A brief review of loop quantum black hole models

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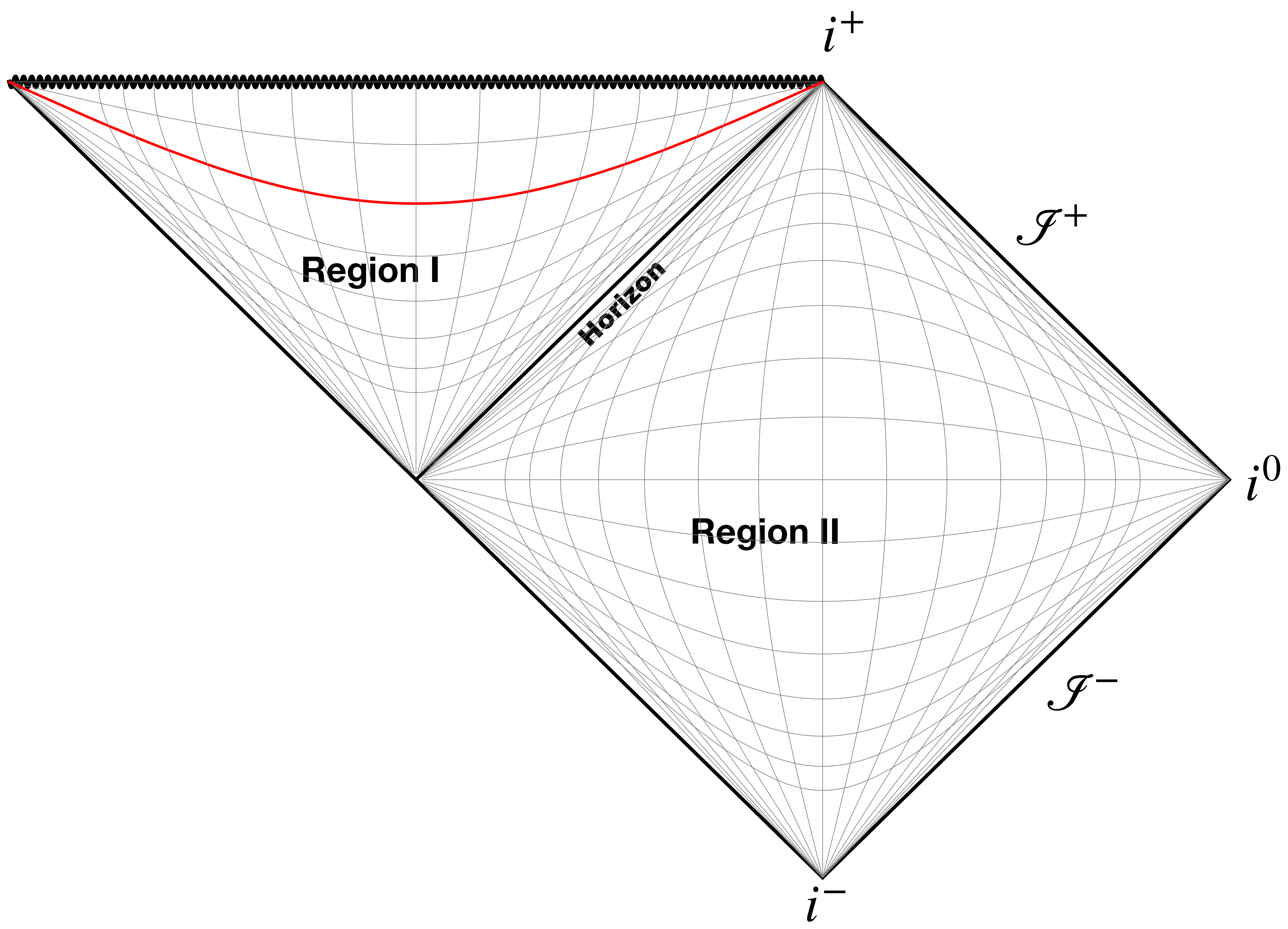
Motivation and background

The black holes of nature are the most perfect macroscopic objects there are in the Universe

—Subrahmanyan Chandrasekhar

- **Black holes have been observed by gravitational wave and EHT; It sets the stage to probe quantum gravity;**
- **Spherically symmetric BH is a good candidate for QG research; It is neither as simple as cosmology nor very complicated.**
- **In Loop quantum gravity, our answer on quantum BH haven't form a unique picture; There are, e.g., Ashtekar-Bojowald paradigm** [Ashtekar & Bojowald 05'], **the SF qBH model** [Rovelli, Harggard, Christodoulou, Speziale, Vilensky etc. 15', 16', Han, Qu, CZ 24'] **and other models by the people in the community** [Ashtekar, Bojowald, Bodendorfer, Boehmer, Chiou, Giesel, Gambini, Han, Liu, Modesto, Ma, Mehdi, Olmedo, Pullin, Singh, Vandersloot, Yang, Zhang and so on]

Classical theories



Region I is a spacetime in its own right. The homogenous Cauchy slice has topology $\mathbb{R}\times \mathrm{S}^2$:

- **Standard Schwarzschild interior $\Longleftarrow$ EOM + general metric:**

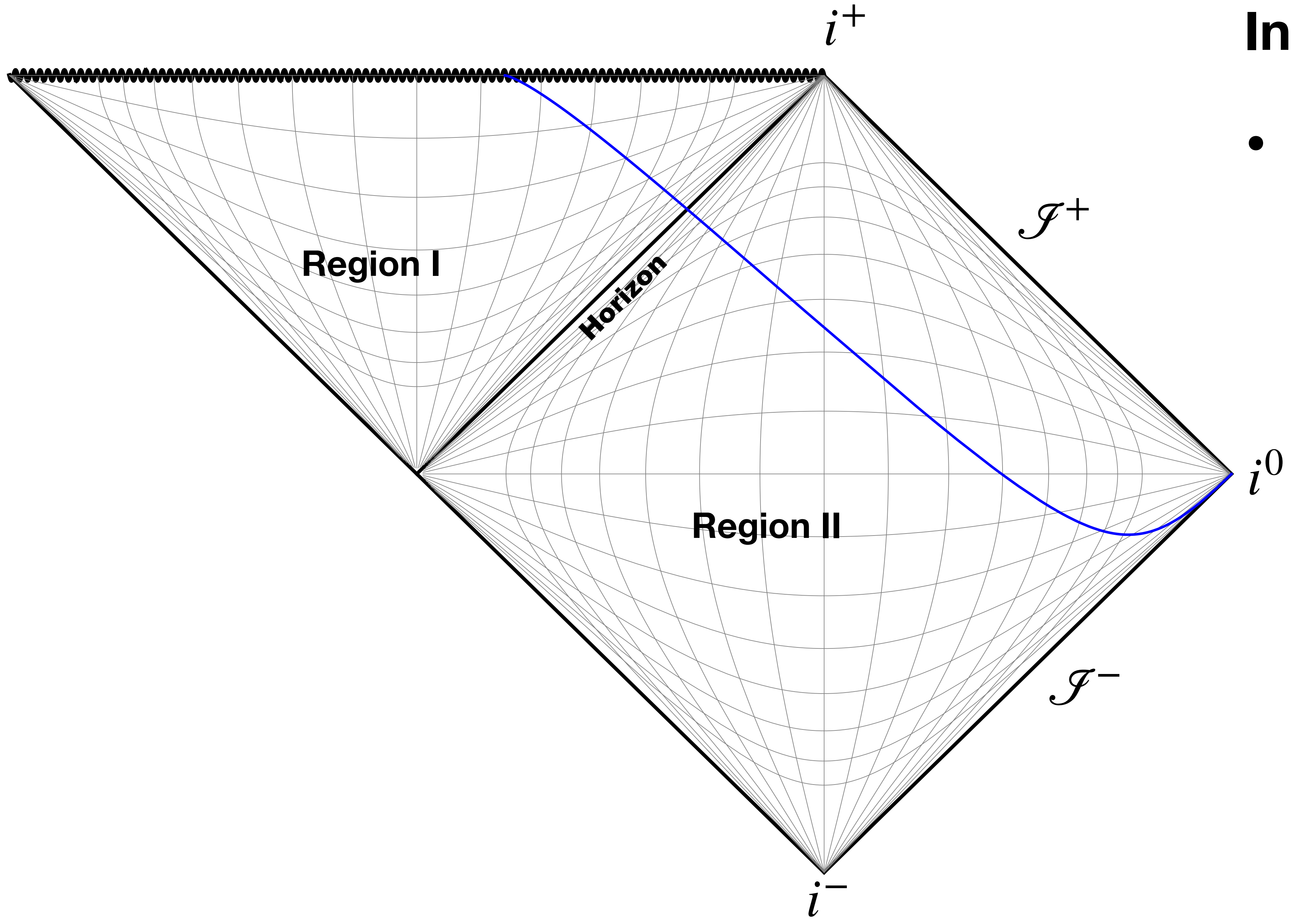
$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{p_b^2}{|p_c|L_0^2}\mathrm{d}x^2 + |p_c|\mathrm{d}\Omega^2$$

[Ashtekar, Olmedo & Singh 19', Zhang &CZ, 19']

This fact motives us to do (polymer) quantization on the surface $\mathbb{R}\times \mathrm{S}^2$ with symmetry $\mathbb{T}\times \mathrm{SO}(3)$

- **Leads to a system with finite D.O.F,**
- **However, the exterior need further assumption.**

Classical theories



In the entire spacetime, the Cauchy surface have only the $\mathrm{SO}(3)$ symmetry

