New perspectives on O-plane

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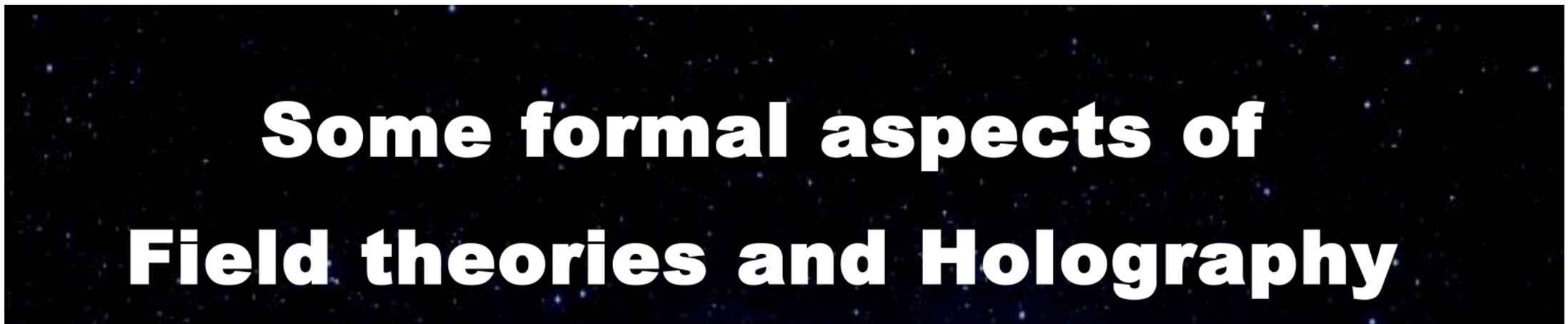
2025-04-19

Hirotaka Hayashi, Kimyeong Lee, Futoshi Yagi [2306.11631]
Xiaobin Li, Satoshi Nawata, Futoshi Yagi [2403.12525]
Xiaobin Li, Futoshi Yagi, Rui-Dong Zhu [2412.19655]



Orientifold planes (O-planes) are fixed planes under orientifold projections, and at these planes, open strings are identified with their orientation-reversed images.

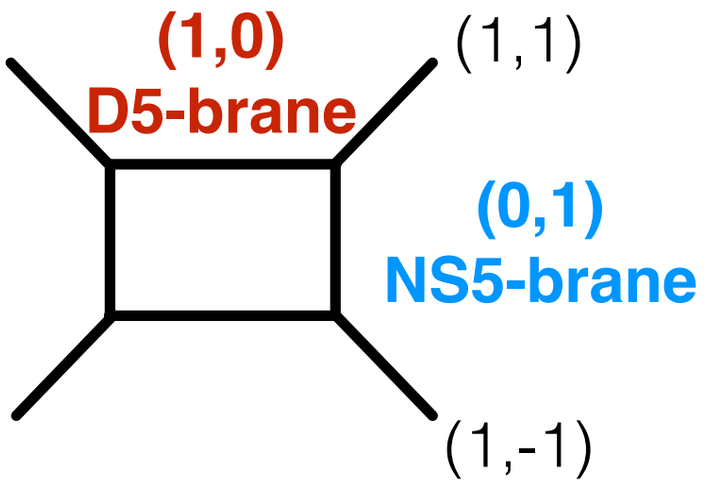
O-planes play a crucial role in constructing supersymmetric gauge theories:

- Engineering Gauge Group: $\mathrm{SO}(N)$ or $\mathrm{USp}(2N) = \mathrm{Sp}(N)$
- Matter content:
  Antisymmetric (AS) and symmetric (Sym) representations
- Duality realization:
  Hanany-Witten moves and Seiberg duality

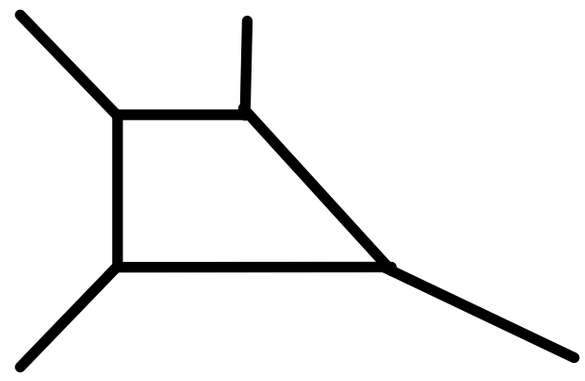
IIB 5-brane setup leads to new perspectives on O-plane

In type IIB string theory, a gauge theory is embedded into a brane configuration:
5-brane web

