- Is this model better (holographic) ?

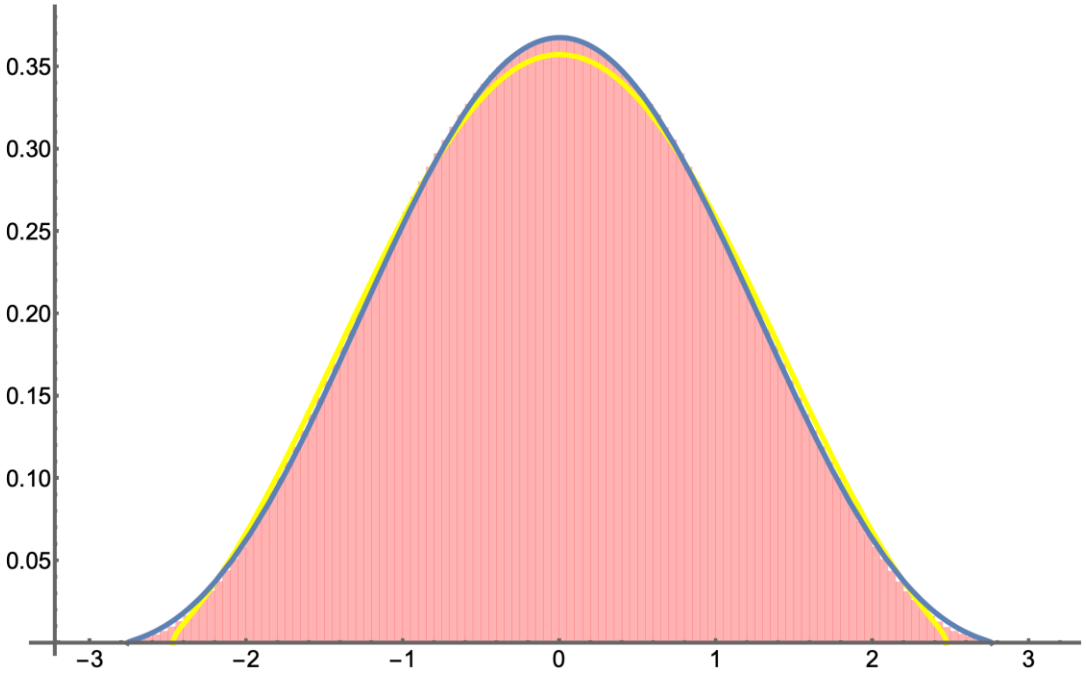
MORE IS DIFFERENT !

- Spectrum

d = 1, Gaussian

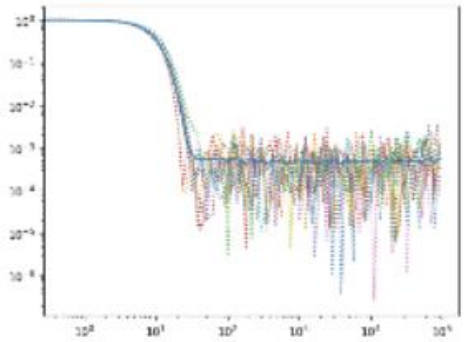


d > 1, very similar to the regular SYK

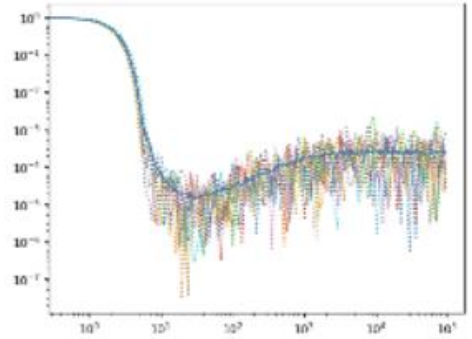


MORE IS DIFFERENT !

