The Curious Story of the Photon

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- S. Klein, A. Mueller, BX, F. Yuan, 1811.05519; 2003.02947;
- BX, F. Yuan, J. Zhou, 2003.06352
- Y. Shi, L. Wang, S. Y. Wei, BX, L. Zheng, 2008.03569

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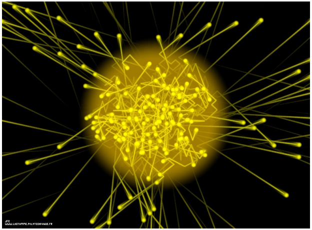
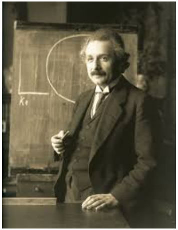
Outline

Early History of the Photon

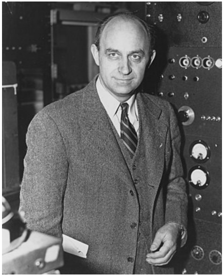
As always, the story begins with Einstein.

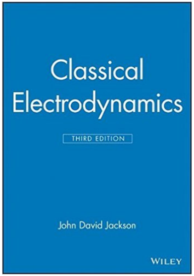




- Photons are quanta of EM radiation and represent the particle nature of light.
- A. Einstein was awarded the Nobel Prize in Physics (1921) “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect."
- This led us into the quantum world and the concept of Wave-Particle duality.

Classical Electrodynamics and Virtual Quanta

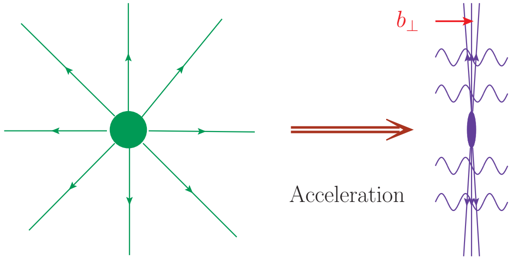




- Following Fermi[24], Weizsäcker [34] and Williams [35] discovered that the EM fields of a relativistically moving charged particle are almost transverse.
- This is equivalent to say that the charged particle carries a cloud of quasi-real photons, which are ready to be radiated if perturbed.
- Weizsäcker-Williams method of virtual quanta (Equivalent Photon Approximation).
- Gravitational case and t' Hooft Amplitude. [Jackiw, Kabat and Ortiz, 92]
- Application in QCD: WW gluon distribution. [McLerran, Venugopalan, 94; Kovchegov, 96; Jalilian-Marian, Kovner, McLerran and Weigert, 97]

EPA and Weizsäcker-Williams Photon Distribution

Boost the static potential to the infinite momentum frame ($\gamma \to \infty$):
[Jackiw, Kabat and Ortiz, 92] and HW problem (P11.18) in [Jackson]



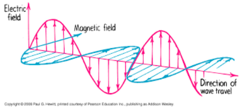
$\beta = 0$ $\beta = 1$

$$A^{+}_{Cov} = -\frac{q}{\pi}\ln(\lambda b_\perp)\delta(t-z),$$

$$\vec{E} = \frac{q}{2\pi}\frac{\vec{b}_\perp}{b_\perp^2}\delta(t-z),$$

$$\vec{B} = \frac{q}{2\pi}\frac{\hat{v}\times\vec{b}_\perp}{b_\perp^2}\delta(t-z),$$

$$\vec{A}^{LC}_{\perp} = -\frac{q}{2\pi}[\vec{\nabla}_\perp \ln(\lambda b_\perp)]\theta(t-z).$$



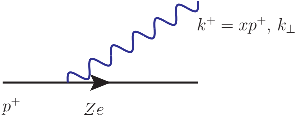
Static electric fields $\Rightarrow$ Electro-Magnetic Wave
$\Rightarrow$ The EM pulses are equivalent to a lot of photons

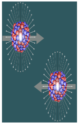
- The gauge potentials A_μ in Covariant gauge and LC gauge are related by a gauge transformation. λ is an irrelevant parameter setting the scale.
- Classical EM: transverse EM fields $\Leftrightarrow$ QM: Co-moving Quasi-real photons.