- **Schwarzschild solution** $\Longleftarrow$ **E.O.M + general metric**

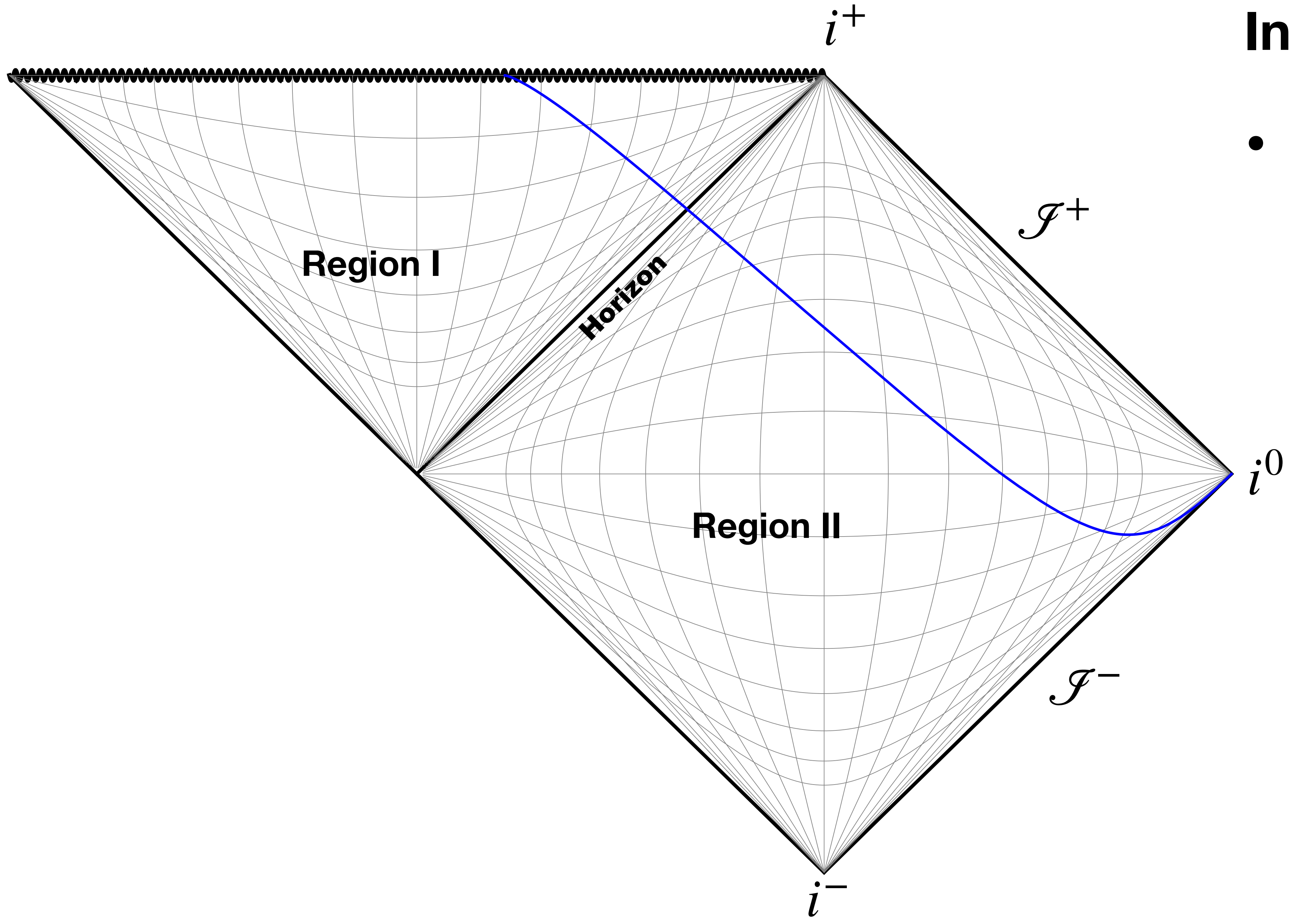
$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{(E^b)^2}{E^c}(\mathrm{d}x + N^x\mathrm{d}t)^2 + E^c\mathrm{d}\Omega^2$$

[Gambini, Olmedo & Pullin, 2015]

We are motived to do (polymer) quantization on the surface $\mathbb{R}\times\mathrm{S}^2$ with symmetry $\mathrm{SO}(3)$

- **The entire spacetime is obtained,**
- **However, it leads to a system with infinity D.O.F.**

Classical theories



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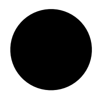
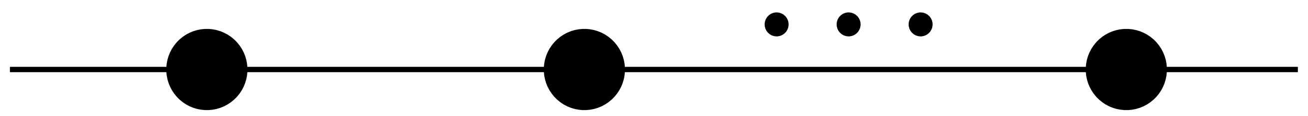
We are motived to do (polymer) quantization on the surface $\mathbb{R} \times \mathrm{S}^2$ with symmetry $\mathrm{SO}(3)$

- **The entire spacetime is obtained,**
- **However, it leads to a system with infinity D.O.F.**

It isn't guaranteed that the two quantum theories are the same even though their classical correspondences do so.

Quantum theories

	$\mathbb{R}\times S^2$ **with symmetry** $\mathbb{T}\times SO(3)$ **:**	$\mathbb{R}\times S^2$ **with symmetry** $SO(3)$ **:**
Graph:	**a vertex** v	**1-D lattice**
Hilbert space basis:	 $\vert p_b, p_c\rangle$	 $\vert E^b(v_1), E^c(v_1)\rangle \otimes \vert E^b(v_2), E^v(v_2)\rangle \otimes \cdots \otimes \vert E^b(v_n), E^c(v_n)\rangle$
Classical correspondence:	$E_i^a \tau^i \partial_a = p_c \tau_3 \sin\theta \partial_x + \frac{p_b}{L_0}\tau_2 \sin\theta \partial_\theta - \frac{p_b}{L_0}\tau_1 \partial_\phi$	$E_j^a \tau^j \frac{\partial}{\partial \sigma^a} = E^c(x)\sin(\theta)\tau_1 \partial_x + E^b(x)\tau_2 \sin(\theta)\partial_\theta + E^b(x)\tau_3 \partial_\varphi$

[CZ, Ma, Song, Zhang, 21', CZ, 21']