5d SCFTs



$\mathrm{SU}(2)_0$



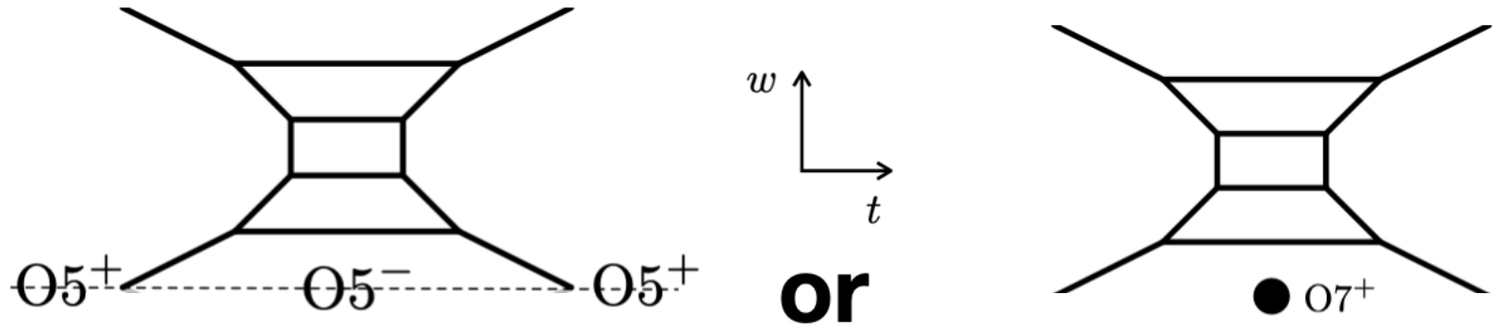
$\mathrm{SU}(2)_\pi$

$$\pi_4(\mathrm{SU}(2)) = \mathbb{Z}_2$$

5-brane webs with O-planes

describe Sp and SO gauge groups

Two O-planes: (1) O5-plane (2) O7-plane



5-brane webs with O-planes

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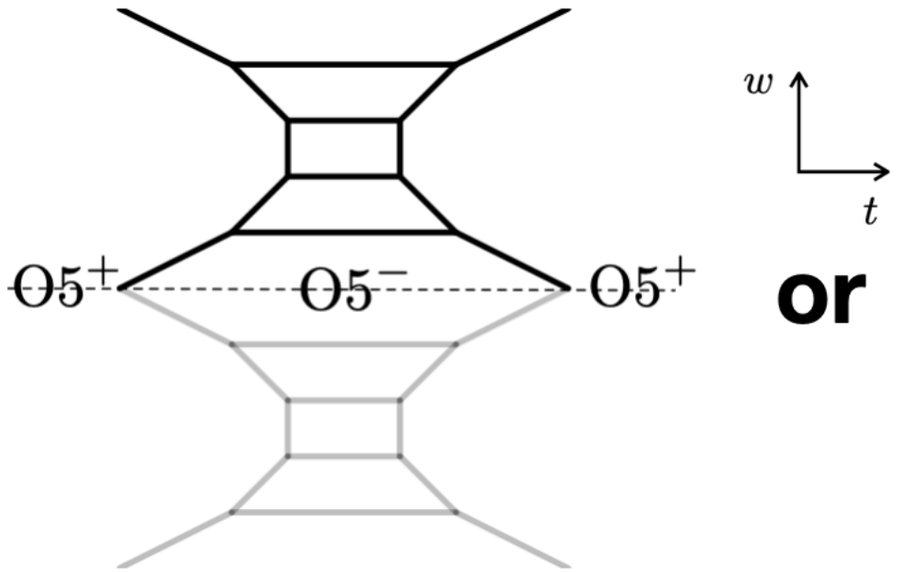
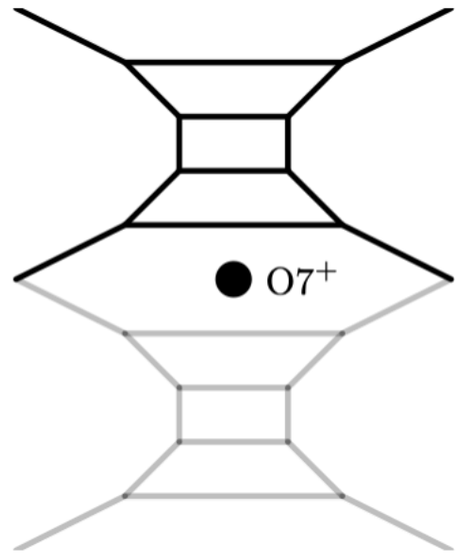
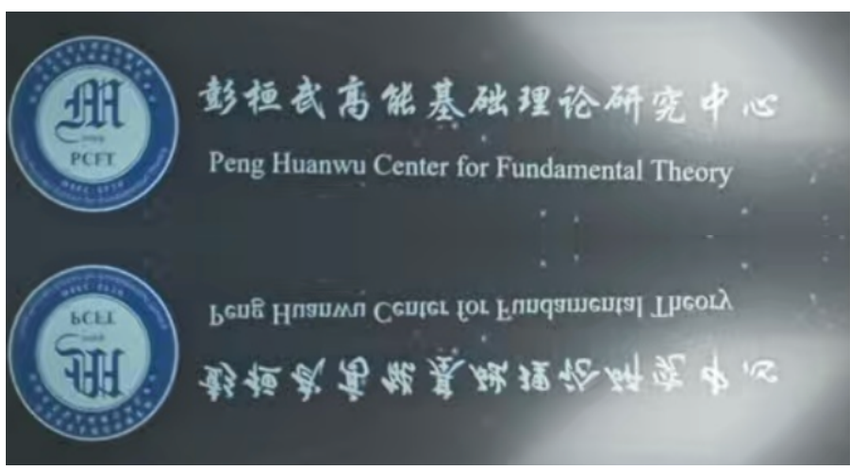
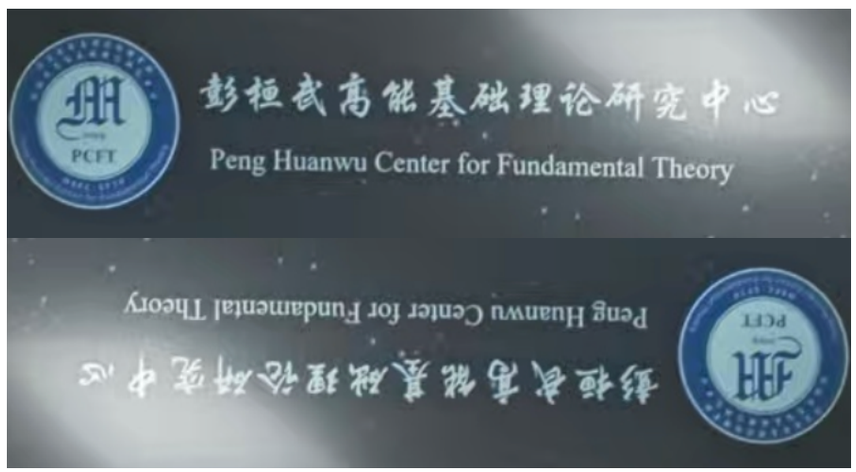


or



Both have $\mathbb{Z}_2$ projection



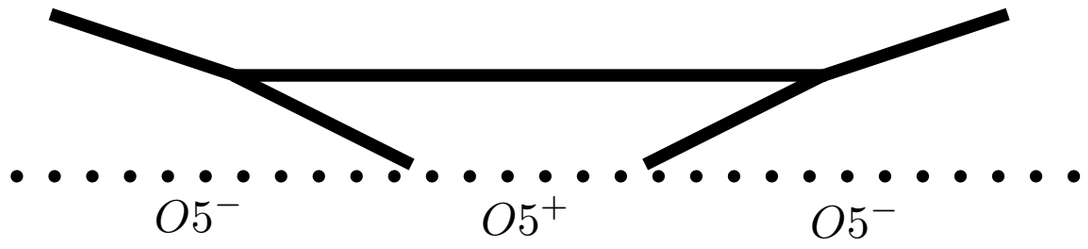


Each one has distinct features when describing matter (hypermultiplets)

New progress made along O5-planes

$Sp(1)_0$ vs $Sp(1)_\pi$

As SU(2) = Sp(1), Sp(1) should has two distinct configurations for the discrete θ angle:



$Sp(1)_0$ vs $Sp(1)_\pi$

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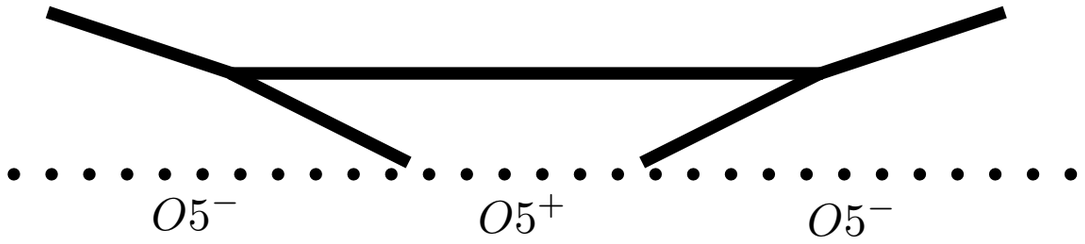
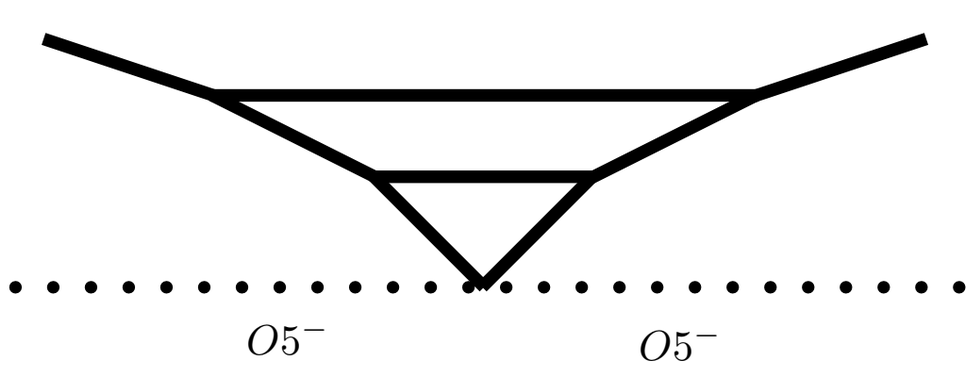
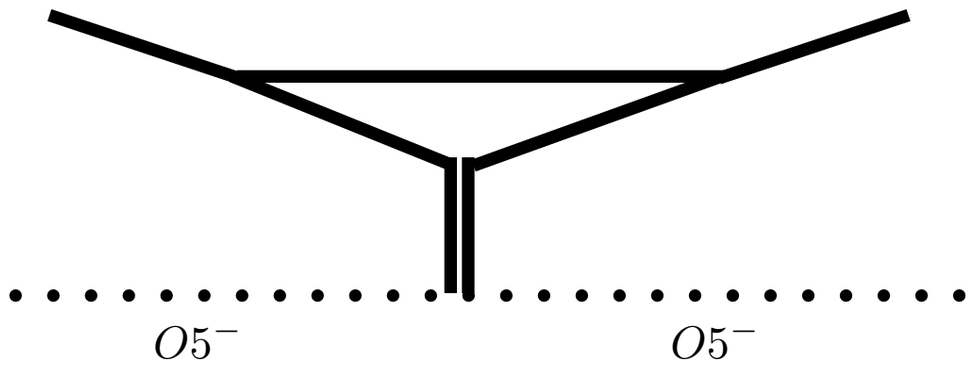


[Hayashi, SK, Lee, Yagi]