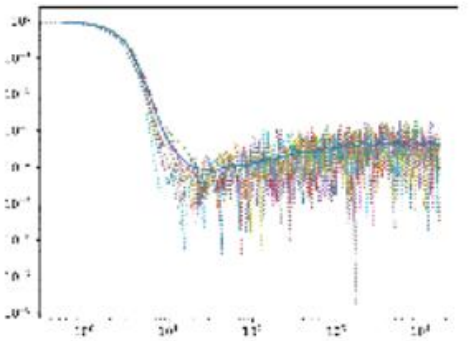
- Spectral form factor



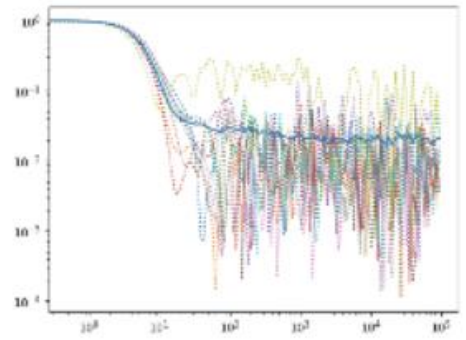
(a) $d = 1, \beta = 0$



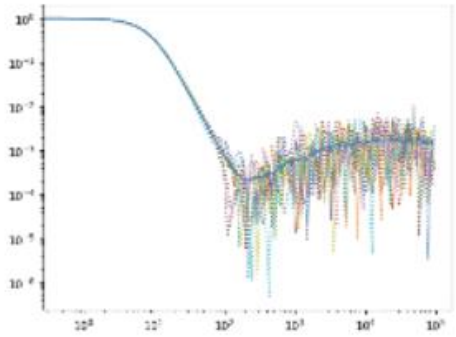
(b) $d = 2, \beta = 0,$ $k = 0.77$



(c) $d = 2, \beta = 2,$ $k = 0.45$



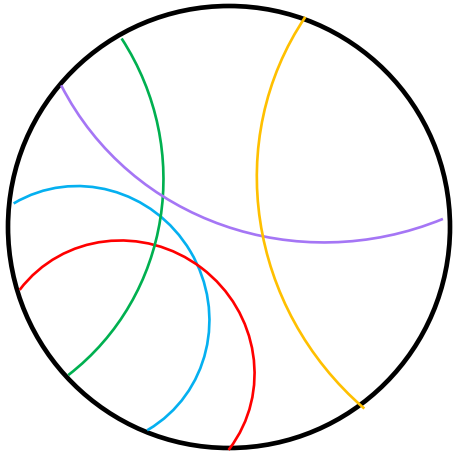
(d) $d = 2, \beta = 10$



(e) $d \to \infty, \beta = 10$

In all the plots, the colorful dashed curves are different realizations, and the solid blue curve is the averaged SFF. k is the slope of the ramp if it exists.

$d \to \infty$: $\cong$ REGULAR SYK



- We can first take the extremal limit $d \to \infty$
- Solve the model in the double scaling limit

$$N \to \infty, \qquad p \to \infty, \qquad \lambda = \frac{4p^2}{N}$$

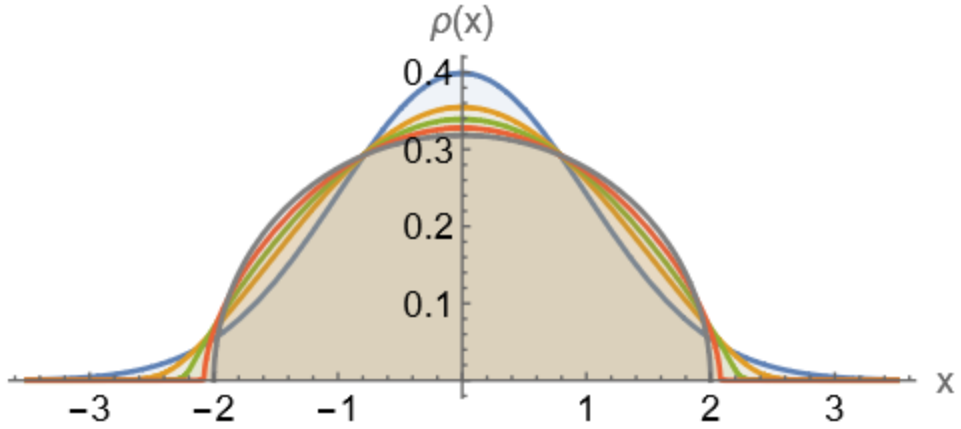
- Chord diagrams, each intersections contributes a factor of $q = e^{-\lambda}$
- A typical chord diagram in the limit $d \to \infty$