Quantum theories

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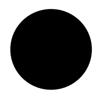
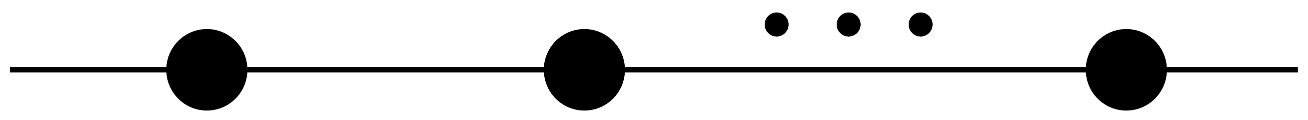
Hilbert space basis:	 $\lvert p_b, p_c\rangle$	 $\lvert E^b(v_1), E^c(v_1)\rangle \otimes \lvert E^b(v_2), E^v(v_2)\rangle \otimes \cdots \otimes \lvert E^b(v_n), E^c(v_n)\rangle$
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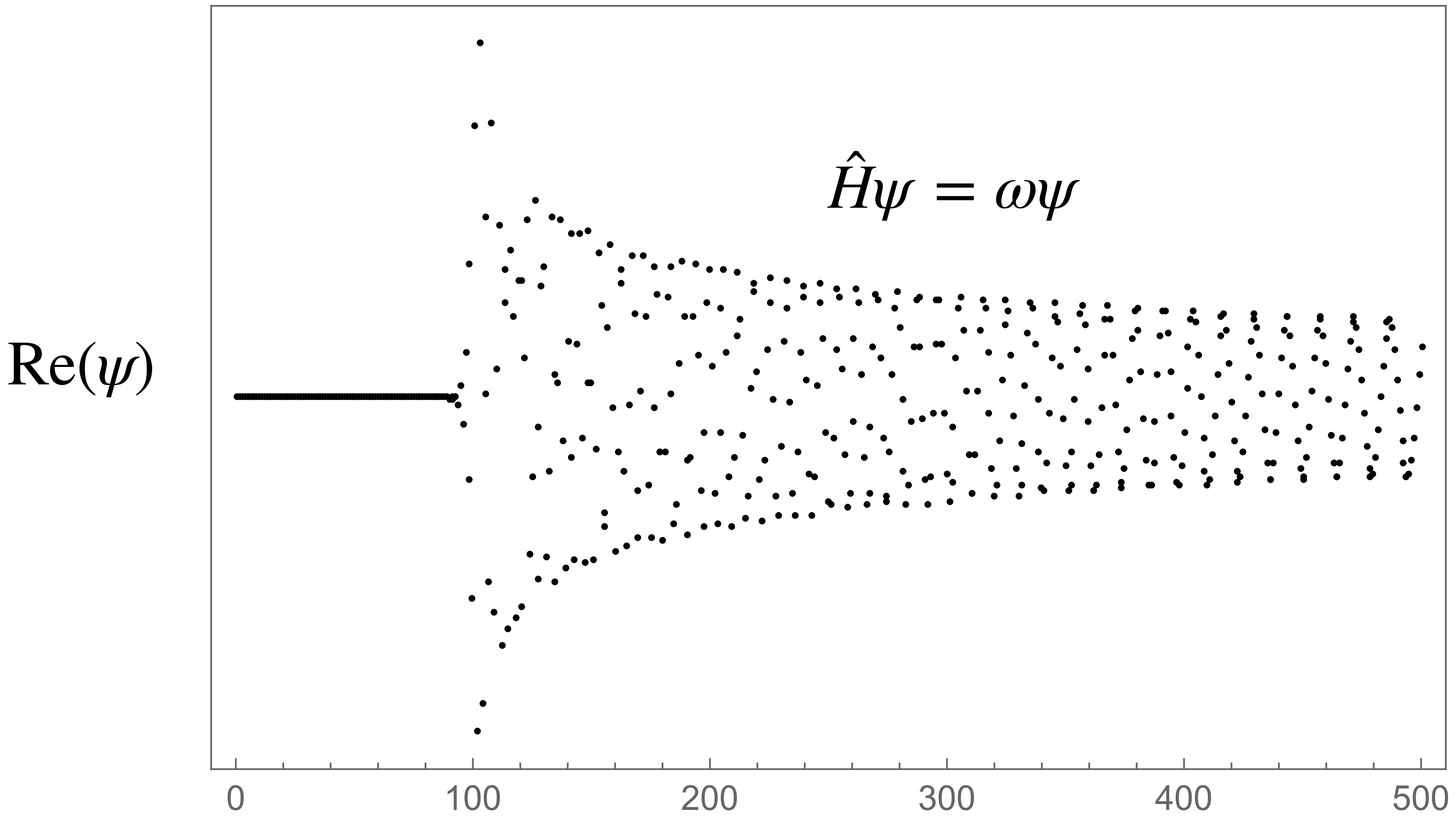
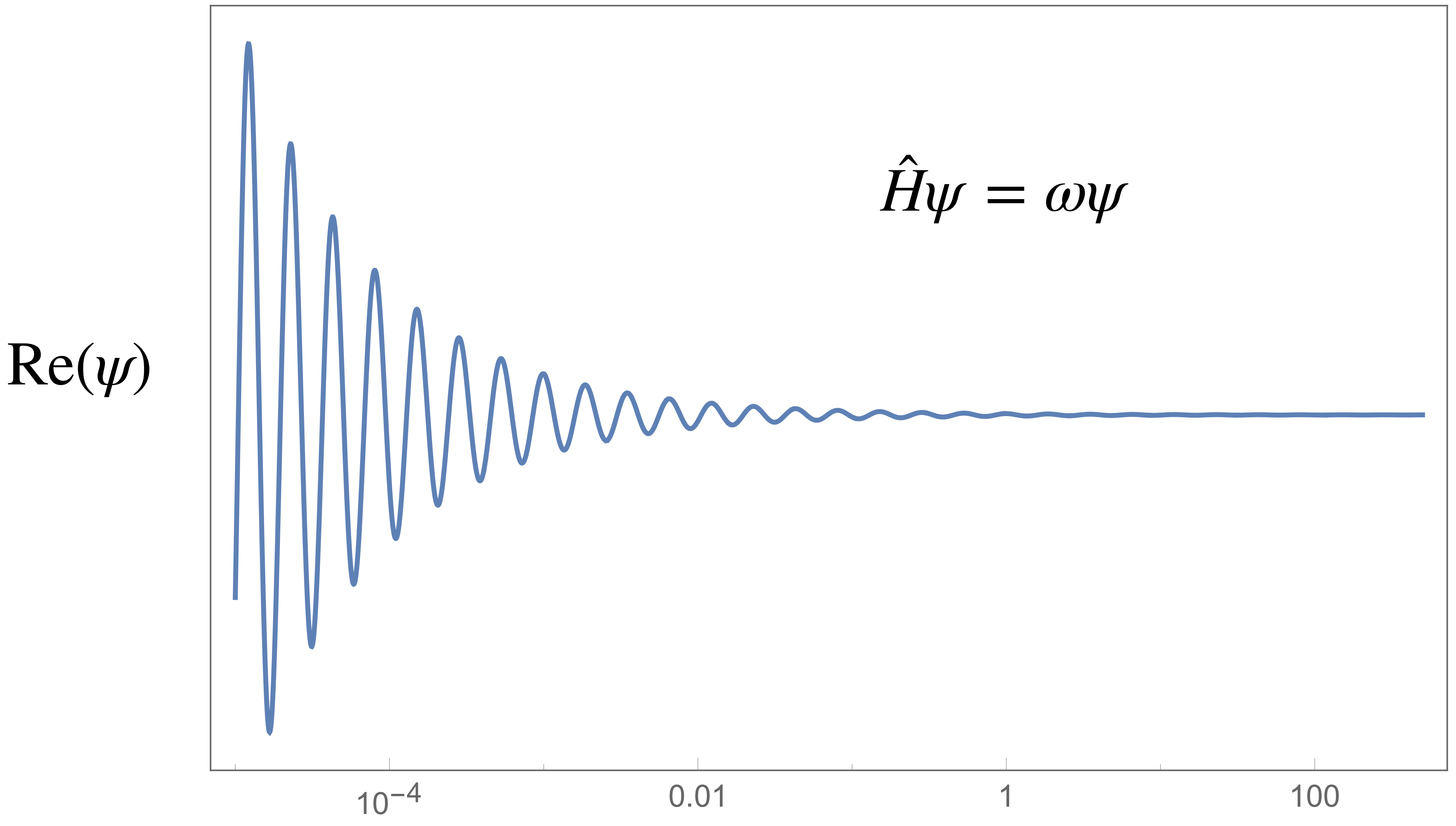
“Holonomy” operator	$\widehat{e^{i\lambda b}}\,\lvert p_b, p_c\rangle = \lvert p_b + \lambda, p_c\rangle,\quad \widehat{e^{i\lambda c}}\,\lvert p_b, p_c\rangle = \lvert p_b, p_c + \lambda\rangle$	$\widehat{e^{i\lambda(v)K_b(v)}}\,\lvert E^b(v), E^c(v)\rangle = \lvert E^b(v) + \lambda(v), E^c(v)\rangle$, $\widehat{e^{i\lambda(v)K_c(v)}}\,\lvert E^b(v), E^c(v)\rangle = \lvert E^b(v), E^c(v) + \lambda(v)\rangle$

In contract to the Schordinger quantization, we have no differential operators like $\hat{b}, \hat{c}, \hat{K}_b(v), \hat{K}_c(v)$**. However, the Hamiltonian involves the variables** $b, c, K_b(v), K_c(v)$**. We thus have to regularize them to be operators.**

- **Regularization leads to difference operator as the Hamiltonian, key to singularity resolution;** [CZ, Ma, Song, Zhang, 21’, CZ, 21’]

$$-i\frac{\partial}{\partial t}\psi = \hat{H}\psi, \quad \hat{H} = \sqrt{p}\hat{x}\sqrt{\hat{p}}$$





$$(\hat{x}\psi)(p) \to i(\psi(p+1) - \psi(p-1))$$

$$\hat{x}\psi \to i\frac{\mathrm{d}}{\mathrm{d}p}\psi$$

[CZ, Lewandowski, Ma 19']

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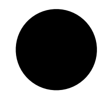
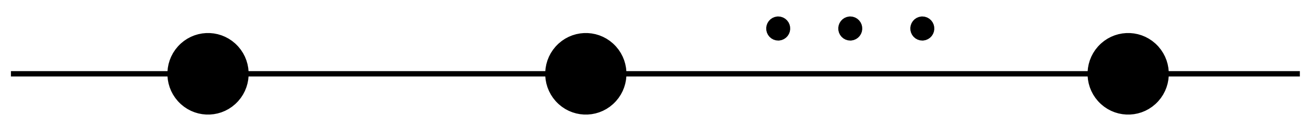
Hilbert space basis:	 $\vert p_b, p_c\rangle$	 $\vert E^b(v_1), E^c(v_1)\rangle \otimes \vert E^b(v_2), E^v(v_2)\rangle \otimes \cdots \otimes \vert E^b(v_n), E^c(v_n)\rangle$
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In contract to the Schordinger quantization, we have no differential operators like $\hat{b}, \hat{c}, \hat{K}_b(v), \hat{K}_c(v)$. However, the Hamiltonian involves the variables $b, c, K_b(v), K_c(v)$. We thus have to regularize them to be operators.

- **Regularization leads to difference operator as the Hamiltonian, key to singularity resolution;**
- **Causes the ambiguities in LQBH.**

[CZ, Ma, Song, Zhang, 21', CZ, 21']

Quantum theories

The quantum dynamics of the homogeneous model:

$$H[V] = -E_i^a E_j^b \left[\epsilon^{ij}{}_k F_{ab}^k - 2(1+\gamma^2) K_{[a}^i K_{[a)}^i\right] = 2p_b b c p_c + p_b^2 b^2 + \gamma^2 p_b^2$$

Regularization gives [Ashtekar, Bojowald 05', CZ, Ma, Song, Zhang 21']

$$H[V]^{(\tilde{\delta}_b,\tilde{\delta}_c)} = 2p_b \frac{\sin(\tilde{\delta}_b b)}{\tilde{\delta}_b} p_c \frac{\sin(\tilde{\delta}_c c)}{\tilde{\delta}_c} + p_b^2 \frac{\sin^2(\tilde{\delta}_b b)}{\tilde{\delta}_b^2} + \gamma^2 p_b^2$$

Classically, $H[V] = \lim_{\tilde{\delta}_b,\tilde{\delta}_c \to 0} H[V]^{(\tilde{\delta}_b,\tilde{\delta}_c)}$ **but in quantum theory,** $\widehat{H[V]} = \lim_{\tilde{\delta}_b,\tilde{\delta}_c \to \delta_b,\delta_c} \widehat{H[V]}^{(\tilde{\delta}_b,\tilde{\delta}_c)}$