$Sp(1)_0$



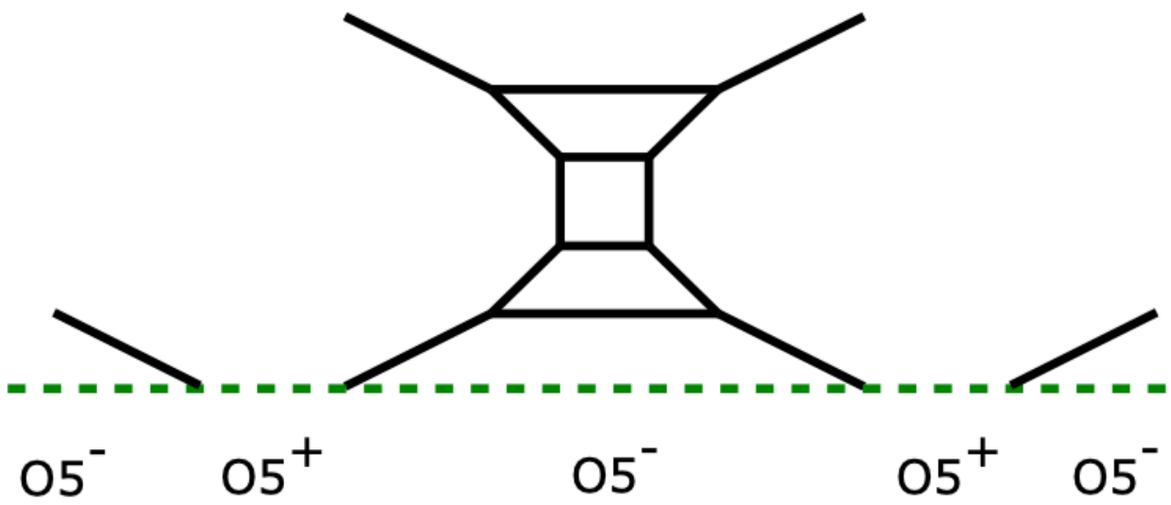
$Sp(1)_\pi$

SO(N) theory with spinor matter

It was a challenge to introduce hypermultiplets other than those in the vector representation.

From the Higgsing of [1]-Sp(1) x SU(2N)-[1],
we can introduce hypers in the spinor representation of SO(N)

[Zafrir '15]



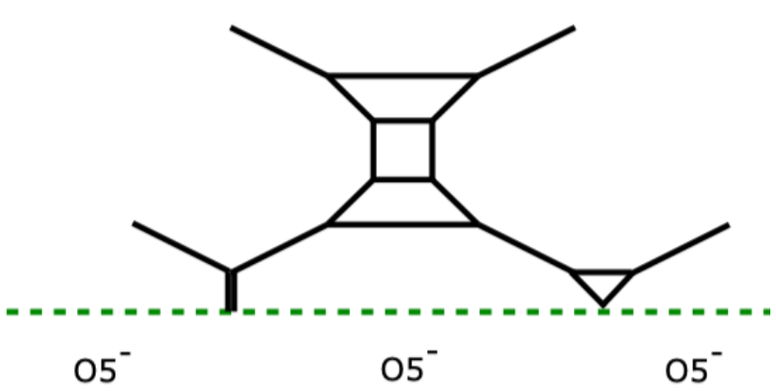
$$SO(8) + 1S + 1C \text{ (or } 2S)$$

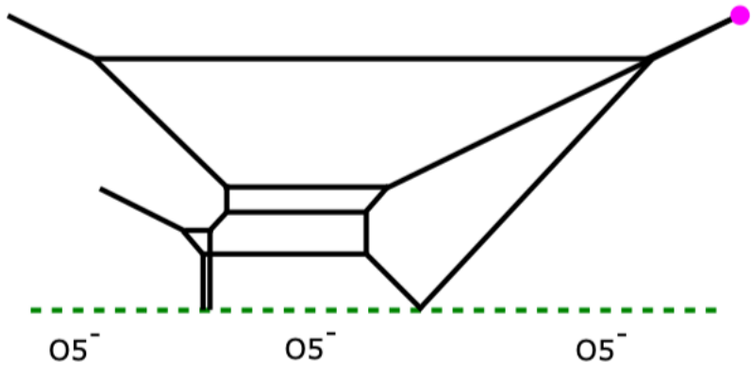
Exceptional gauge group

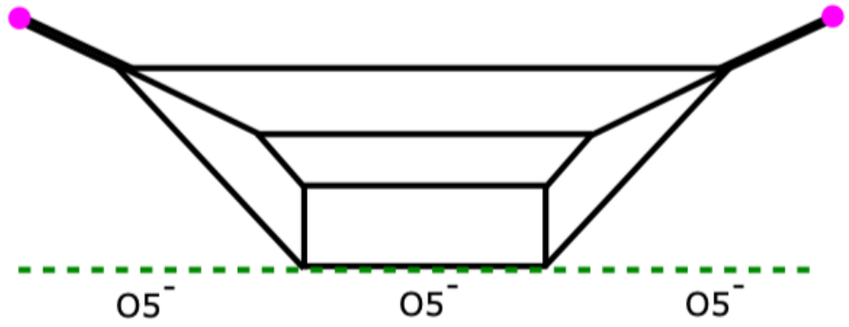
Branes have advantages visualizing the strings on branes also others, but it was believed that brane webs are not useful for gauge theories with exception gauge symmetry, $G_2, F_4, E_{6,7,8}$

However, there is a Higgs brach which leads to G_2 theory! [Hayashi, SK, Lee, Yagi]

$$SO(8) + 1C + 1S \quad \rightarrow \quad SO(7) + 1S \quad \rightarrow \quad G_2$$







Topological vertex

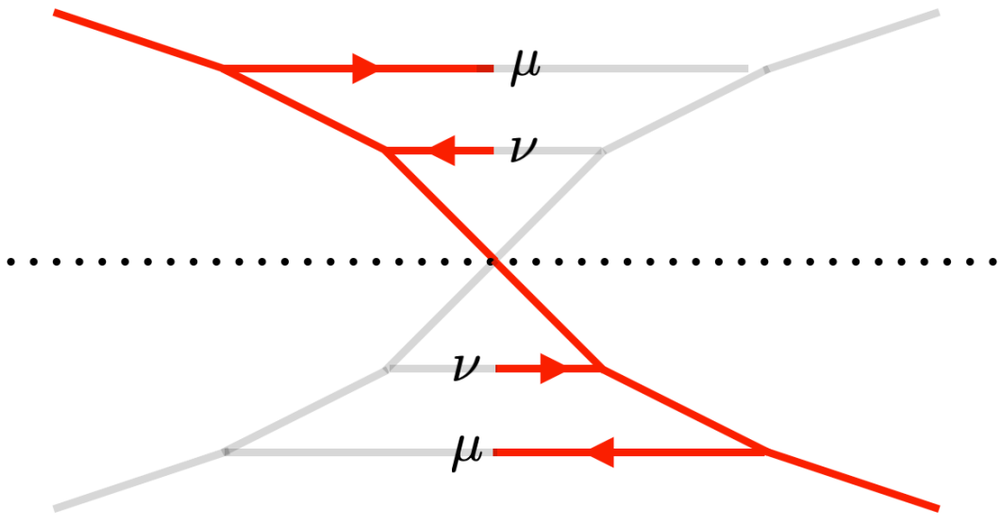
Topological vertex method is a systematic way to compute the instanton partition function.

For toric web diagrams, it works well.

O-planes are non-toric, nevertheless there are ways to apply the topological vertex formalism with some modifications **[Futoshi's talk]**

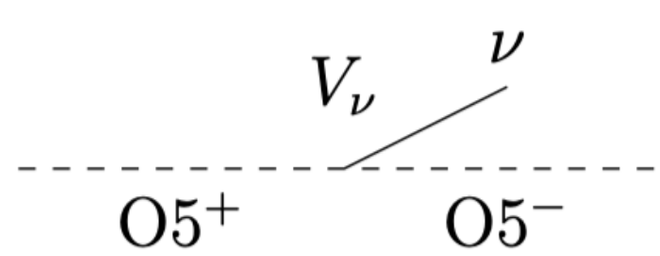
Topological vertex with O5

[SK, Yagi]