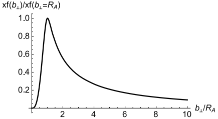
Transverse Momentum Dependent (TMD) Photon Distribution

The photon distribution (flux) for a point particle can be computed from $\vec{A}_\perp^{LC}$

$$xf_\gamma(x, b_\perp) = \frac{Z^2\alpha}{\pi^2} x^2 m^2 K_1^2(xm\, b_\perp) = \frac{Z^2\alpha}{\pi^2 b_\perp^2}\bigg|_{m\to 0} \quad \text{with} \quad q = Ze.$$





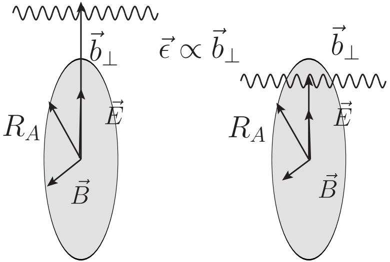


The photon distribution in the transverse momentum space

$$\begin{aligned} xf_\gamma(x, \vec{k}_\perp) &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-ixP^+\xi^- - ik_\perp\cdot\xi_\perp} \left\langle A \middle| F^{+i}\left(\frac{\xi}{2}\right) F^{+i}\left(-\frac{\xi}{2}\right) \middle| A \right\rangle \\ &\simeq \frac{Z^2\alpha}{\pi^2} \frac{k_\perp^2}{(k_\perp^2 + x^2M^2)^2} F_A^2(k^2). \end{aligned}$$

- $F_A(k^2)$ is the charge form factor with $k^2 = k_\perp^2 + x^2M^2$. $F_A = 1$ for point charge.
- Wood-Saxon or Gaussian models for realistic nuclei. (*Pb* is very bright!)
- Typical transverse momentum of the photon is $1/R_A$, which is 30MeV for *Pb*.

Linearly Polarized Photon



- E is linearly polarized along the impact parameter $b_\perp$ direction;
- $\vec{B} \perp \vec{E}$;
- The LC gauge potential $A_\perp \propto \vec{b}_\perp$;
- Polarization vector $\vec{\epsilon}_\perp = \vec{b}_\perp / b_\perp$.
- Similar case in momentum space.

- WW photon distribution is maximumly polarized, since $xf_\gamma = xh_\gamma$.

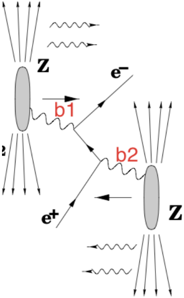
$$xf_\gamma^{ij}(x; b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \langle A, -\frac{\Delta_\perp}{2} | F^{+i} F^{+j} | A, \frac{\Delta_\perp}{2} \rangle \,,$$

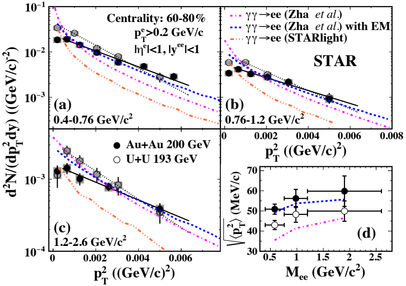
$$xf_\gamma^{ij}(x; b_\perp) = \frac{\delta^{ij}}{2} xf_\gamma(x; b_\perp) + \left(\frac{b_\perp^i b_\perp^j}{b_\perp^2} - \frac{\delta^{ij}}{2} \right) xh_\gamma(x; b_\perp) = \frac{b_\perp^i b_\perp^j}{b_\perp^2} xf_\gamma,$$

$$xh_\gamma(x, b_\perp) = xf_\gamma(x, b_\perp) = 4Z^2\alpha \left| \int \frac{d^2k_\perp}{(2\pi)^2} e^{ik_\perp \cdot b_\perp} \frac{\vec{k}_\perp}{k^2} F_A(k^2) \right|^2$$

The Need of Photon Wigner Distribution from Experiments

STAR[1806.02295], ATLAS ([CONF-2019-051]) and CMS[PAS-HIN-19-014] collaborations observe $\gamma\gamma \to l^+ l^-$ azimuthal angular correlations in *AA* collisions with different impact parameter $b_\perp = b_{1\perp} - b_{2\perp}$

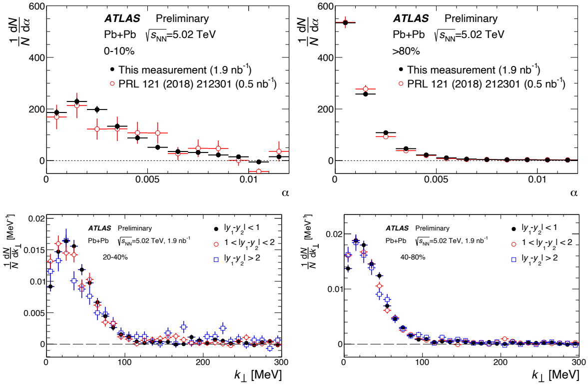




- Need the incoming photon $k_\perp$ distribution at fixed impact parameter $b_\perp$.
- Earlier calculations: [Vidovic, *et al*, 93; Hencken, *et al*, 94; Zha, *et al*, 18; Li, Zhou, Zhou, 19; and many]