all chords are different in color $\lim_{d\to\infty} \binom{d}{n} n!/d^n = 1$

- The contributions are the same as those in regular SYK

GENERAL d , $q \to 0$

- Small d>1, but more complicated
- Again try to solve these models in the double scaling limit
- One solvable limit is $q \to 0$ ($q = e^{-\lambda}$)
- Different color crossing is forbidden, color clusters
- The model can be solved using free probability with the seed spectrum of a single copy



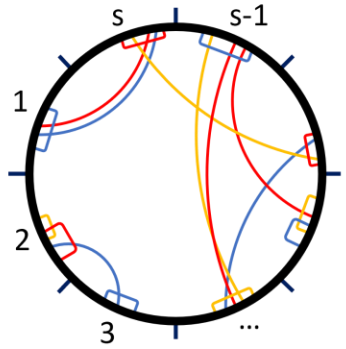
$$\rho_c(x) = \sqrt{\frac{d}{2\pi}} e^{-dx^2/2}$$

GENERAL d , $q \to 1$

- Coarse graining

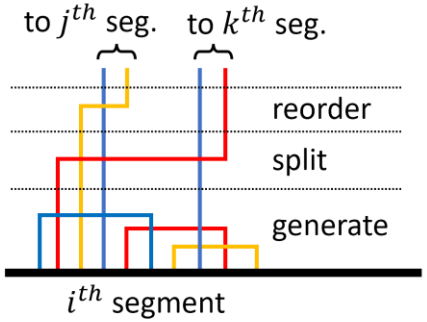
Berkooz, Brukner, Jia, Mamround, 24'

$$Z(\beta) = \sum_{\{n_{ij}^a \geq 0\}} q^{\sum_{a\neq b}\sum_{i>k>j>l} n_{ij}^a n_{kl}^b} Z_s(\beta)$$



- s segments, take the $s \gg 1$, $\sqrt{\lambda}\beta/s \ll 1$, $\lambda \to 0$ limit

$$Z(\beta) = \sum_{\{n_{ij}^a\}} q^{\sum_{a\neq b}\sum_{i>k>j>l} n_{ij}^a n_{kl}^b} \frac{e^{\beta^2/(2s)}}{\prod_{i>j}\prod_a (n_{ij}^a)!} \frac{\beta^{2n}}{(ds^2)^n} \qquad n = \sum_{a=1}^{d}\sum_{i>j} n_{ij}^a$$



GENERAL d , $q \to 1$

Gao, Lin, CP, 2411.12806

- Integrate out the n_{ij}^a variable, we get

$$Z(\beta) = \int d\mathbf{J} \frac{e^{\frac{1}{2\lambda}\mathbf{J}\cdot M^{-1}\cdot\mathbf{J}} e^{\left(\frac{1}{s}+\frac{2}{s^2 d}\sum_{i>j;a} e^{J_{ij}^a}\right)\beta^2/2}}{\sqrt{\det(-2\pi\lambda M)}} .$$

- Take the continuous limit $s \to \infty$, $\tau_i = \beta_r i/s$, $(\beta = \beta_r/\sqrt{\lambda})$

$$Z(\beta) \sim \int d\mathbf{J} e^{S/\lambda} ,$$

$$S = \int_{\tau_1>\tau_2}^{\beta_r} d\tau_{1,2} \left[\frac{1}{4} J^a K_{ab}^{-1} \partial_1 \partial_2 J^b + \frac{1}{d}\sum_a e^{J^a}\right], \qquad K_{ab} = 1-\delta_{ab}$$