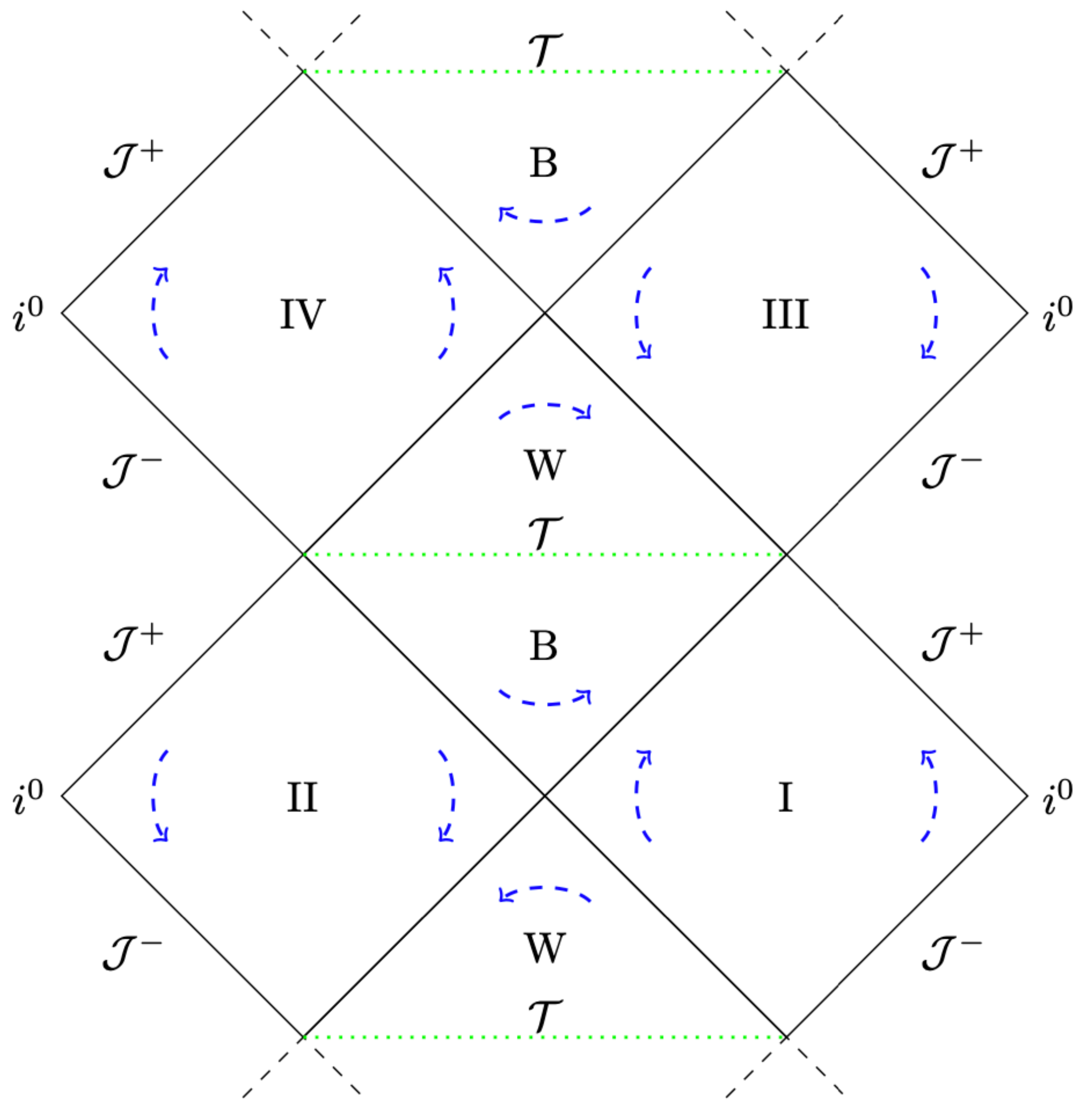
Ambiguities arise due to various choices of δ_b, δ_c **:**

- μ_0**—scheme, constant** δ_b, δ_c **;** [Boehmer Vandersloot 07', Chiou 08']
- $\bar{\mu}$**—scheme,** δ_b, δ_c **being phase space function;** [Chiou 08']
- **New scheme,** δ_b, δ_c **being function of dynamical trajectories.** [Corichi, Singh 16', Ashtekar, Olmedo Singh 18']

Quantum theories

Some results:

- **Effective dynamics: singularity resolution, BH-WH transition, etc.**
  [Boehmer Vandersloot 07', Chiou 08', Corichi, Singh 16', Ashtekar, Olmedo Singh 18']
- **Quantum dynamics: discreteness of BH mass at the dynamical level;** [CZ, Ma, Song, Zhang 20' & 21']



[Ashtekar, Olmedo Singh 18']

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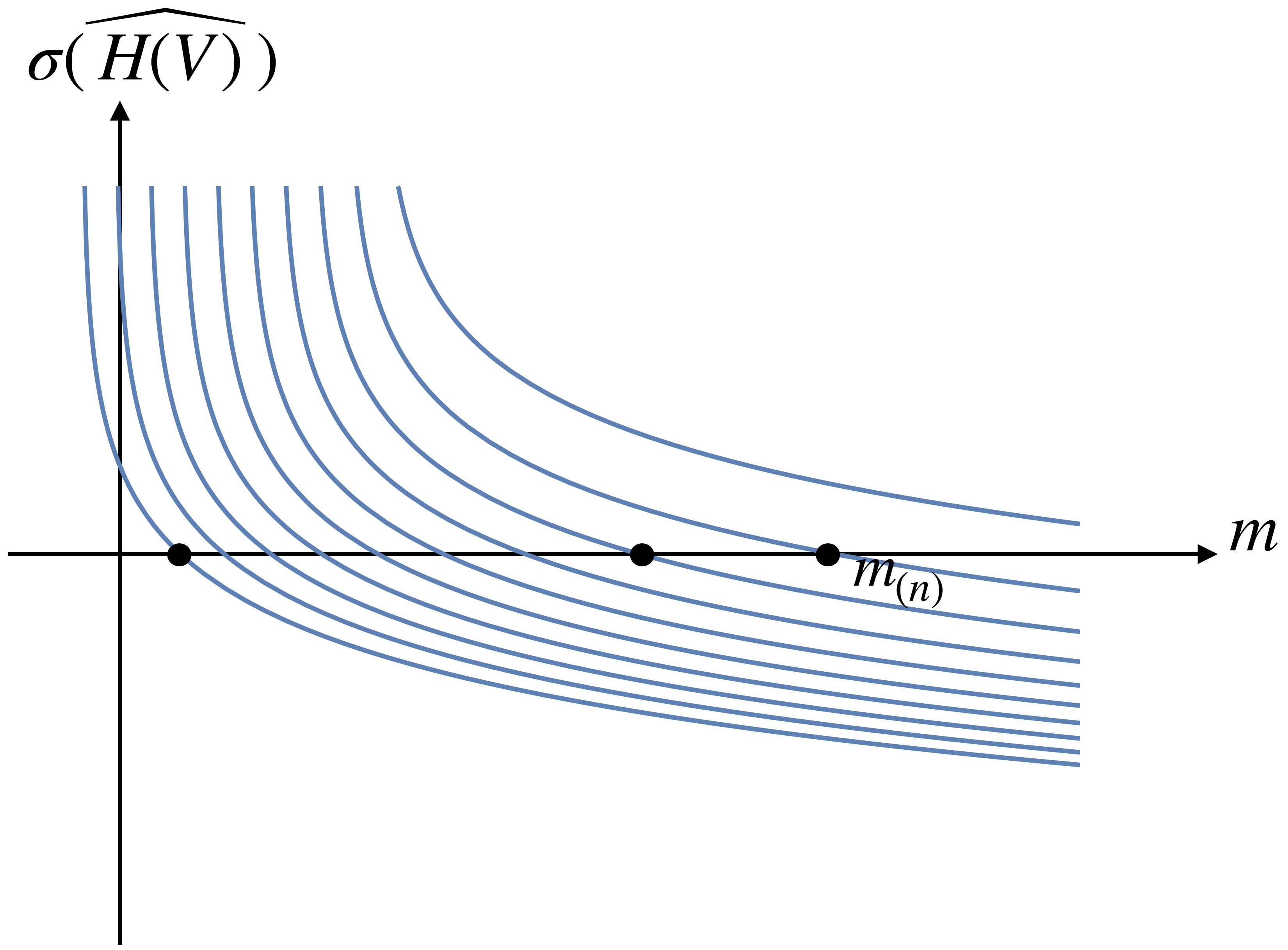

Constraint equation: $\widehat{H[V]} = 0$:

- **Only for countably many values $m_{(n)}$, one can obtain solutions;**
- **The minimal value $m_{(0)}$ is not vanishing.**

Quantum theories

Some results:

- **Effective dynamics: singularity resolution, BH-WH transition, etc.**
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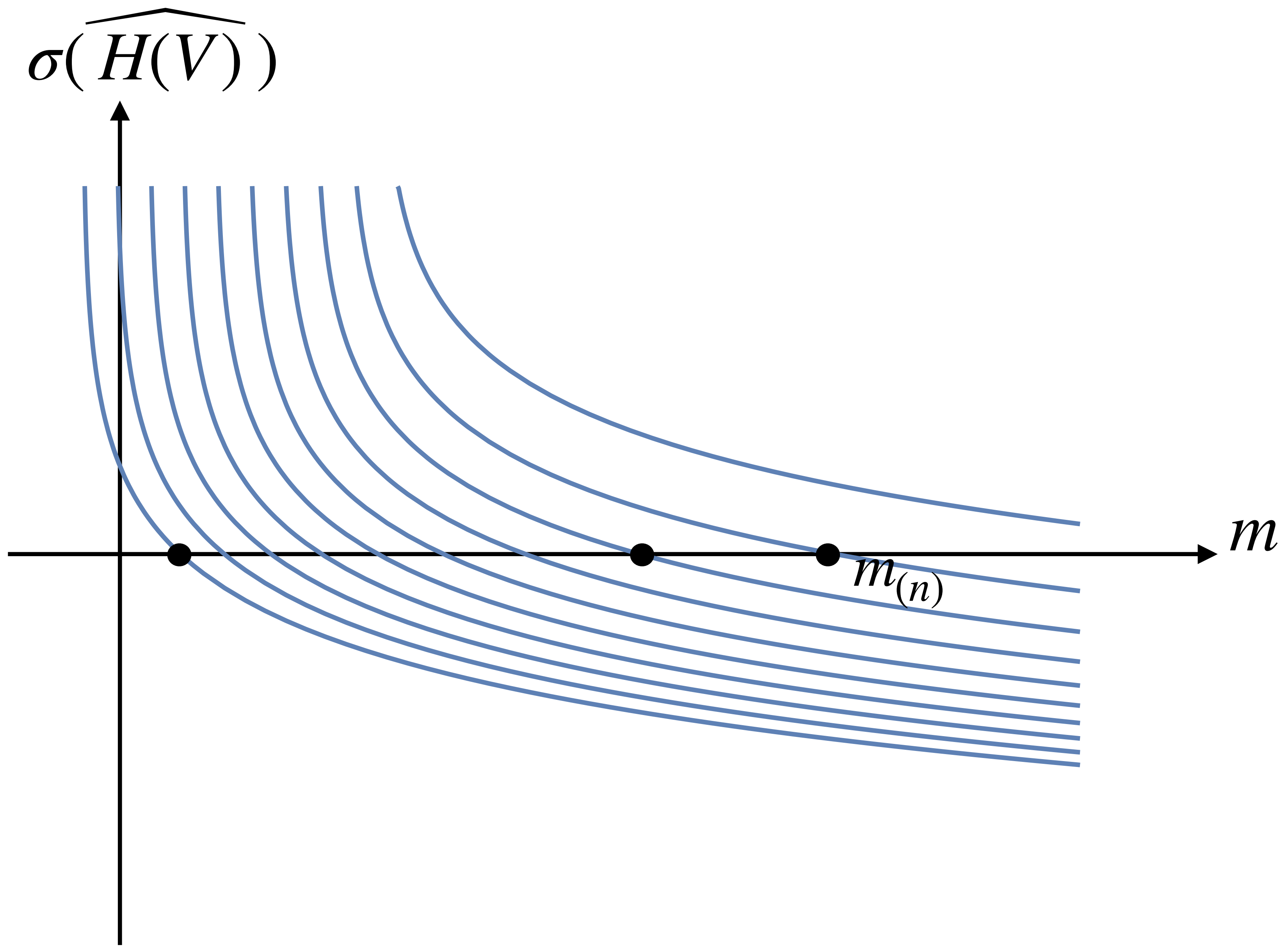
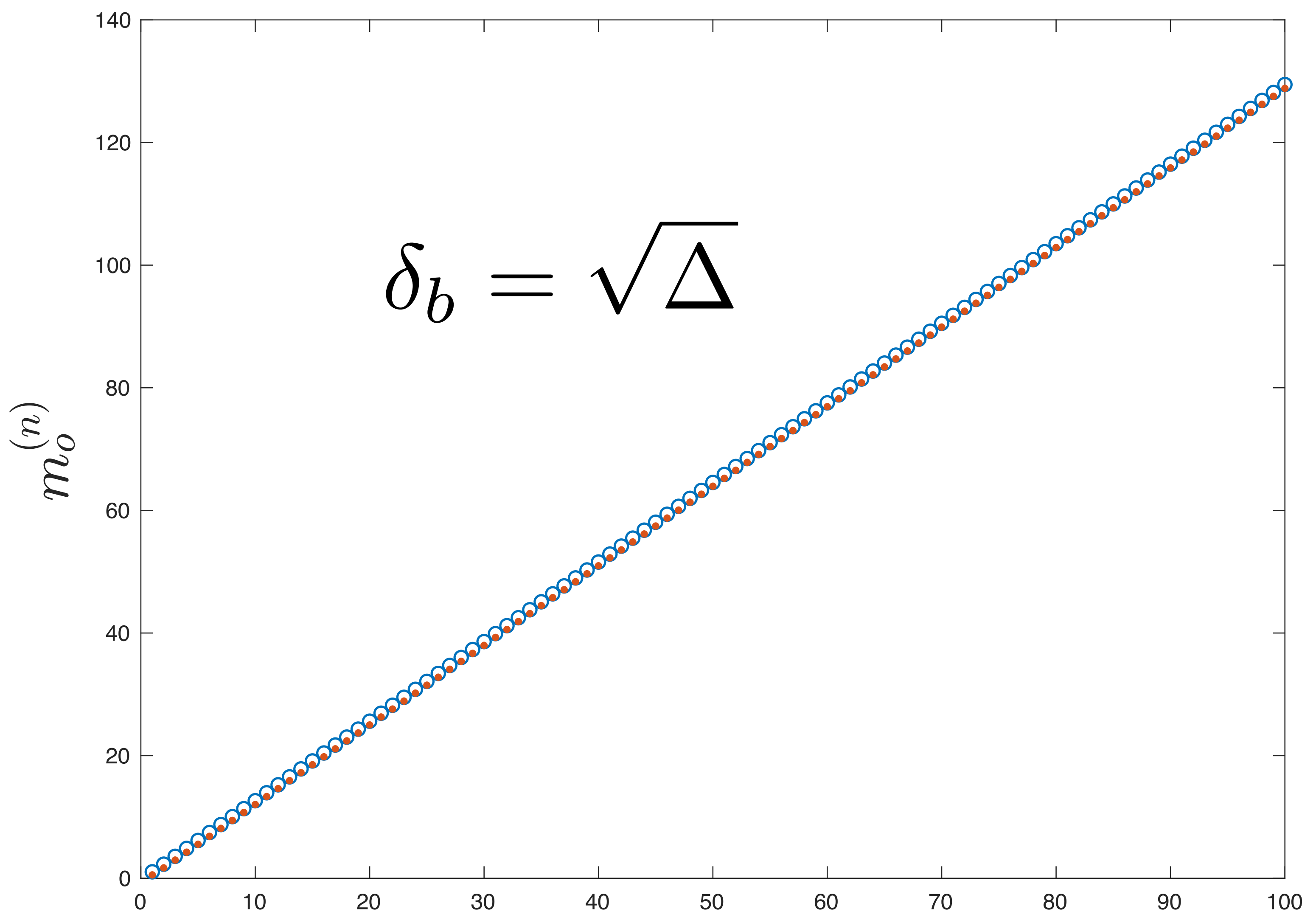
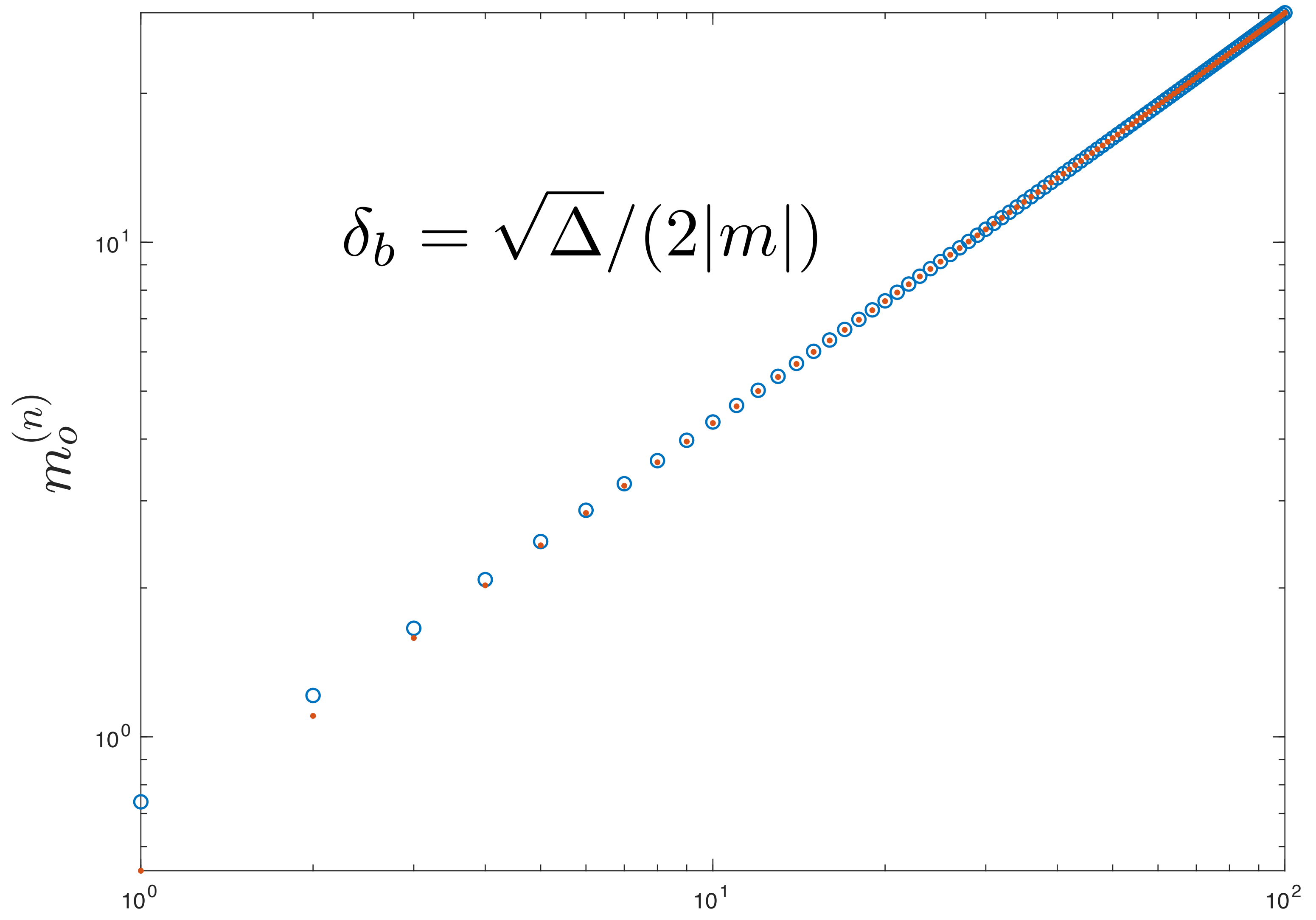


Constraint equation: $\widehat{H[V]}$

- **Only for countably many v**
- **The minimal value $m_{(0)}$ is r**

Not for $\bar{\mu}$ scheme!