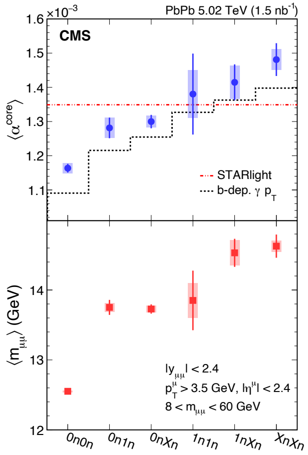
The Need of Photon Wigner Distribution from Experiments

ATLAS ([CONF-2019-051]) measures the acoplanarity $\alpha \equiv 1 - |\Delta\phi|/\pi$ ($\Delta\phi = \phi_{l+} - \phi_{l\_}$) and the total momentum imbalance $k_\perp$ of the muon pair in *PbPb* collisions with different centralities (impact parameter $b_\perp$)



- Need the incoming photon $k_\perp$ distribution at fixed impact parameter $b_\perp$.
- Mysterious and interesting displaced peaks (dips) in central collisions.

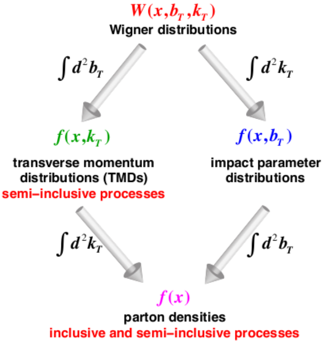
CMS Results

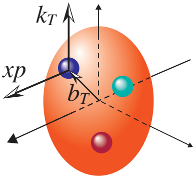


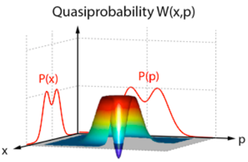
- CMS[PAS-HIN-19-014] measures the number of neutrons in the very forward region in UPC.
- Higher neutron multiplicity corresponds to smaller $\langle b \rangle$ on average, and vice versa.
- This measurement demonstrates the transverse momentum imbalance and energy of photons (invariant mass of the dilepton) emitted from relativistic ions have impact parameter dependence.
- Great way to select events with $\langle b \rangle$. Even measure various asymmetries.
- Black dotted line (QED calculation) [Brandenburg, Li, Ruan, Tang, Xu, Yang and Zha, 20]

Wigner distribution

Wigner distributions [Ji, 03; Belitsky, Ji, Yuan, 2004] ingeniously encode all quantum information of how partons are distributed inside hadrons.







- Quasi-probability distribution; Not positive definite.
- Need to smear over k_T and $b_\perp$ $\rightarrow$ Husimi distribution. [Hagiwara, Hatta, 14]

Photon Wigner Distribution and Generalized TMD

Def. of Wigner distribution:

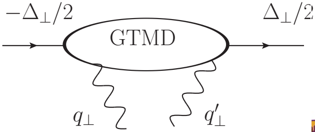
$$xf_\gamma(x,\vec{k}_\perp;\vec{b}_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ixP^+\xi^- - ik_\perp\cdot\xi_\perp}$$
$$\times \quad \left\langle A, +\frac{\Delta_\perp}{2} \middle| F^{+i}\left(\vec{b}_\perp + \frac{\xi}{2}\right) F^{+i}\left(\vec{b}_\perp - \frac{\xi}{2}\right) \middle| A, -\frac{\Delta_\perp}{2} \right\rangle ,$$

Def. of GTMD

$$xf_\gamma(x,k_\perp,\Delta_\perp) \equiv \int d^2b_\perp e^{-i\Delta\cdot b_\perp} xf_\gamma(x,\vec{k}_\perp;\vec{b}_\perp).$$

- For a heavy nucleus with charge Ze, the GTMD reads

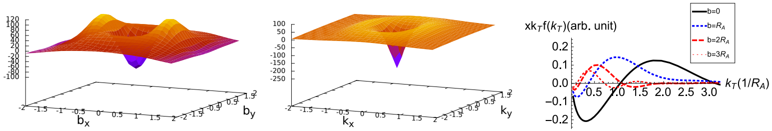
$$xf_\gamma(x,k_\perp;\Delta_\perp) = xh_\gamma(x,k_\perp;\Delta_\perp)$$
$$= \frac{4Z^2\alpha}{(2\pi)^2} \frac{q_\perp \cdot q'_\perp}{q^2 q'^2} F_A(q^2) F_A(q'^2) ,$$
$$q_\perp = k_\perp - \frac{\Delta_\perp}{2}, \quad \text{and} \quad q'_\perp = k_\perp + \frac{\Delta_\perp}{2}$$



- $\int d^2b_\perp xf_\gamma(x,k_\perp,b_\perp) \Rightarrow TMD$; $\int d^2k_\perp xf_\gamma(x,k_\perp,b_\perp) \Rightarrow b_\perp$ distribution.
- EPA $\rightarrow$ Generalized EPA.