GENERAL d , $q \to 1$

Gao, Lin, CP, 2411.12806

- Saddle point equation

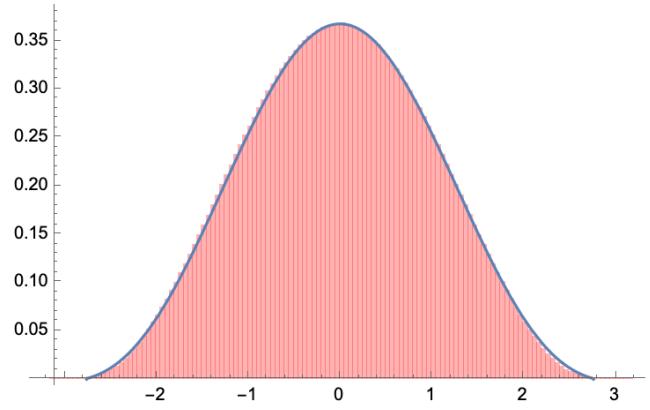
$$\partial_1 \partial_2 J^a = -\frac{2}{d} \sum_{b \neq a} e^{J^b(\tau_1, \tau_2)} \qquad J^a(\tau, \tau) = J^a(\beta_r, 0) = 0$$

- (Translational invariant) Saddle point solution

$$e^J = \frac{\cos^2 \omega \beta_r / 2}{\cos^2 \omega(\tau_{12} - \beta_r/2)}, \quad \omega = \sqrt{1 - 1/d} \cos \omega \beta_r / 2$$

- The on-shell partition function

$$Z(\beta) \sim e^{-E_0 \beta + \frac{\pi^2}{[\lambda(1-1/d)]^{\frac{3}{2}} \beta}}, \quad E_0 = -2/\sqrt{\lambda(1-1/d)}$$



GENERAL d , $q \to 1$

Gao, Lin, CP, 2411.12806

- The 1-loop fluctuation around the above solutions

$$S_2 = \int_{\tau_1>\tau_2}^{\beta_r} d\tau_{1,2} \left[\frac{1}{4(d-1)} \delta J \partial_1 \partial_2 \delta J + \frac{1}{2d} e^J \delta J \delta J + \sum_{i=1}^{d-1} \left(-\frac{1}{4} \delta J^i \partial_1 \partial_2 \delta J^i + \frac{1}{2d} e^J \delta J^i \delta J^i \right) \right]$$

where δJ and δJ^i are fluctuations of the d-1 and -1 modes of K_{ab} respectively

- The δJ and δJ^i obey the same type of equation

$$\left[\frac{1}{4} \partial_\tau^2 - \omega^2 \partial_\theta^2 + \frac{h(h-1)\omega^2}{\sin^2 \theta} \right] f(\tau, \theta) = \eta f(\tau, \theta) \qquad f(\tau, \theta) = f(\tau + \beta_r/2, \pi - \theta)$$

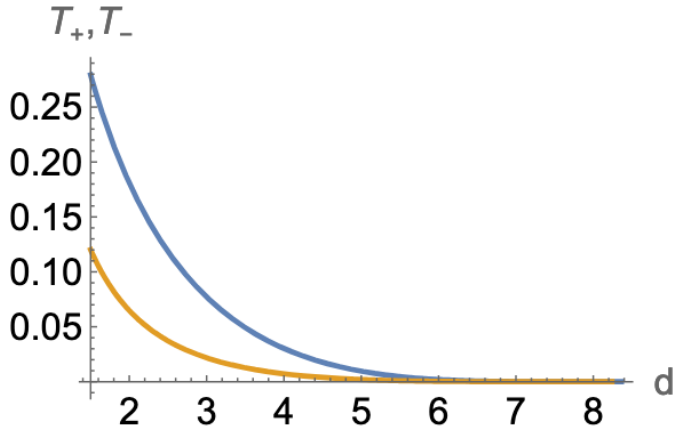
where $h = 2$ for δJ and $h = \frac{1}{2}\left(1 \pm \sqrt{(d-9)/(d-1)}\right)$ with multiplicity $d-1$ for δJ^i

and $\theta = \pi/2 - \omega(\tau_{12} - \beta_r/2)$, $\tau = (\tau_1 + \tau_2)/2$