Quantum theories

The quantum dynamics of the non-homogeneous model:

$$C = \frac{-1}{|E^b|\sqrt{|E^c|}}\left(8E^cE^bK_cK_b + 2E^{b2}K_b^2 + 2E^{b2}\right) - \frac{E^{c\prime 2}}{2|E^b|\sqrt{|E^c|}} + \left(\frac{2E^cE^{c\prime}}{|E^b|\sqrt{|E^c|}}\right)^{\prime}$$

$$\mathscr{V} = E^bK_b' - E^{c\prime}K_c$$

$$H_T = \int \mathrm{d}x(N(x)C(x) + N^x(x)\mathscr{V}(c))$$

$\bar{\mu}$-scheme is chosen to quantize the Hamiltonian constraint. However, there still are ambiguities caused by different approached to deal with the constraint algebra:

- **Gambini-Olmedo-Pullin model:** choose phase space dependent lapse function and shift vector to modify the constraint algebra to get a "true" Lie algebra; **[Gambini, Olmedo, Pullin 14' & 20]**
- **Kelly-Santacruz-Wilson-Ewing model:** Introduce the Areal gauge to do gauge fixing; **[Kelly, Santacruz, Wilson-Ewing 20'&22', see Giesel, Li Singhm, Weigl 21' for further discussion on gauge fixing]**
- **Han-Liu model:** consider the phase-space reduce quantization where the constraints are solve at the classical level. **[Han, Liu 20', CZ 21', see, e.g., Giesel, Tambornino, Thiemann 09' for phase-space reduced quantization]**

Some results in the Han-Liu model: [CZ 21']

- **The quantum dynamics is studied by path integral formulation,**
- **The effective dynamics is coincide with the heuristic one where one just replaces K_a, K_b in the Hamiltonian by sine functions and solve the Hamiton's equations,**
- **For small BH mass, the effective dynamics is different from the heuristics one.**

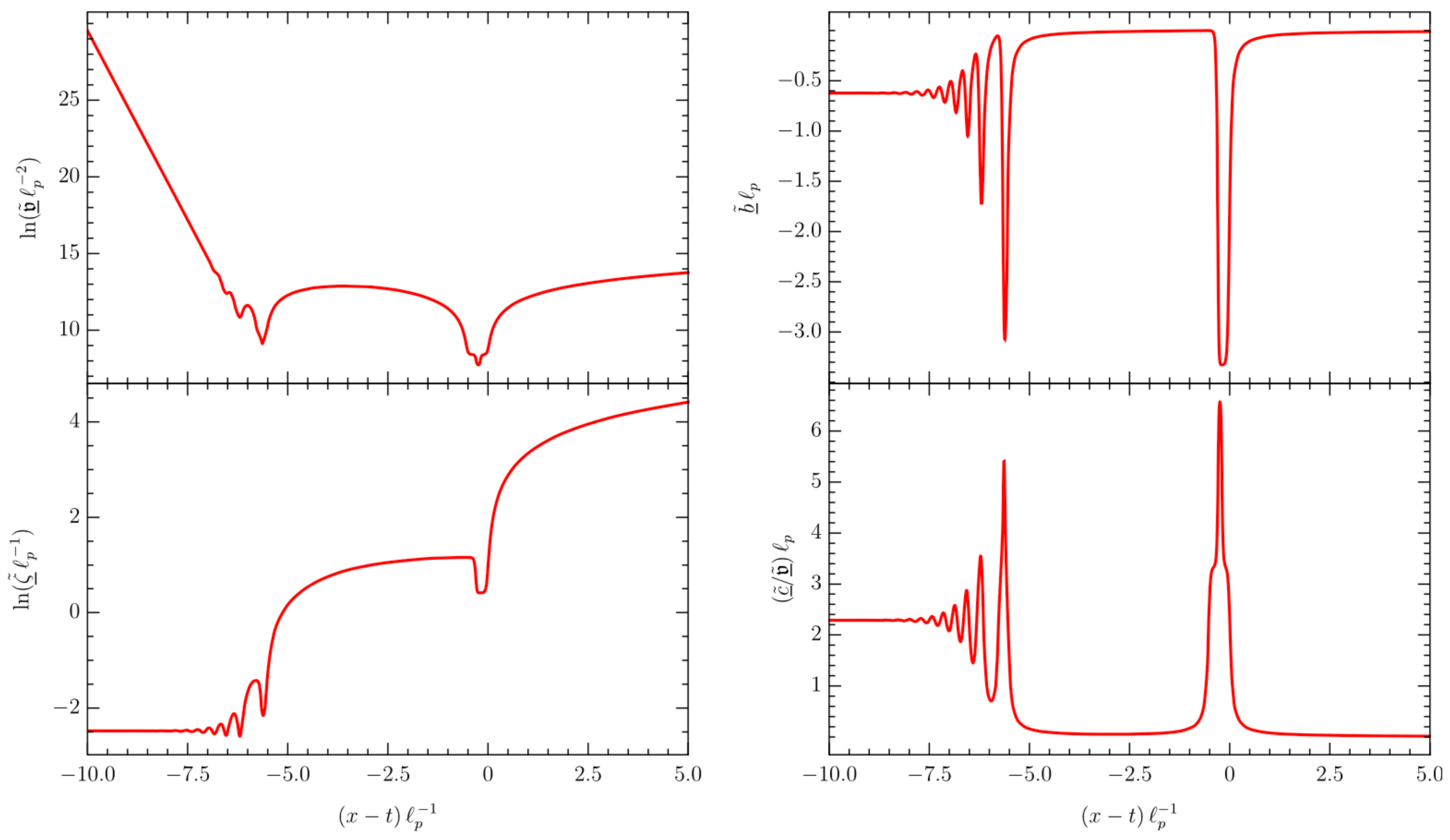


FIG. 1. Solutions to (6.20), (6.22), (6.23), and (6.24). The initial data are chosen such that $\tilde{\underline{\mathfrak{v}}}(y_0) = 6\pi F_0 y_0$, $\tilde{\underline{b}}(y_0) = -\frac{1}{6\pi y_0}$, $\tilde{\underline{\zeta}}(y_0) = (\frac{3}{2}\sqrt{F_0}y_0)^{2/3}$, $\tilde{\underline{c}}(y_0) = \frac{3F_0}{2}$, and $\tilde{\underline{\zeta}}'(y_0) = \frac{2}{3}(\frac{3}{2}\sqrt{F_0}y_0)^{-1/3}$ with $y_0 = 10^5\ell_p$ and $F_0 = 10^4\ell_p$. The initial data are choose by considering the Schwarzschild solution with $2GM = F_0$ in Lemâitre coordinate at $x - t = y_0$. The parameters are set to be $\Delta = 0.1$ and $\beta = 0.2375$.

The spacetime ends up in a Nariai spacetime $\mathrm{ds}_2 \times \mathrm{S}_2$

[Han, Liu 20', CZ 21']

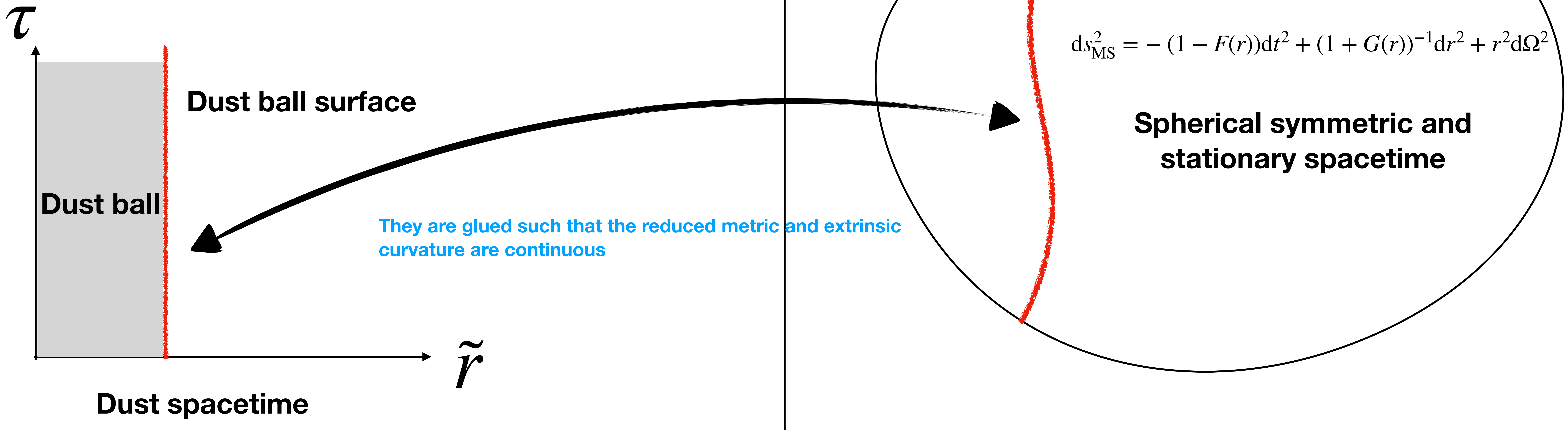
Matter collapsing in LQG

Consider LQC dust to determine the exterior spacetime by junction condition. [Lewandowski, Ma, Yang, CZ 23']

Gravity coupled to homogeneous pressureless dust

$$\mathrm{d}s^2_{\mathrm{APS}} = -\,\mathrm{d}\tau^2 + a(\tau)(\mathrm{d}\tilde{r}^2 + \tilde{r}^2\mathrm{d}\Omega^2)$$

Dynamics: $\mathbb{H}^2 = \frac{8\pi G}{3}\rho(1 - \frac{\rho}{\rho_c}),\ \partial_\tau(\rho a^3) = 0$