O-vertex

[Rui-dong Zhu, Hayashi, Nawata]



$$V_\nu = (-1)^{|\nu|} \langle 0 | \, \mathbb{O}(q) \, | \nu \rangle$$

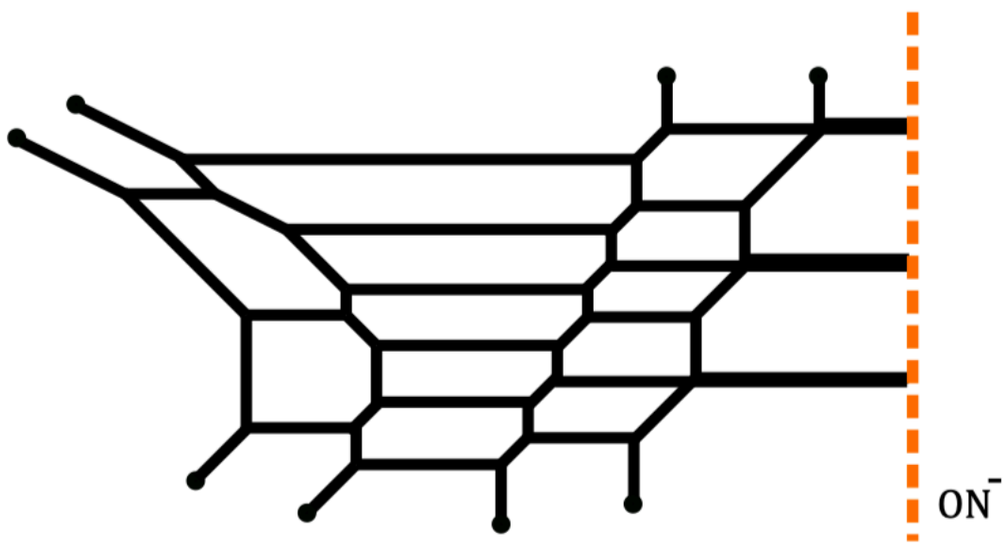
$$\mathbb{O}(q) := \exp\left(\sum_{n=1}^{\infty} -\frac{1+q^n}{2n(1-q^n)} J_{2n} + \frac{1}{2n} J_n J_n \right) ,$$

where J_n's are free boson oscillators satisfying

$$[J_n, J_m] = n \, \delta_{n+m,0} \, .$$

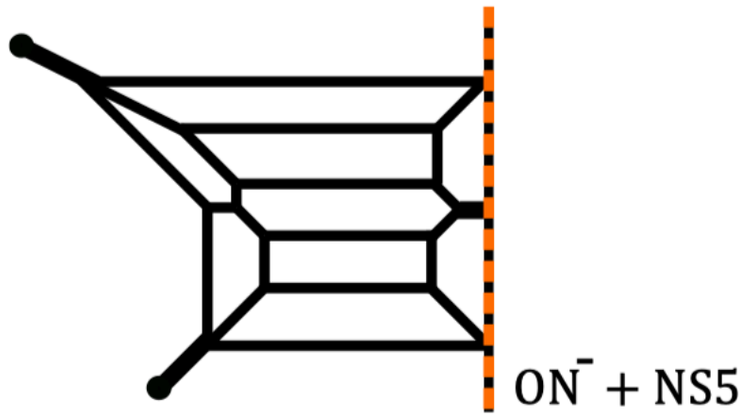
Rank-3 antisymmetric (TAS) matter

Hypermultiplets in the rank-3 antisymmetric representation are possible form certain gauge theories.



(a)

$SU(6)_{\frac{1}{2}} - [SU(3)_0]^3$ quiver theory.



(b)

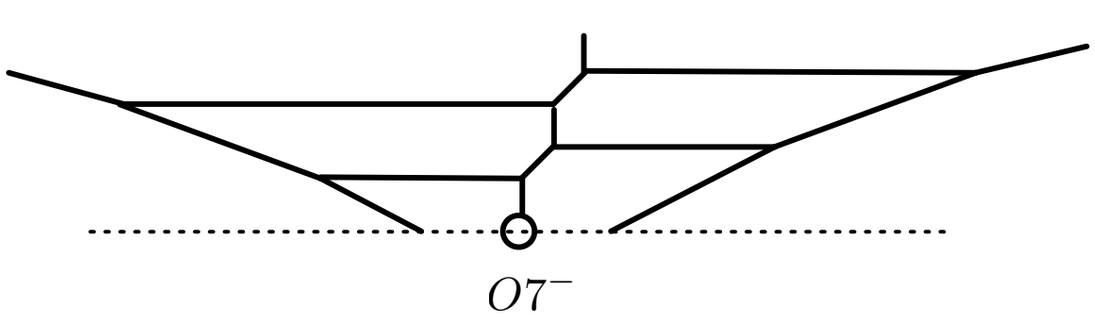
$SU(6)_{\frac{1}{2}}$ gauge theory with $N_{\mathbf{TAS}} = \frac{3}{2}$

New progress made along **O7-planes**

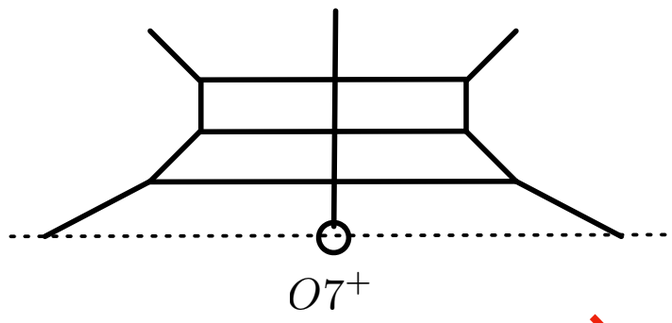
O7-plane leads to AS / Sym

With O7-planes, 5-brane webs describe
SU(*N*) + **AS** or **Sym** and also Sp / SO gauge theories

SU + 1**AS**

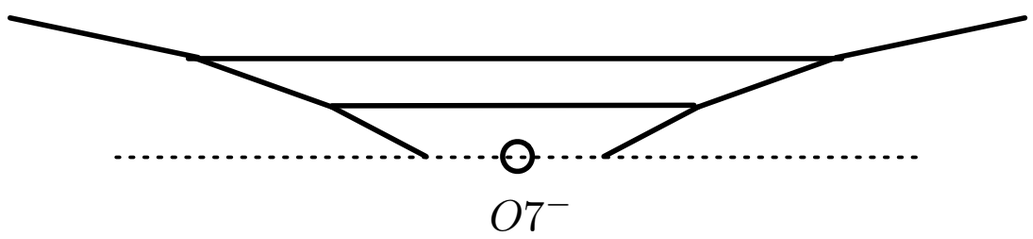


SU + 1**Sym**

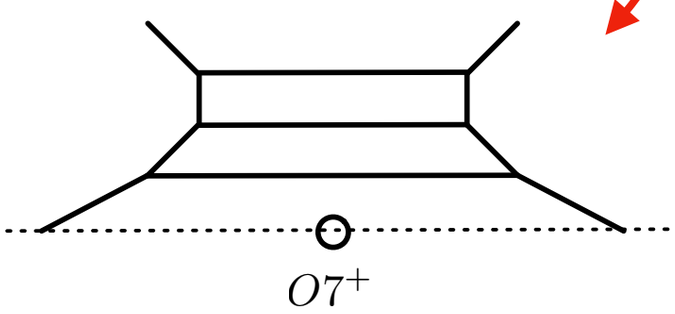


Higgsing
NS5-brane

Sp



SO



$O7^-$ resolution and UV duality

An $O7^-$-plane can be resolved into a pair of 7-branes of charges,
either (2,-1), (0,1) or (1,-1), (1,1).