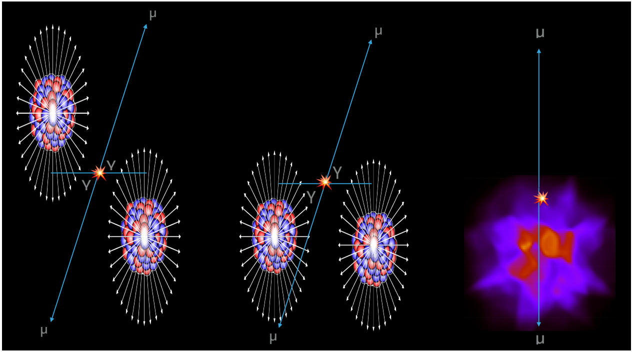
The Oscillating Behavior of Wigner Distributions

Models of Wigner[Lorcé, Pasquini, 11; Lorcé, Pasquini, Xiong, Yuan, 11]
[Hagiwara, Hatta, 14] [S. Klein, A. Mueller, BX, F. Yuan, 20]



- Due to the uncertainty principle, Wigner distributions often has the oscillating behavior when one tries to measure $b_\perp$ and $k_\perp$ simultaneously.
- Will the negative region of the Wigner distribution cause a serious problem?
- Two observations: diffractive dijets in DIS and $\gamma\gamma \to l^+ l^-$ in $PbPb$ collisions.
- Opinion: No, it seems that the LO cross-sections are always positive-definite. (It will be interesting if one can prove this conjecture.)

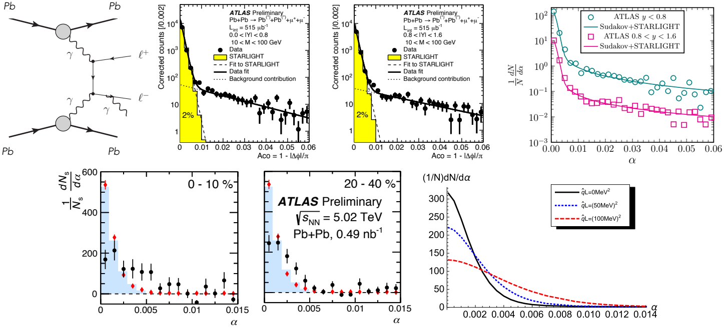
Dilepton productions in AA collisions



- Ultra-peripheral Collisions (UPC) has no hadronic background.
- Strong hadronic background in non-UPC collisions.
- QGP medium interactions and magnetic fields may be present as well.

Dilepton productions in AA collisions

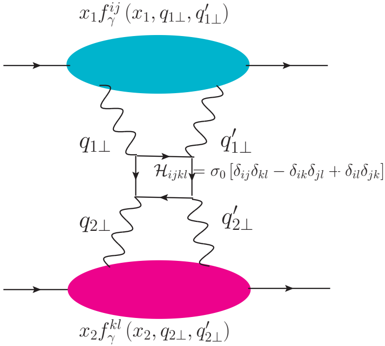
[ATLAS, 1806.08708; 2019 Conf-51]; [Klein, Mueller, Xiao, Yuan, 18; 20]



- Acoplanarity $\alpha \equiv 1 - \Delta\phi/\pi$ and asymmetry $A \equiv ||p_T^+| - |p_T^-||/||p_T^+| + |p_T^-||$.
- Initial photon $k_\perp$ distribution plus photon radiation describes the background.
- Sudakov resummation of QED radiative corrections. (Similar to QCD parton shower)