GENERAL d , $q \to 1$

Gao, Lin, CP, 2411.12806

- The $h = 2$ mode gives the $1/\beta^{3/2}$ contribution
- The $h = \frac{1}{2}\left(1 \pm \sqrt{(d-9)/(d-1)}\right)$ modes are new, we focus on the d<9 case.
- The 1-loop determinant around these solutions has a peculiar feature that as we decrease the temperature to some critical temperature $T=T_+$, additional zero modes appear. Therefore new physics should kick in below this temperature.



- Above this temperature, these modes give a contribution to the partition function

$$(\beta_+ \lambda^{-1/2} - \beta)^{-(d-1)/2}$$

GENERAL d , $q \to 1$

Gao, Lin, CP, 2411.12806

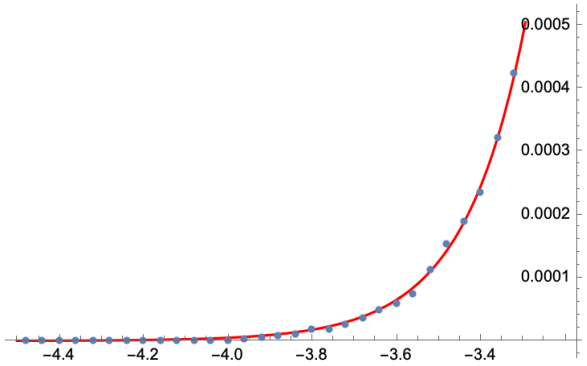
- Therefore the total contribution to the partition function is

$$Z(\beta) \sim \frac{1}{(1/T_c - \beta)^{(d-1)/2}\beta^{3/2}} e^{-\beta E_0 + \frac{\pi^2}{[\lambda(1-1/d)]^{\frac{3}{2}}\beta}}, \qquad T_c = T_+ \lambda^{1/2}$$

- Fit the tail part of the numerical results with the inverse Laplace transform of the above

$T_c/\lambda^{1/2}$	$d=2$	$d=3$	$d=4$
Theory	0.1809	0.07717	0.03025
Numerics	0.1787	0.07667	0.03291
Error	1.2%	0.6%	8.8%

$\tilde{E}_0$	$d=2$	$d=3$	$d=4$
Theory	-3.4926	-3.0247	-2.8517
Numerics	-3.5554	NA	-3.3000
Error	1.8%	NA	15.7%



SUMMARY

- Chaos-integrability transition.

  Not from changing the couplings

- Possibly more interesting phase structures

- Experimental realizations ?

  A typical realization should be enough, without the need of Machine Learning to get a special set of copulings

Thank You!

COMMUTING SYK

Gao JHEP 01 (2024)

- Terms in the Hamiltonian commute
- A more general representation of such models

$$H = \sum_{i_k} \mathcal{J}_{i_1 \cdots i_{q/2}} X_{i_1} \cdots X_{i_{q/2}} \qquad X_i = \psi_{2i-1}\psi_{2i}$$