[Ashoke Sen]

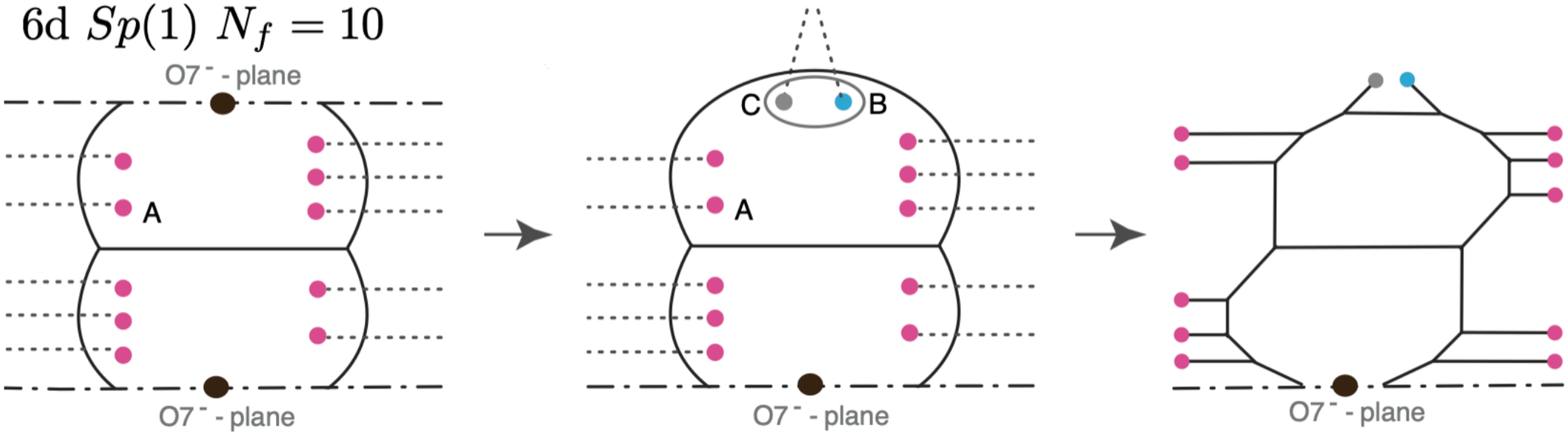
If we do not resolve $O7^-$-plane, then it is an Sp theory.

UV-dual

If we resolve $O7^-$-plane, it is a configuration without O7-plane,
which is an SU theory.

D7-brane
[1,1] 7-brane
[1,-1] 7-brane

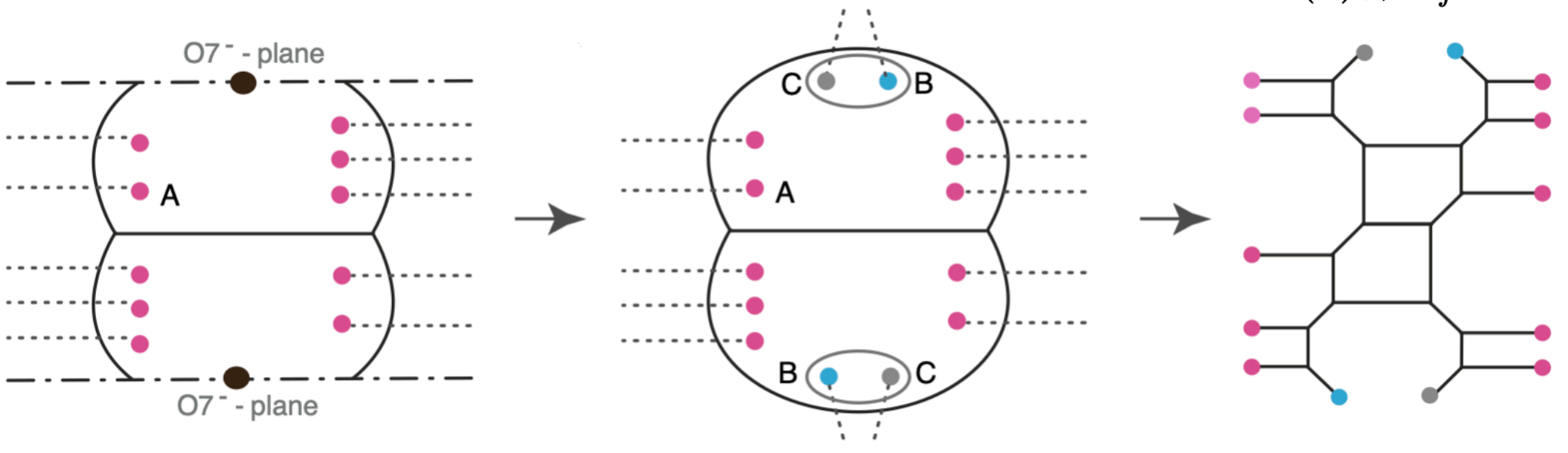
6d $Sp(1)$ $N_f = 10$



5d $Sp(2)$ $N_f = 10$

UV-dual

5d $SU(3)_0, N_f = 10$



Their partition functions agree. [Hayashi, SK, Lee, Yagi]

What is new is ..

$$\text{“O7}^+ \sim \text{O7}^- + \text{8D7”}$$

[Hirotaka Hayashi, SK, Kimyeong Lee, Futoshi Yagi ’23]

[Hee-Cheol Kim, Minsung Kim, SK, Gabi Zafrir ’23]

[SK, Xiaobin Li, Satoshi Nawata, Futoshi Yagi ’23]

[SK, Xiaobin Li, Futoshi Yagi, Rui-dong Zhu ’24]

What is new is ..

$$\text{“}O7^+ \sim O7^- + 8\text{D}7\text{”}$$

Freezing 8 D7s at the position of $O7^-$, in a particular way.

SU(*N*)+1**Sym**		SU(*N*)+1**AS**+8**F**
SO(2*N*)	$\longleftrightarrow$	Sp(*N*)+8**F**

(i) monodromies

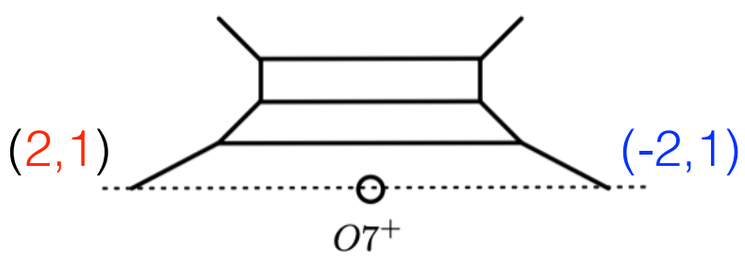
It is well-known that the monodromy of $O7^+$ is the same as $O7^- + 8D7$

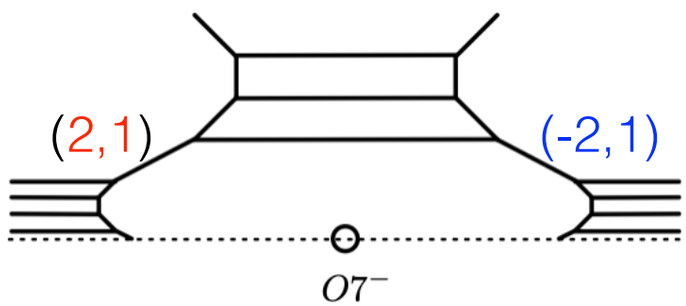
$$O7^+ \quad \longleftrightarrow \quad O7^- + 8D7$$

Monodromy:

$$\begin{pmatrix} -1 & -4 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$$

This explains, in fact, why and how branes bend when they go across O7.





It is also well known that their brane RR-charges are the same

$$(\mathrm{O}p^+ \sim \mathrm{O}p^- + 2^{p-4}\mathrm{D}p)$$

In particular,

$$O7^+ \longleftrightarrow O7^- + 8D7$$

$$+4 \qquad -4 \qquad +1 \times 8$$

(ii) Cubic prepotential

On Coulomb branch

- Prepotential characterizing the abelian low energy effective theory

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij}\phi^i\phi^j + \frac{\kappa}{6} d_{ijk}\phi^i\phi^j\phi^k + \frac{1}{12}\left(\sum_{\text{Roots}} |R\cdot\phi|^3 - \sum_f \sum_{w\in W_f} |w\cdot\phi + m_f|^3 \right)$$

[96 Morrison-Seiberg]
[97 Intriligator-Morrison-Seiberg]

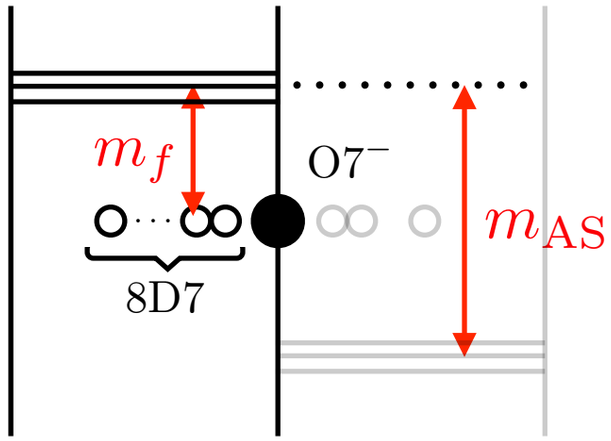
g_0 : Gauge coupling
κ : Chern-Simons level
m_f : Mass
ϕ : VEV of adjoint scalar
Δ : Root system
w :weight of the representation R_f

$$h_{ij} = \mathrm{Tr}(T_i T_j), \quad d_{ijk} = \frac{1}{2}\mathrm{Tr}(T_i\{T_j, T_k\})$$