The application of Wigner distribution in the lepton pair production

Assuming the GEPA factorization at LO, and compute the hard factor $\mathcal{H}$



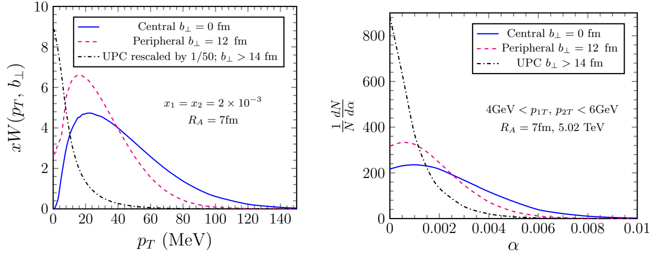
$$\frac{d\sigma(AB[\gamma\gamma] \to \mu^+\mu^-)}{dy_1 dy_2 d^2p_{1T} d^2p_{2T} d^2b_\perp}$$
$$= \int d^2k_{1T} d^2k_{2T} \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp}$$
$$\times x f_\gamma^{ij}(k_{1T}; \Delta_\perp) x_2 f_\gamma^{kl}(k_{2T}; \Delta_\perp) H_{ijkl}$$
$$\times \delta^{(2)}(p_T - k_{1T} - k_{2T})$$
$$\propto \int d^2b_{1\perp} d^2b_{2\perp} \delta^{(2)}(b_\perp - b_{1\perp} + b_{2\perp})$$
$$\times x f_\gamma^{ij}(k_{1T}; b_{1\perp}) x_2 f_\gamma^{kl}(k_{2T}; b_{2\perp}) H_{ijkl}$$

- Notations for the momenta: $q_\perp = k_T - \frac{\Delta_\perp}{2}$, and $q'_\perp = k_T + \frac{\Delta_\perp}{2}$.
- Need the off-diagonal momenta $\Delta_\perp$ to access impact parameter $b_\perp$.
- Will negative region of Wigner Dist $x f_\gamma^{ij}(k_T; b_\perp)$ ever be catastrophic?

Results of GEPA

If we define $G^{ik} = \int \frac{d^2 k_{1T}}{(2\pi)} e^{ik_{1T}\cdot b_\perp} k^i_{1T} k^k_{2T} \frac{F(k_1^2)}{k_1^2}\frac{F(k_2^2)}{k_2^2}$, and note $k_{1T} + k_{2T} = p_T$

$$\frac{d\sigma}{dy_1 dy_2 d^2 p_{1T} d^2 p_{2T} d^2 b_\perp} = \sigma_0 \left[(G^{11} - G^{22})(G^{11*} - G^{22*}) + (G^{12} + G^{21})(G^{12*} + G^{21*}) \right] \geq 0$$

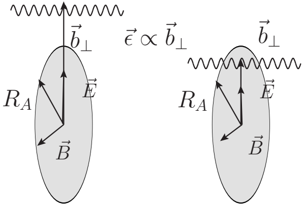


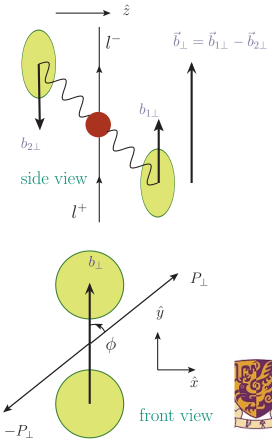
- The cross section $= 0$ when $b_\perp = 0$ and $p_T = 0$ ($G11 = G_{22}$ and $G_{12} = -G_{21}$)
- Explains the dip (displaced peak, ATLAS) in central AA. $\sigma \sim xW(p_T, b_\perp)$
- However, the dip becomes much less significant after averaging over momenta.
- Qualitatively explains recent ATLAS CONF-2019-51 data. Still a bit puzzling.

Electromagnetic Flow of Leptons in Heavy Ion Collisions $\cos 4\phi$

Integrate over the momentum imbalance and measure P_T at fixed $b_\perp$

[BX, F. Yuan, J. Zhou, 20]





$$|\mathcal{M}|^2 = 2e^4 \left[\left(\frac{u}{t} + \frac{t}{u} \right) - 2\cos 2(\phi_1 + \phi_2) \right]$$

$$\frac{d\sigma}{d^2P_\perp dy_1 dy_2 d^2b_\perp} = \frac{2\alpha_e^2}{Q^4} \left[\mathcal{A} + \mathcal{C} \cos 4\phi \right]$$

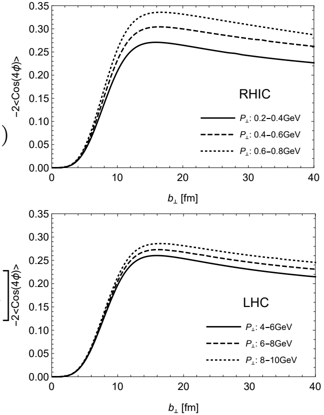
- Novel and large v_4 due to linear polarized photon.
- $v_2 \sim m^2/P_\perp^2$, since it requires spin flip which is suppressed by the lepton mass.