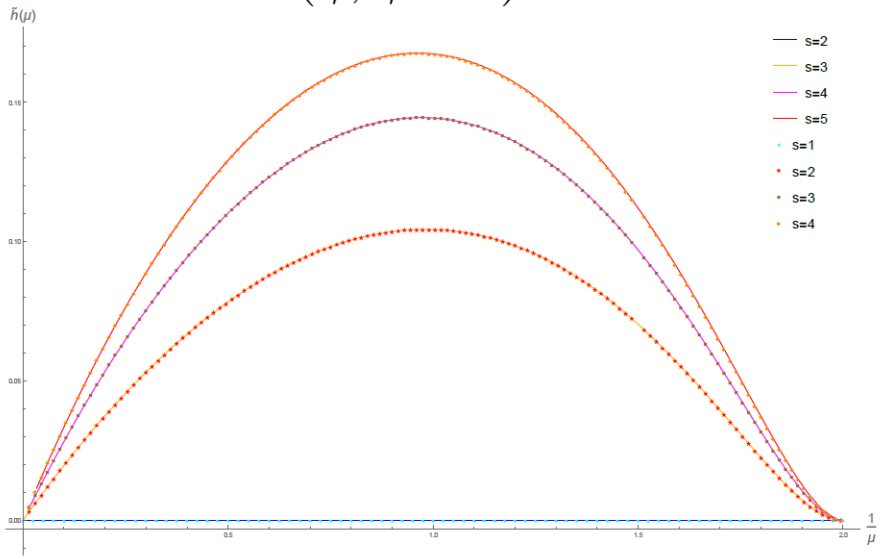
- Also known as the Sherrington-Kirkpatrick (SK) model for q=4
  - Integrable, should not be chaotic
  - The partition function is Gaussian, not dense near the edge, not holographic

$$\overline{Z} = \int dE e^{-\beta E}\rho(E) \implies \rho(E) = \frac{1}{\mathcal{J}}\sqrt{\frac{q}{\pi N}}\exp(-qE^2/(N\mathcal{J}^2))$$

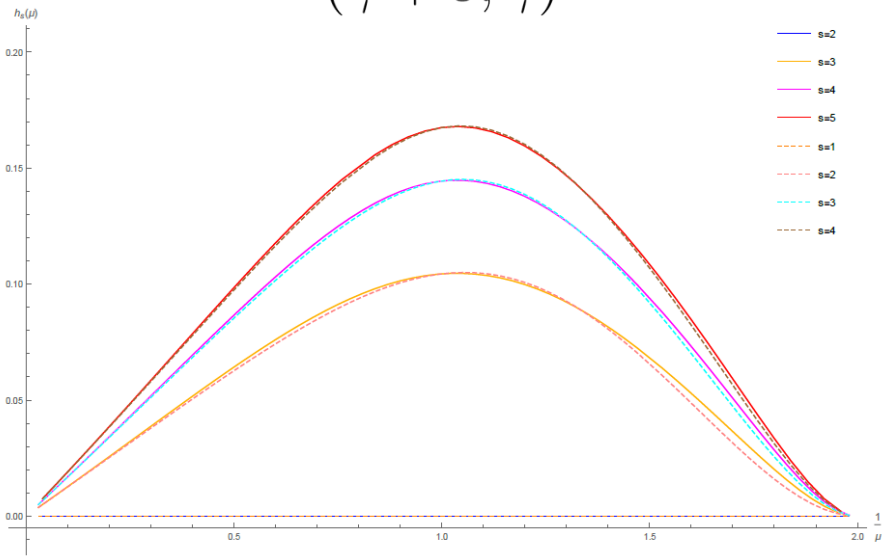
TOWERS OF SPINNING OPERATORS

- A tower of operators has conformal dimension (0,s) or (s,0) in the special limit.
- Emergent higher-spin conserved operators in the two limits!
- Generate large symmetry ➡ nonchaotic

$(\gamma, \gamma + s)$



$(\gamma + s, \gamma)$



s: spin

γ: anomalous dimension

HIGH-DIMENSIONAL DISORDERED MODELS

CP, *2018,* Chang, Colin-Ellerin, **CP**, Rangamani, 2021, 2022, and W.I.P.