Freezing at the level cubic prepotential

$$\mathcal{F}_{SU(N)+1\mathrm{AS}+8F} = \frac{1}{2{g_0}^2}\sum_{i=1}^{N}{a_i}^2 + \frac{\kappa}{6}\sum_{i=1}^{N}{a_i}^3 + \frac{1}{6}\sum_{i<j}^{N}(a_i - a_j)^3$$
$$- \frac{1}{12}\sum_{i<j}^{N}|a_i + a_j + m_{\mathrm{AS}}|^3 - \frac{1}{12}\sum_{f=1}^{8}\sum_{i=1}^{N}|a_i + m_f|^3$$

$$(a_1 > a_2 > \cdots a_N)$$



$$m_f = \frac{1}{2}m_{\mathrm{AS}} \quad (f = 1, 2, \cdots, 8)$$

$$\mathcal{F}_{SU(N)+1\mathrm{Sym}} = \frac{1}{2{g_0}^2}\sum_{i=1}^{N}{a_i}^2 + \frac{\kappa}{6}\sum_{i=1}^{N}{a_i}^3 + \frac{1}{6}\sum_{i<j}^{N}(a_i - a_j)^3$$
$$- \frac{1}{12}\sum_{i<j}^{N}|a_i + a_j + m_{\mathrm{AS}}|^3 - \frac{1}{12}\sum_{i=1}^{N}|2a_i + m_{\mathrm{AS}}|^3$$

$$(m_{\mathrm{Sym}} = m_{\mathrm{AS}})$$

In Summary,

$$\mathcal{F}_{SU(N)+1\mathrm{Sym}} = \mathcal{F}_{SU(N)+1\mathrm{AS}+8F}\Big|_{m_f=\frac{1}{2}m_{\mathrm{AS}}\ (f=1,\cdots,8)}$$

Similarly,

$$\mathcal{F}_{SO(2N)} = \mathcal{F}_{Sp(N)+8F}\Big|_{m_f=0\ (f=1,\cdots,8)}$$

The equivalence is realized at the level of cubic prepotential!

(iii) Partition function

Nekrasov's instanton partition function on $R^4 \times S^1$

Given by index for ADHM quantum mechanics

$$Z_{\text{inst}}\left(\epsilon_1, \epsilon_2, a_i, m\right) = \text{Tr}\left[(-1)^{\mathsf{F}} e^{-\beta\left\{Q, Q^\dagger\right\}} e^{-\epsilon_1(J_1+J_R)} e^{-\epsilon_2(J_2+J_R)} e^{-a_i C_i} e^{-m\cdot F}\right] .$$

$$\begin{aligned} J_1, J_2 &: \text{Cartans of spacetime symmetry SO(4)} \\ J_R &: \text{Cartan of SU(2)}_R \\ a_i &: \text{Coulomb branch parameters} \\ m_i &: \text{Mass parameters} \end{aligned}$$

Instanton Partition functions on $\mathbb{R}^4 \times S^1$

AHDM contour integral

[Nekrasov, Shadchin '04]
[Shadchin, '05 thesis]
[Hee-Cheol Kim, SSK, Kimyeong Lee 1206.]
[Chiung Hwang, Joonho Kim, Seok Kim, Jaemo Park 1406.]

$$Z^{\rm vec}_{{\rm SU}(N),k} = \frac{\prod_{I\neq J}{\rm sh}(\phi_I-\phi_J)\cdot\prod_{I,J}{\rm sh}(2\epsilon_+-\phi_I+\phi_J)}{\prod_{I,J}{\rm sh}(\epsilon_{1,2}+\phi_I-\phi_J)\prod_{I=1}^{k}\prod_{s=1}^{N}{\rm sh}(\epsilon_+\pm(\phi_I-a_s))}$$

$$Z^{\rm fund}_{{\rm SU}(N),k}(m_l) = \prod_{I=1}^{k}\prod_{l=1}^{N_f}{\rm sh}(\phi_I+m_l)$$

$$Z^{\rm anti}_{{\rm SU}(N),k}(m) = \prod_{I=1}^{k}\frac{\prod_{s=1}^{N}{\rm sh}(\phi_I+a_s+m)}{{\rm sh}(-\epsilon_+\pm(2\phi_I+m))}\prod_{I<J}^{k}\frac{{\rm sh}(\phi_I+\phi_J+m\pm\epsilon_-)}{{\rm sh}(-\epsilon_+\pm(\phi_I+\phi_J+m))}$$

$$Z^{\rm sym}_{{\rm SU}(N),k}(m) = \prod_{I=1}^{k}{\rm sh}(2\phi_I+m\pm\epsilon_-)\prod_{s=1}^{N}{\rm sh}(\phi_I+a_s+m)\prod_{I<J}^{k}\frac{{\rm sh}(\phi_I+\phi_J+m\pm\epsilon_-)}{{\rm sh}(-\epsilon_+\pm(\phi_I+\phi_J+m))}$$

$${\rm sh}(x) := e^{\frac{x}{2}}-e^{-\frac{x}{2}}\ , \qquad {\rm ch}(x) := e^{\frac{x}{2}}+e^{-\frac{x}{2}}$$

At *k*-instanton order, we require

$$Z^{\rm vec}_{{\rm SU}(N),k}Z^{\rm sym}_{{\rm SU}(N),k}(m) \;=\; Z^{\rm vec}_{{\rm SU}(N),k}Z^{\rm anti}_{{\rm SU}(N),k}(m)Z^{\rm fund}_{{\rm SU}(N),k}$$

Instanton Partition functions on $\mathbb{R}^4 \times S^1$

AHDM contour integral

$$Z^{\text{vec}}_{\text{SU}(N),k} = \frac{\prod_{I\neq J}\text{sh}(\phi_I-\phi_J)\cdot\prod_{I,J}\text{sh}(2\epsilon_+-\phi_I+\phi_J)}{\prod_{I,J}\text{sh}(\epsilon_{1,2}+\phi_I-\phi_J)\prod_{I=1}^{k}\prod_{s=1}^{N}\text{sh}(\epsilon_+\pm(\phi_I-a_s))}$$

$$Z^{\text{fund}}_{\text{SU}(N),k}(m_l) = \prod_{I=1}^{k}\prod_{l=1}^{N_f}\text{sh}(\phi_I+m_l)$$

$$Z^{\text{anti}}_{\text{SU}(N),k}(m) = \prod_{I=1}^{k}\frac{\prod_{s=1}^{N}\text{sh}(\phi_I+a_s+m)}{\text{sh}(-\epsilon_+\pm(2\phi_I+m))}\prod_{I<J}^{k}\frac{\text{sh}(\phi_I+\phi_J+m\pm\epsilon_-)}{\text{sh}(-\epsilon_+\pm(\phi_I+\phi_J+m))}$$

$$Z^{\text{sym}}_{\text{SU}(N),k}(m) = \prod_{I=1}^{k}\text{sh}(2\phi_I+m\pm\epsilon_-)\prod_{s=1}^{N}\text{sh}(\phi_I+a_s+m)\prod_{I<J}^{k}\frac{\text{sh}(\phi_I+\phi_J+m\pm\epsilon_-)}{\text{sh}(-\epsilon_+\pm(\phi_I+\phi_J+m))}$$

$$\text{sh}(x) := e^{\frac{x}{2}}-e^{-\frac{x}{2}}\,, \qquad \text{ch}(x) := e^{\frac{x}{2}}+e^{-\frac{x}{2}}$$