Electromagnetic Flow of Leptons in Heavy Ion Collisions $\cos 4\phi$

$$\frac{d\sigma}{d^2P_\perp dy_1 dy_2 d^2b_\perp} = \frac{2\alpha_e^2}{Q^4}\left[\mathcal{A} + \mathcal{C}\cos 4\phi\right]$$

$$\mathcal{A} = \frac{Q^2 - 2P_\perp^2}{P_\perp^2}\int d^2b_{1\perp} d^2b_{2\perp}\delta^2(\vec{b}_\perp - \vec{b}_{1\perp} + \vec{b}_{2\perp}) \times x_1 f_\gamma(x_1, b_{1\perp}^2) x_2 f_\gamma(x_2, b_{2\perp}^2)$$

$$\mathcal{C} = -2\int d^2b_{1\perp} d^2b_{2\perp}\delta^2(\vec{b}_\perp - \vec{b}_{1\perp} + \vec{b}_{2\perp}) \times \left[2\left(2(\hat{b}_{2\perp}\cdot\hat{b}_\perp)(\hat{b}_{1\perp}\cdot\hat{b}_\perp) - \hat{b}_{1\perp}\cdot\hat{b}_{2\perp}\right)^2 - 1\right] \times x_1 h_\gamma(x_1, b_{1\perp}^2) x_2 h_\gamma(x_2, b_{2\perp}^2)\,.$$



- A and C depend on unpolarized and linearly polarized photon distributions, respectively.
- CMS will be able to use neutron tagging to probe this interesting EM flow.
- Crucial information on the production mechanism of the dilepton.

Proposed EIC Facilities Across the Globe

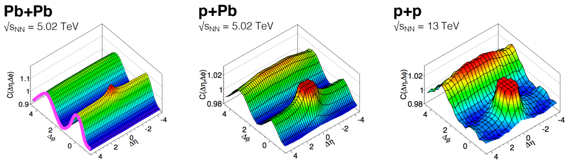
- Electron-Ion colliders will become the cutting-edge high-energy and nuclear physics research facilities in the near future.

Collectivity (correlation, flow) is everywhere!

In high multiplicity events, large azimuthal angle correlations are observed:

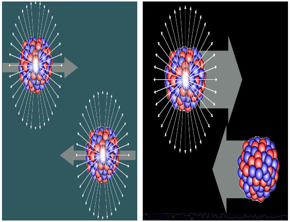
$$C_n\{2\} \equiv \{e^{in(\phi_1-\phi_2)}\} = \frac{\int d\phi_1 d\phi_2 e^{in(\phi_1-\phi_2)} \frac{dN}{d\phi_1}\frac{dN}{d\phi_2}}{\int d\phi_1 d\phi_2 \frac{dN}{d\phi_1}\frac{dN}{d\phi_1}}$$

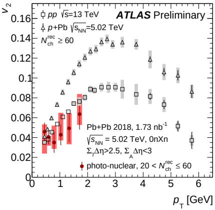
$$= \{e^{in(\phi_1-\phi_{RP})}\}\{e^{in(\phi_{RP}-\phi_2)}\} = v_n^2\{2\}.$$



- Collectivity is used to describe the particle correlation. It is observed in both large and small systems and for light and heavy hadrons!
- The origin of the collectivity phenomenon is still not clear. Initial vs Final?

Collectivity at EIC?



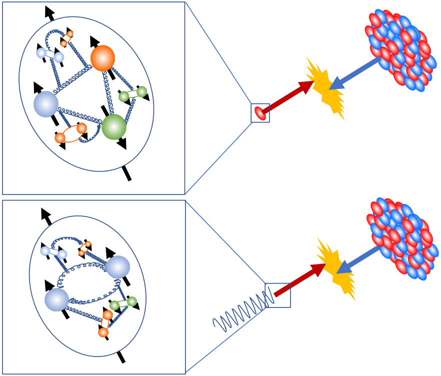


- Collectivity is used to describe the particle correlation. It is observed in both large and small systems and for light and heavy hadrons!
- New exciting results for UPC in PbPb collisions. (Mini-EIC)
- What about predictions for the collectivity at the EIC on the horizon?

The Structure of Photons

Photons can have a very rich QCD structure

$$|\gamma\rangle = |\gamma_0\rangle + \sum_{m,n} |m\, q\bar{q} + n\, g\rangle + \sum_{\rho,\omega,\cdots} |V\rangle + \cdots ,$$



- Point like (high Q^2)
- Partonic
- VMD [Sakurai, 60]

Strong similarity between $\gamma^* A$ and pA collisions when γ^* has a long lifetime.