1. If there exist high dimensional covariant disordered models ?

   2D and 3D models with different numbers of SUSY and tunable parameters

2. Do they share similar nice features as their low dimensional counterparts ?

   Solvable in the large-N limit, analytically in the IR and numerically in general

3. Do they fulfill the usual requirements obeyed by conventional QFTs ?

   Consistent with various **bootstrap** bounds, hence compatible with many requirements

4. **If there are clear connections with other well-known conventional QFTs ?**

   Observe **higher-spin limits** in different models, which sets up clear connections with higher-spin theories and probably string theory

A 3D SYK MODEL

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- It turns out possible to construct an N=2 Supersymmetry SYK model

$$\mathrm{L} = -\int d^2\theta d^2\bar{\theta}\left(\bar{\Phi}_i(y^\dagger)\Phi_i(y)\right) - \left[\int d^2\theta \frac{1}{3} g_{ijk}\Phi_i(y)\Phi_j(y)\Phi_k(y) + \mathrm{c.c}\right]$$

$$\mathrm{P}(g_{ijk}) \propto e^{-N^2\frac{g_{ijk}\bar{g}_{ijk}}{J}}, \qquad \langle g_{ijk}\rangle = 0, \qquad \langle g_{ijk}\bar{g}_{ijk}\rangle = \frac{J}{N^2}.$$

with N flavors of chiral multiplets

$$\Phi(X) = \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^2 F(y) \qquad \bar{\Phi}(X^\dagger) = \bar{\phi}(y^\dagger) + \sqrt{2}\bar{\theta}^\alpha\bar{\psi}_\alpha(y^\dagger) + \bar{\theta}^2\bar{F}(y^+)$$

- In components:

$$\mathrm{L} = -i\bar{\psi}_i\not{\partial}\psi_i + \partial_\mu\bar{\phi}_i\partial_\mu\phi_i - \bar{F}_i F_i - g_{ijk}\left(\phi_i\phi_j F_k - \psi_i\psi_j\phi_k\right) - \bar{g}_{ijk}\left(\bar{\phi}_i\bar{\phi}_j\bar{F}_k - \bar{\psi}_i\bar{\psi}_j\bar{\phi}_k\right)$$

- The model is again solvable, and its properties indicates that the disordered theory flow to a normal IR fixed point that has no obvious difference from the other conventional models.

SPECTRUM: THE 3D SYK MODEL v.s. $\mathcal{N}$=2 BOOTSTRAP

Chang, Colin-Ellerin, **CP**, Rangamani, *JHEP* 11 (2021) 211

- The IR spectrum is within the bounds obtained from numerical bootstrap

Operators	l	Δ	Bootstrap bound
$(\bar{\Phi}\Phi)$	0	1.6994	<1.9098
$(\bar{\Phi}\Phi)'$	0	3.4295	<5.3
J'	1	4.2676	<5.25

Bobev, El-Showk, Mazac, Paulos, *Phys. Rev. Lett.* 115 (2015) 051601

- Anomalous dimension $\tau = \Delta - \ell = 2\Delta_\phi + 2m + \gamma(m,\ell)$

The large-spin limit, ie fixed m, large ℓ limit

$$\gamma(m,\ell) = (-1)^{\ell+1} \frac{g_3(\Delta_\phi)}{\ell^{\Delta_\phi}} \frac{\Gamma(m-\Delta_\phi+1)}{\Gamma(m+1)}, \qquad \ell \gg 1$$

agrees with results from the light-cone analytic bootstrap

$$\gamma(m,1) = (-1)^1 \frac{C_m}{1^{\tau_{\min}}} \qquad \tau_{\min} = \tau_\phi = \Delta_\phi$$

Fitzpatrick, Kaplan, Poland, Simmons-Duffin, *JHEP* 12 (2013) 004

3D DISORDERED MODELS: OTHER PROPERTIES

Chang, Colin-Ellerin, **CP**, Rangamani, Phys.Rev.Lett. 129 (2022) 1, 011603

- The anomalous dimensions

$$\lim_{1 ? 1} \gamma_{\phi,\sigma}(1,n) \sim \frac{1}{1^{2\Delta_\phi}}, \quad \lim_{n ? 1} \gamma_{\phi,\sigma}(1,n) \sim \frac{1}{n^{4\Delta_\phi}}$$

  again agrees with the bootstrap results

- The central charges behaves as normal field theories in the special limits

$$C_T \to N\left(\frac{3}{2} - \frac{20}{3\pi^2}\lambda + \mathrm{L}\right) \qquad \text{as} \qquad \lambda \to 0$$

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