At *k*-instanton order, we require

$$Z^{\text{vec}}_{\text{SU}(N),k}Z^{\text{sym}}_{\text{SU}(N),k}(m) = Z^{\text{vec}}_{\text{SU}(N),k}Z^{\text{anti}}_{\text{SU}(N),k}(m)Z^{\text{fund}}_{\text{SU}(N),k}$$

$$\text{sh}(2\phi_I+m\pm\epsilon_-) = \frac{1}{\text{sh}(-\epsilon_+\pm(2\phi_I+m))}\prod_{l=1}^{N_f}\text{sh}(\phi_I+m_l)$$

$$\mathcal{Z}^{\mathrm{Sym}} = \mathcal{Z}^{\mathrm{AS}}\mathcal{Z}^{\mathrm{F}}\Big|_{m_\ell = \frac{m \pm \epsilon_\pm}{2}(+\pi i)}$$

$$\begin{aligned}
m_1 &= \frac{m+\epsilon_+}{2}, \quad & m_2 &= \frac{m+\epsilon_+}{2} + \pi i \\
m_3 &= \frac{m-\epsilon_+}{2}, \quad & m_4 &= \frac{m-\epsilon_+}{2} + \pi i \\
m_5 &= \frac{m+\epsilon_-}{2}, \quad & m_6 &= \frac{m+\epsilon_-}{2} + \pi i \\
m_7 &= \frac{m-\epsilon_-}{2}, \quad & m_8 &= \frac{m-\epsilon_-}{2} + \pi i
\end{aligned}$$

$$Z_k^{\mathrm{SU}(N)+1\mathrm{Sym}} = Z_k^{\mathrm{SU}(N)+1\mathrm{AS}+8\mathrm{F}}\Big|_{m_\ell = \frac{m \pm \epsilon_\pm}{2}(+\pi i)}$$

[SK, Xiaobin Li, Satoshi Nawata, Futoshi Yagi '23]

We also find

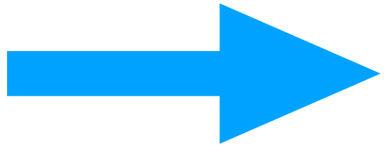
$$Z_k^{\mathrm{SO}(2N)} = Z_{2k}^{\mathrm{Sp}(N)+8\mathrm{F}}\Big|_{m_\ell = \frac{\epsilon_+ \pm \epsilon_\pm}{2}(+\pi i)}$$

$$\begin{aligned}
&m_1 = \epsilon_+, \quad && m_2 = \epsilon_+ + \pi i, \\
&m_3 = 0, \quad && m_4 = \pi i, \\
&m_5 = \frac{\epsilon_1}{2}, \quad && m_6 = \frac{\epsilon_1}{2} + \pi i, \\
&m_7 = \frac{\epsilon_2}{2}, \quad && m_8 = \frac{\epsilon_2}{2} + \pi i.
\end{aligned}$$

c.f. $\quad Z_{2k+1}^{\mathrm{Sp}(N)+8\mathrm{F}}\Big|_{m_\ell = \frac{\epsilon_+ \pm \epsilon_\pm}{2}(+\pi i)} = 0$

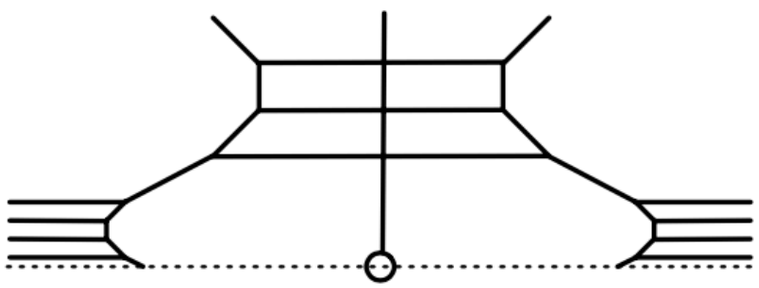
Moreover, Seiberg-Witten curves agree

SW curve for 5d N=1
SU(N) + 1AS + 8F

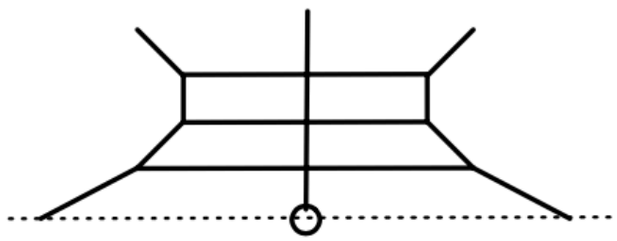


Freezing

SW curve for 5d N=1
SU(N) + 1Sym



$O7^-$



$O7^+$

Conclusion

We conveyed new developments / perspective.
In particular,

$$\mathrm{O7}^+ \sim \mathrm{O7}^- + 8\mathrm{D7}$$

$$(\mathrm{O}p^+ \sim \mathrm{O}p^- + 2^{p-4}\mathrm{D}p)$$

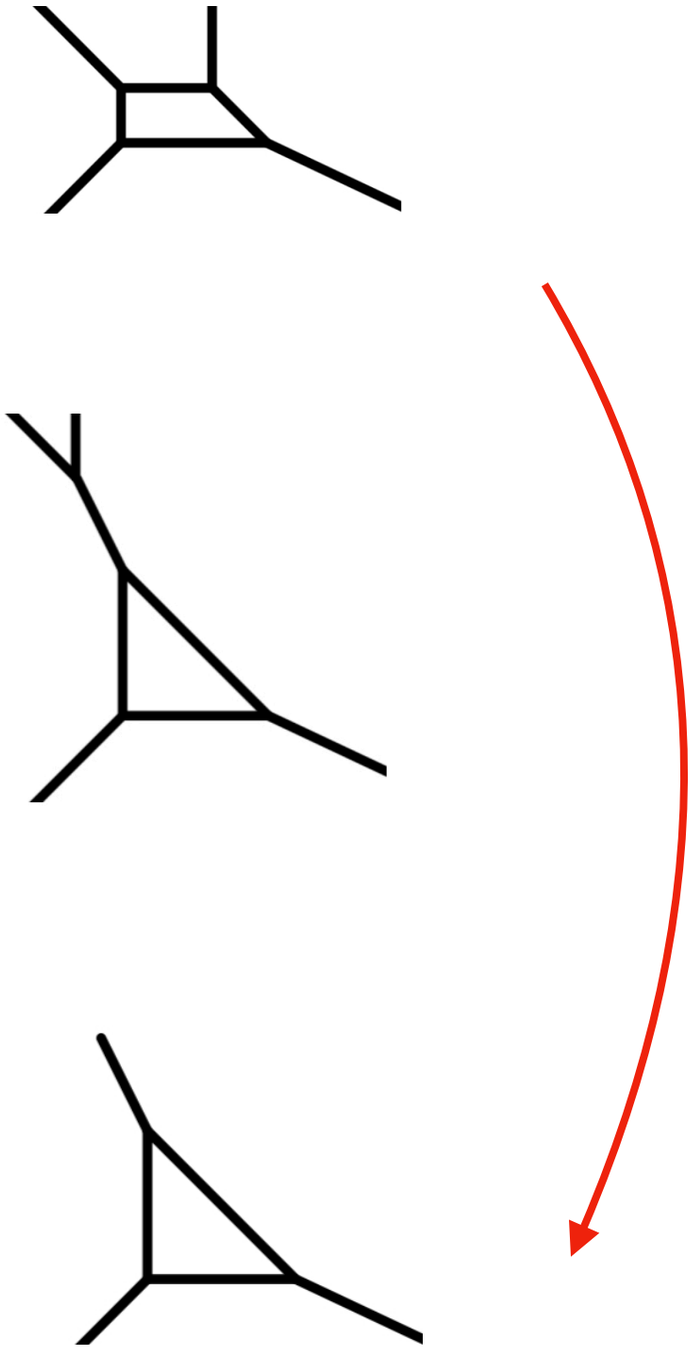
(i) Perturbative observables are consistent.

(ii) Seiberg-Witten curves agree!

(iii) Refined Instanton partition functions also support.

Also useful for ..

computing physical observables for non-Lagrangian theories.