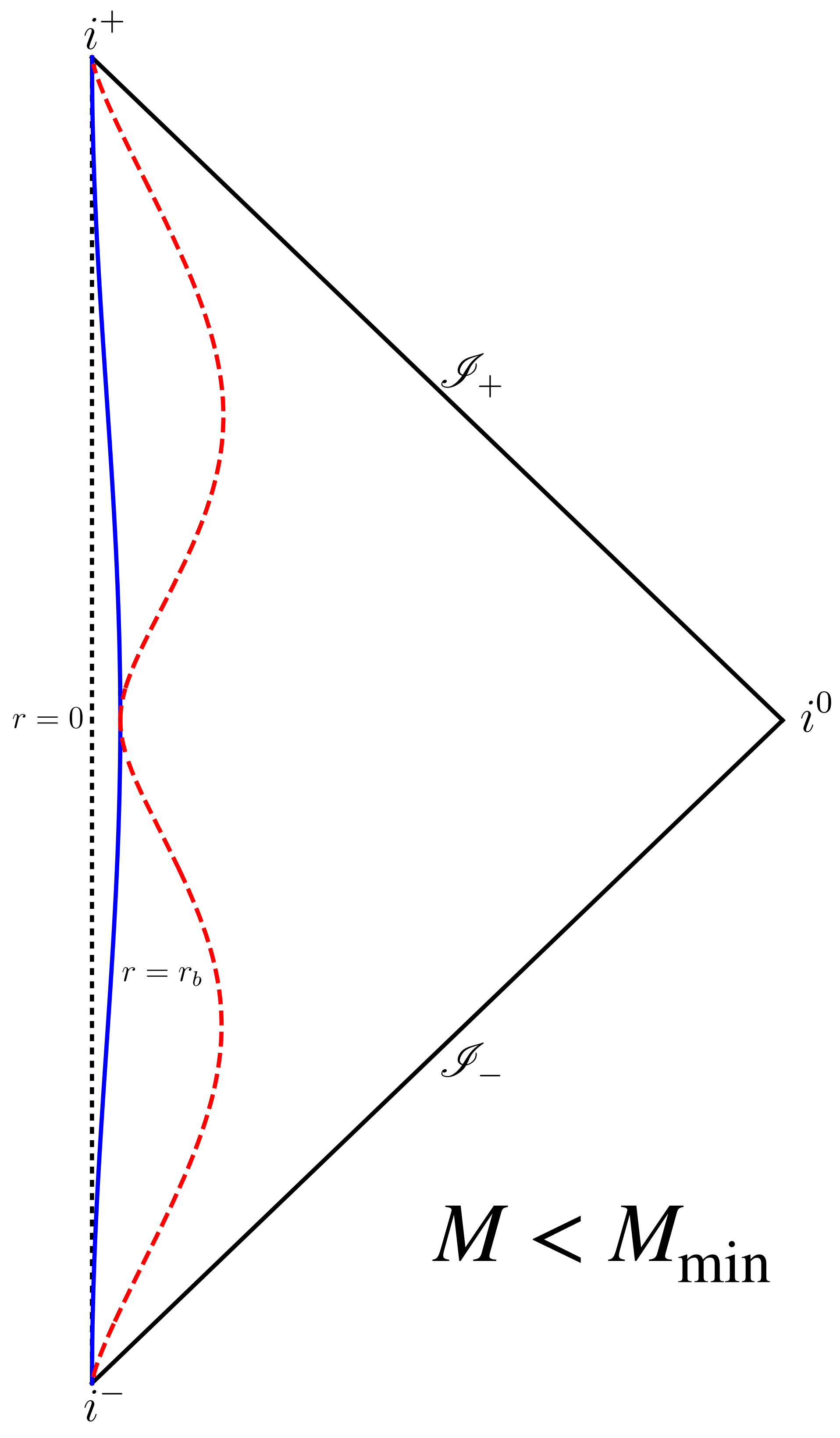


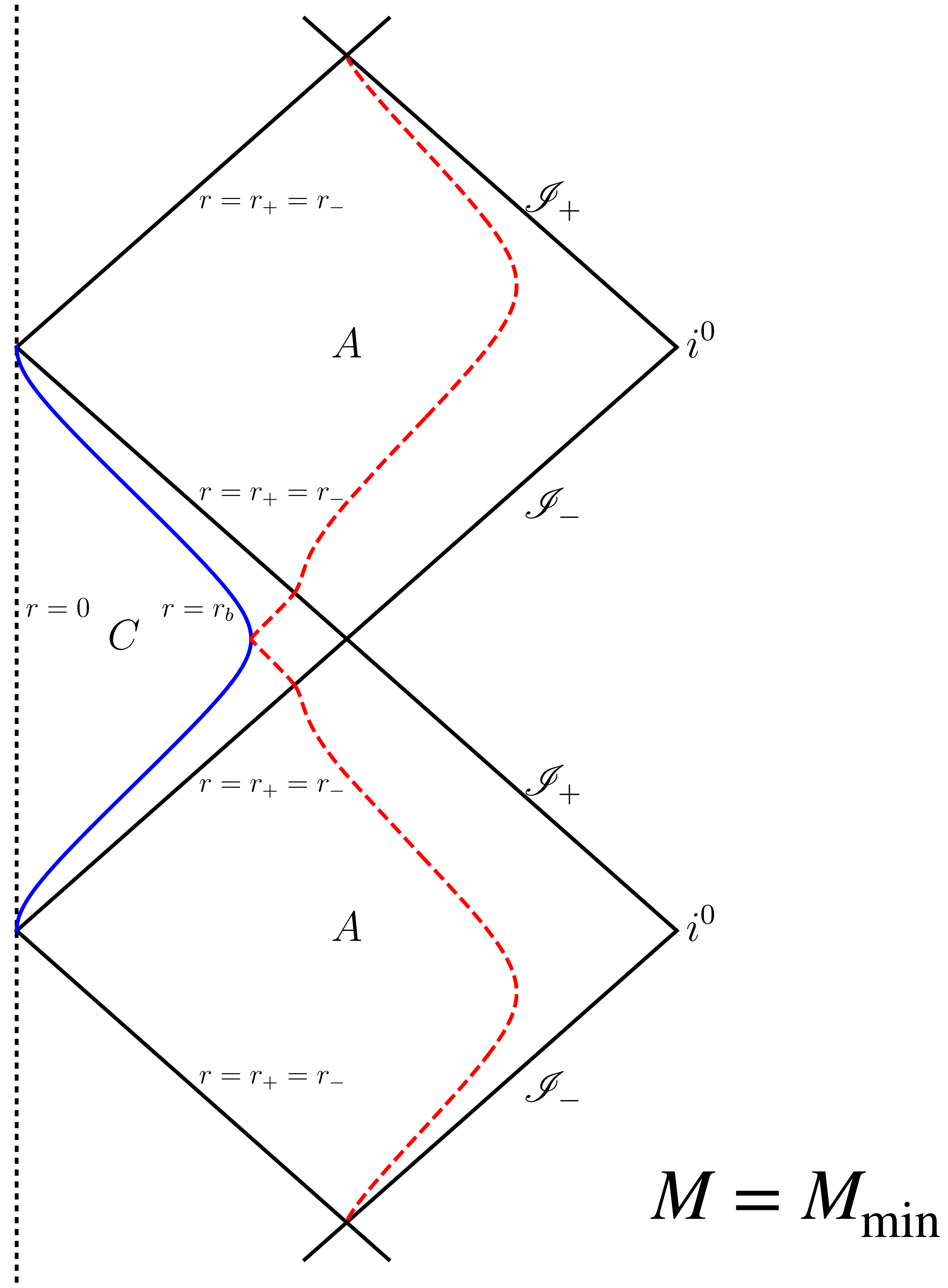
Matter collapsing in LQG

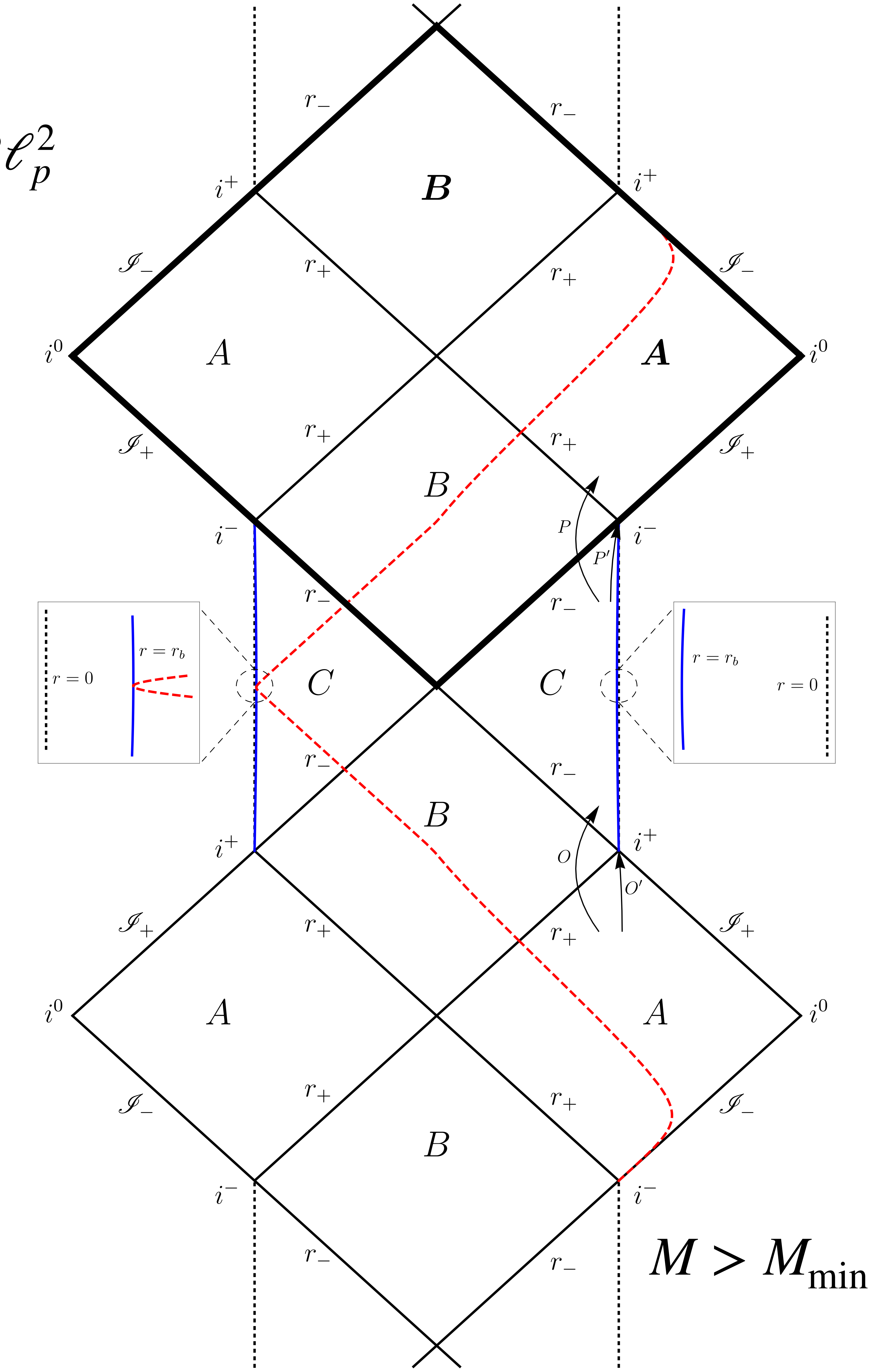
[Lewandowski, Ma, Yang, CZ 22']