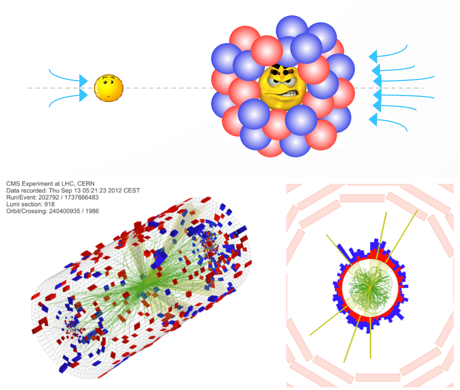
$$t_{\text{lifetime}} \sim \frac{1}{q^-} = \frac{q^+}{Q^2} \gg \frac{m_p}{P^-} R \quad \Rightarrow \quad x_B \ll \frac{1}{m_p R}$$

Opinion: collectivity in $\gamma^* A$ collisions regardless the underlying interpretation.

Collectivity in high multiplicity events in pA collisions

Qualitative understanding of high multiplicity events and correlation.

- Many active partons
  $|P\rangle = |qqq\rangle + |qqq\,ng\rangle + \cdots$
- Fluctuation in parton density
  Stronger Q_s in nuclei.
- Correlated multiple scatterings
  Non-trivial color correlation.
- Possible stronger parton shower.
  Shower produce soft particles due to hard collisions.



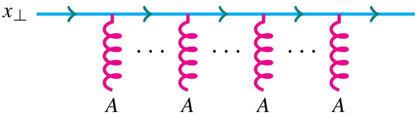
A CGC model for correlation based on the above three pillar in Red.

- Let us pick two initially uncorrelated collinear partons (say $q + q$) from proton, and consider their interactions with the target nucleus.
- Correlation can be generated between them due to multiple interaction.
- Due to Unitarity, the un-observed partons do not affect the correlation of the system.

Wilson Lines in Color Glass Condensate Formalism

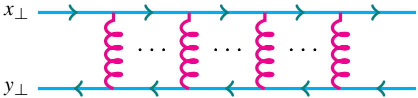
Consider the multiple scattering between a fast quark and target background gluon fields.

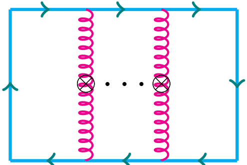
$$U(x_\perp) = \mathcal{P} \exp\left(-ig \int dz^+ A^-(x_\perp, z^+)\right)$$

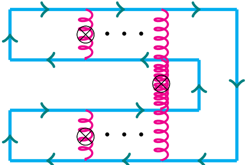


The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$



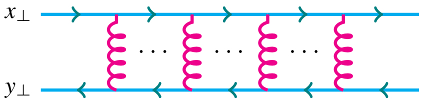

Dipole (DP gluon)


Quadrupole (WW gluon)

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$



- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2 k_\perp} = \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{-ik_\perp \cdot (x_\perp - y_\perp)} \frac{1}{N_c} \left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp) \right\rangle .$$

- Consider multiple particle productions, more complex color structures arise. For example, 2 quark production $\Leftrightarrow$ 2 dipoles $+ \cdots$

$$\left\langle \mathrm{Tr} UU^\dagger \mathrm{Tr} UU^\dagger \right\rangle = \left\langle \mathrm{Tr} UU^\dagger \right\rangle \left\langle \mathrm{Tr} UU^\dagger \right\rangle + \text{correlations}$$

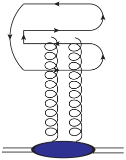
- Quadrupole $\frac{1}{N_c}\left\langle \mathrm{Tr} UU^\dagger UU^\dagger \right\rangle \neq \frac{1}{N_c}\left\langle \mathrm{Tr} UU^\dagger \right\rangle \frac{1}{N_c}\left\langle \mathrm{Tr} UU^\dagger \right\rangle$?

A Tale of Two Gluon Distributions

[F. Dominguez, C. Marquet, Xiao and F. Yuan, 11]

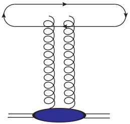
I. Weizsäcker Williams gluon distribution

$$xG_{\rm WW}(x,k_\perp) = \frac{2N_c}{\alpha_S}\int \frac{d^2R_\perp}{(2\pi)^2}\frac{d^2R'_\perp}{(2\pi)^2}\, e^{iq_\perp\cdot(R_\perp-R'_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr}\left[i\partial_i U(R_\perp)\right] U^\dagger(R'_\perp)\left[i\partial_i U(R'_\perp)\right] U^\dagger(R_\perp)\right\rangle_x .$$