$\mathrm{SU}(2)_\pi$

Decoupling instantonic hypermultiplet

$\text{local } \mathbb{P}^2$

$E_0 \text{ theory}$

No global symmetry

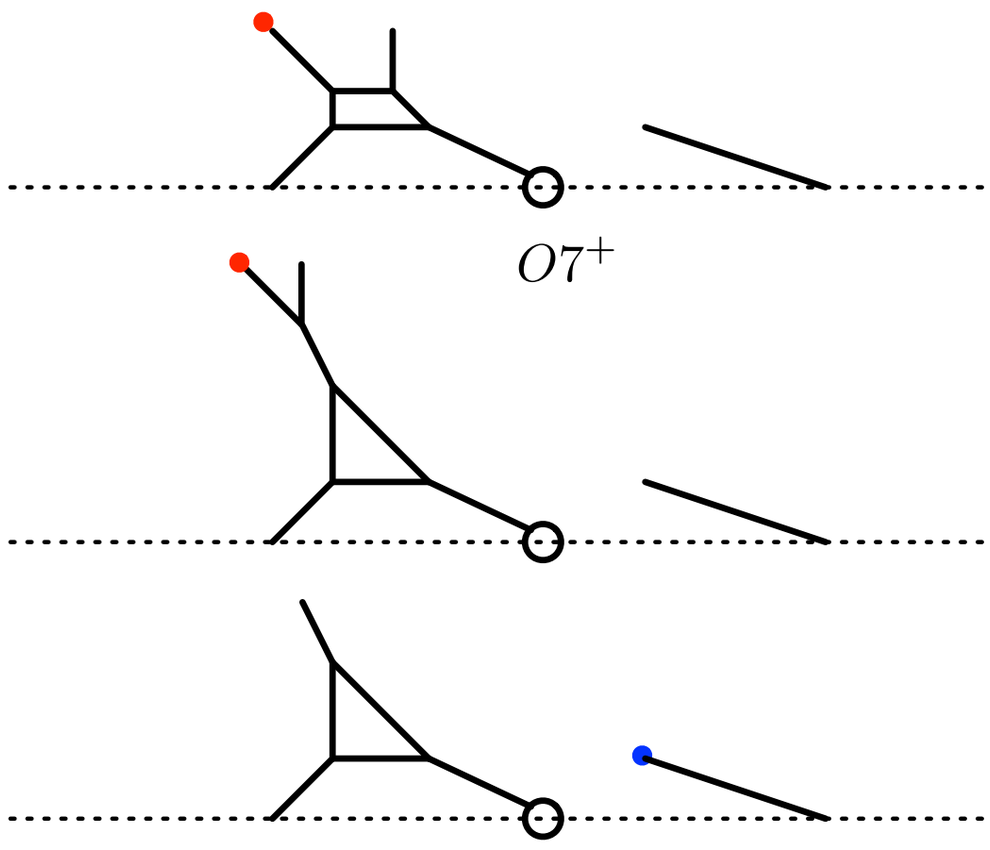
Yet another rank-1 non-Lagrangian theory

$SU(2)_\theta$ + 1**Sym** = $SU(2)_\theta$ + 1**Adj** : KK theories [Bhardwaj '19]

M-string on a circle ($\theta = 0$)

A_2 theory with $\mathbb{Z}_2$ outer automorphism twist ($\theta = \pi$)

Sym $\longrightarrow$ $O7^+$-plane



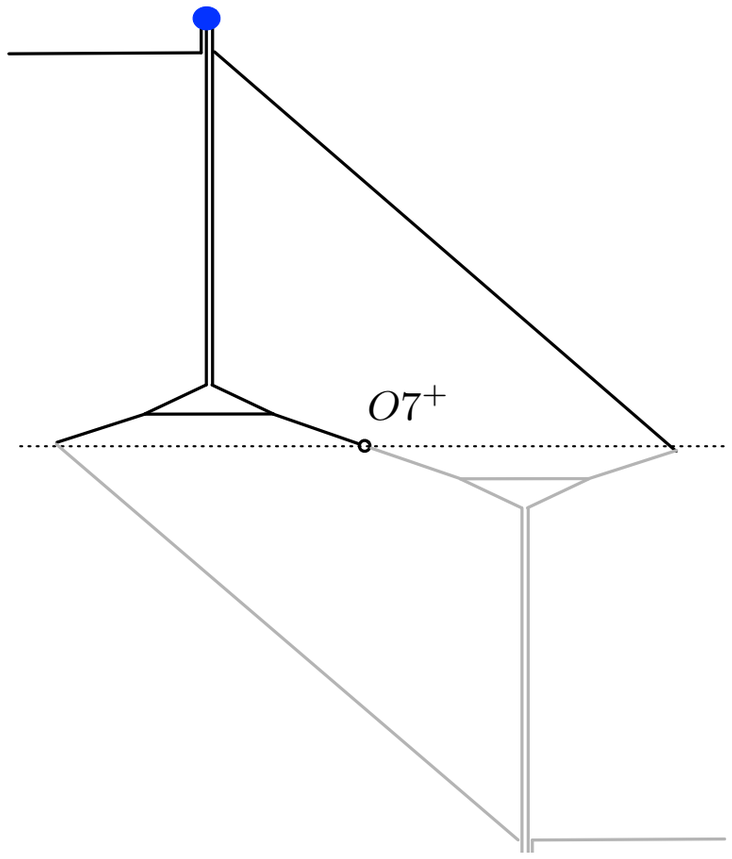
$\mathrm{SU}(2)_\pi + 1\mathbf{Sym}$

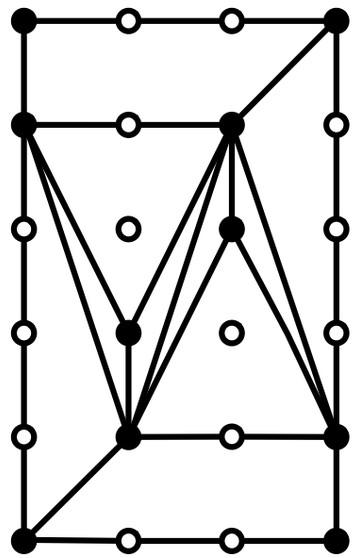
$\mathbb{P}^2 + 1\mathbf{Adj}$ (or $SU(2)_\pi + 1\mathbf{Adj}\text{“}-\text{”}1\mathbf{F}$)

does have flavor symmetry

$\rightarrow$ $\widehat{E_1}$ theory

Seiberg-Witten curve for $\widehat{E}_1$ theory





$$
\begin{aligned}
& w^5(t-M)^3 \\
& - M^3 w^4 (t-M)^2 (t-M^{-5}) \\
& - 2w^3 (t-M)\left(t^2 + (M-U/2)t + M^2\right) \\
& - 2w^2 (1-Mt)\left(1 + (M-U/2)t + M^2 t^2\right) \\
& - M^3 w (1-Mt)^2 (1-M^{-5}t) \\
& + (1-Mt)^3 = 0
\end{aligned}
$$

U : Coulomb branch parameter

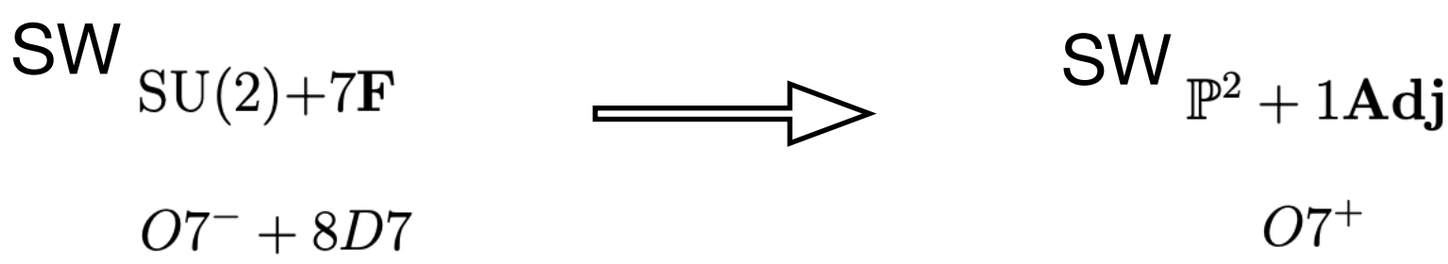
M : mass of Symmetric hyper

$$\mathbb{P}^2 + 1\mathbf{Adj} \text{ (or } SU(2)_\pi + 1\mathbf{Adj} - 1\mathbf{F})$$

by replacing $1\mathbf{Sym}$ with $1\mathbf{AS} + 8\mathbf{F}$

$$\begin{aligned} SU(2)_\pi + 1\mathbf{Adj} - 1\mathbf{F} \quad &\to \quad SU(2)_\pi + 1\mathbf{AS} + 8\mathbf{F} - \mathbf{F} \\ &= \quad SU(2) + 7\mathbf{F} \end{aligned}$$

Freezing: $O7^- + 8D7 \longleftrightarrow O7^+$



Another novel perspective is

$$O7^- + 4D7 \sim \mathbb{Z}_2$$

[SK, Xiaobin Li, Futoshi Yagi, Rui-dong Zhu ’24]

Futoshi’s talk