II. Color Dipole gluon distribution:

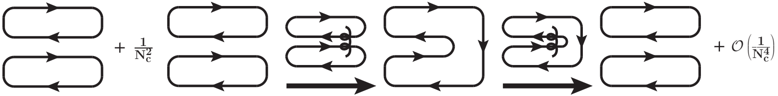
$$xG_{\rm DP}(x,k_\perp) = \frac{2N_c}{\alpha_s}\int \frac{d^2R_\perp d^2R'_\perp}{(2\pi)^4} e^{iq_\perp\cdot(R_\perp-R'_\perp)} \left(\nabla_{R_\perp}\cdot\nabla_{R'_\perp}\right)\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U(R_\perp)\,U^\dagger(R'_\perp)\right]\right\rangle_x ,$$



- Quadrupole ⇒ Weizsäcker Williams gluon; Dipole ⇒ Color Dipole gluon.
- Two fundamental topological color singlets configurations in coordinate space.
- Generalized universality in large N_c in eA and pA collisions for Wilson lines [F. Dominguez, C. Marquet, A. Stasto and BX, 12]

Correlations in CGC

Correlations between uncorrelated incoming quarks (gluons) are generated due to quadrupole as N_c corrections. [Lappi, 15; Lappi, Schenke, Schlichting, Venugopalan, 16; Dusling, Mace, Venugopalan, 17; Davy, Marquet, Shi, Xiao, Zhang, 18]



$$\frac{d^2N}{d^2k_{1\perp}d^2k_{2\perp}} = \int d^2r_{1\perp}d^2r_{2\perp}e^{-ik_{1\perp}\cdot r_{1\perp}}e^{-ik_{2\perp}\cdot r_{2\perp}} \times \frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle$$

where

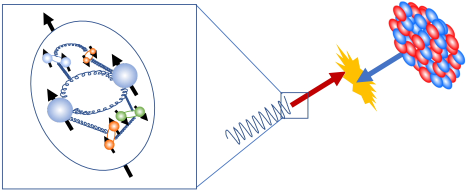
$$\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle \neq \frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V(x_1)V(x_2)^\dagger\right]\right\rangle\left\langle \mathrm{tr}\left[V(x_3)V(x_4)^\dagger\right]\right\rangle$$

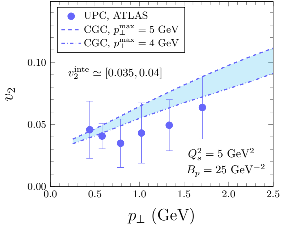
$$= e^{-\frac{Q_s^2}{4}(r_1^2+r_2^2)}\left[1+\frac{(\frac{Q_s^2}{2}\boldsymbol{r}_1\cdot\boldsymbol{r}_2)^2}{N_c^2}\int_0^1 d\xi\int_0^\xi d\eta\, e^{\frac{\eta Q_s^2}{8}[(\boldsymbol{r}_1-\boldsymbol{r}_2)^2-4(\boldsymbol{b}_1-\boldsymbol{b}_2)^2]}\right]$$

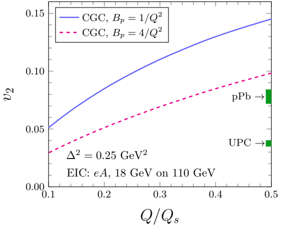
- At leading N_c, $\frac{d^2N}{d^2k_{1\perp}d^2k_{2\perp}} = \left(\frac{dN}{d^2k_{1\perp}}\right)\left(\frac{dN}{d^2k_{2\perp}}\right)$, there are no correlations.
- The correlations only come in as higher order N_c corrections as shown above.

Resulting v_2 in γA collisions from CGC

[Y. Shi, L. Wang, S. Y. Wei, BX, L. Zheng, 2008.03569]



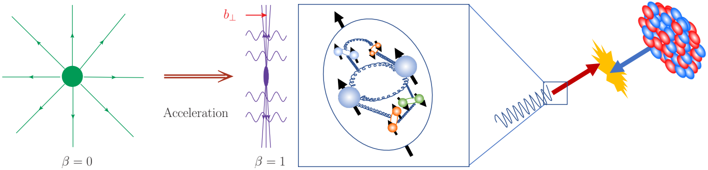




- Assume lifetime is sufficient for photon to build up many partonic contents in its WF.
- The transverse size $r \sim \frac{1}{Q}$ and $r \leq \frac{1}{\Lambda_{QCD}}$ as far as QCD fluctuation is concerned.
- Again multi-gluon correlations can generate correlations for both UPC and pA.

Summary

Several curious and interesting aspects of the photon



- Wigner distribution ⇒ Interesting measurements and theoretical issue.
- Linear polarization ⇒ Non-trivial correlations in final state lepton pairs.
- Rich partonic structure ⇒ Collectivity at the future EIC.
- All of the channels are currently being pursued at the LHC (in particular ATLAS and CMS)!