$$\mathrm{d}s^2_{\mathrm{MS}} = -\left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right)\mathrm{d}t^2 + \left(1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \quad \alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$

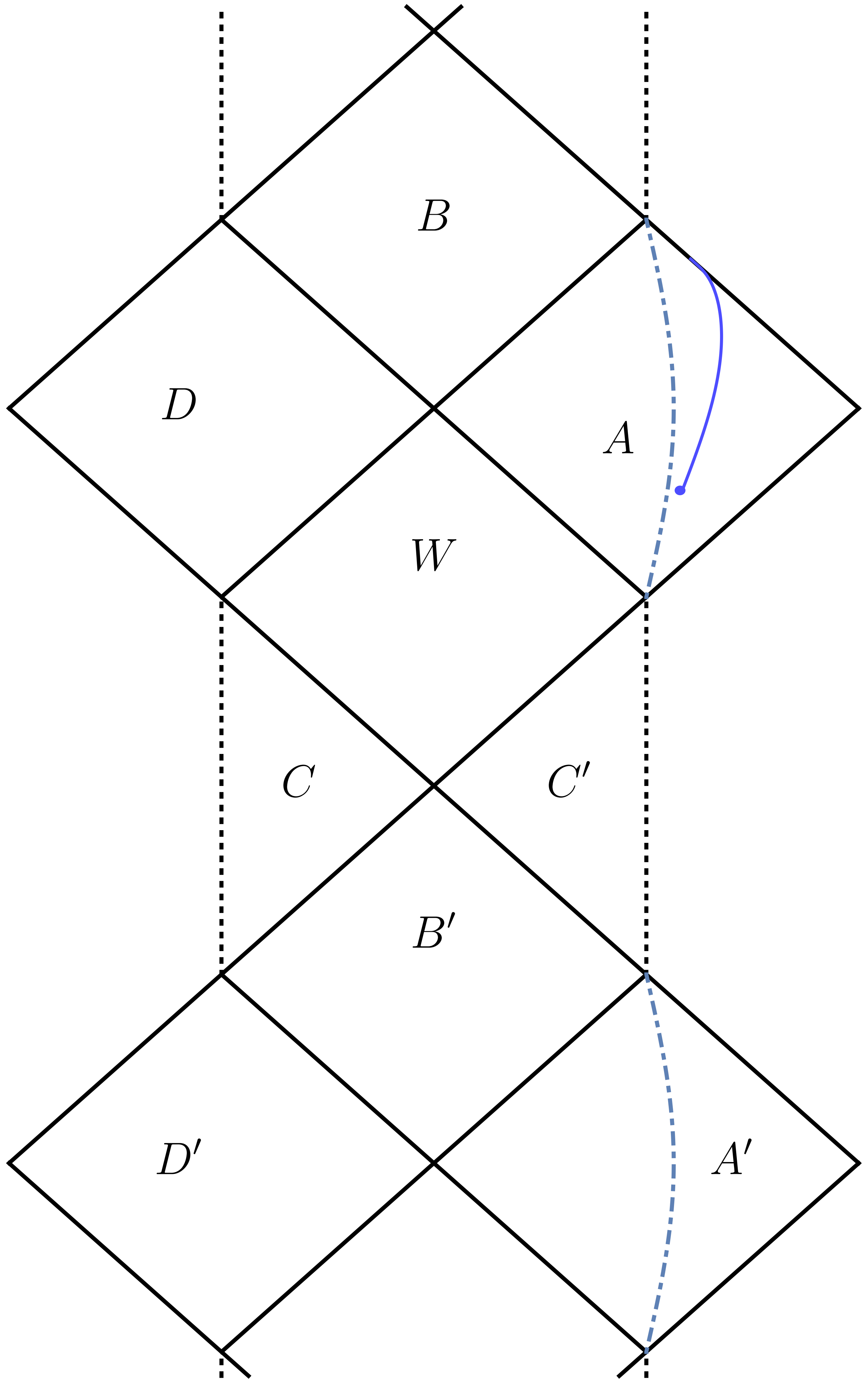
The result is the same as the one given by Kelly, Santacruz, Wilson-Ewing 20' but from loop quantization approach







Observing effect of LQBH



[Yang, CZ, Ma 23']

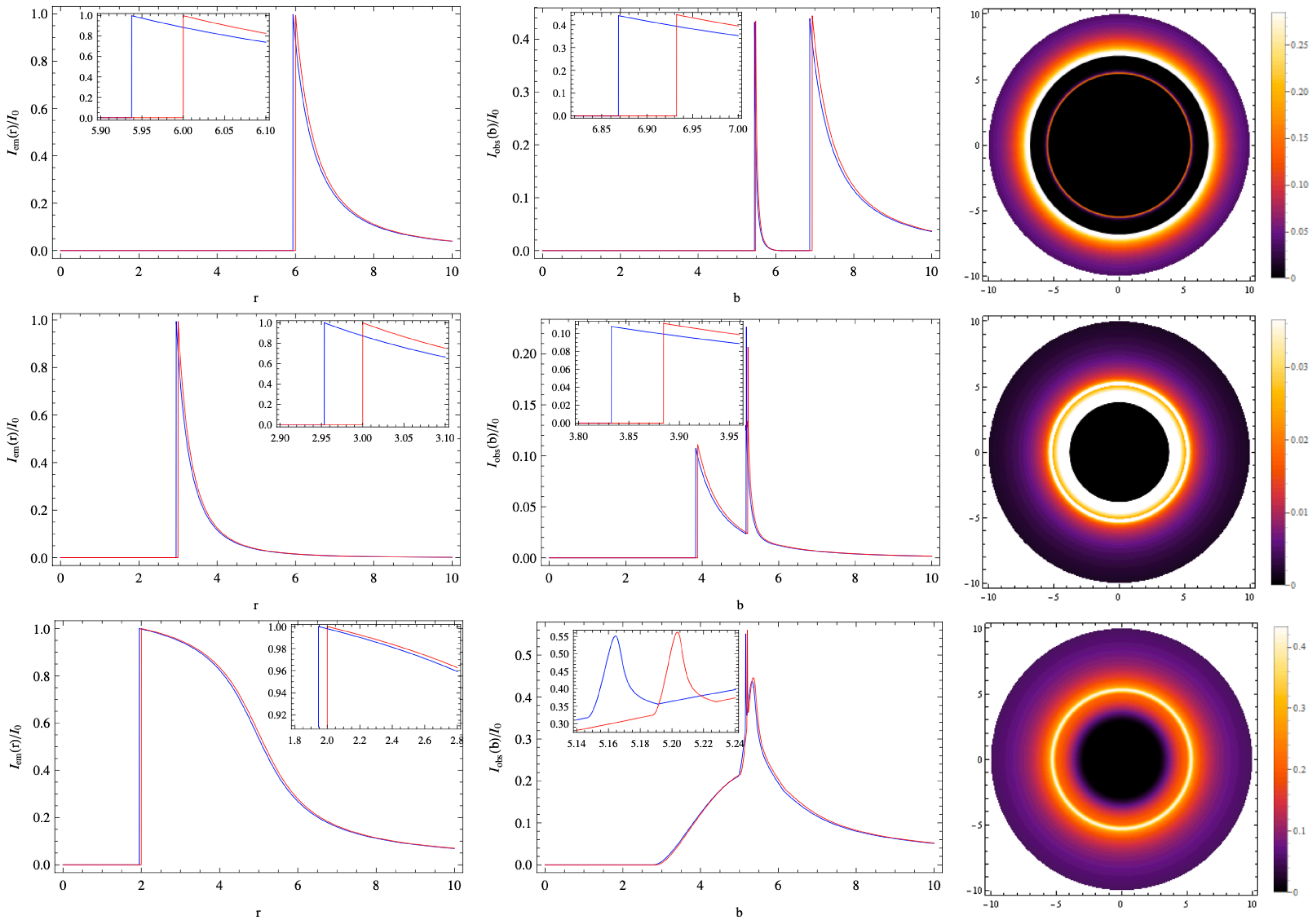
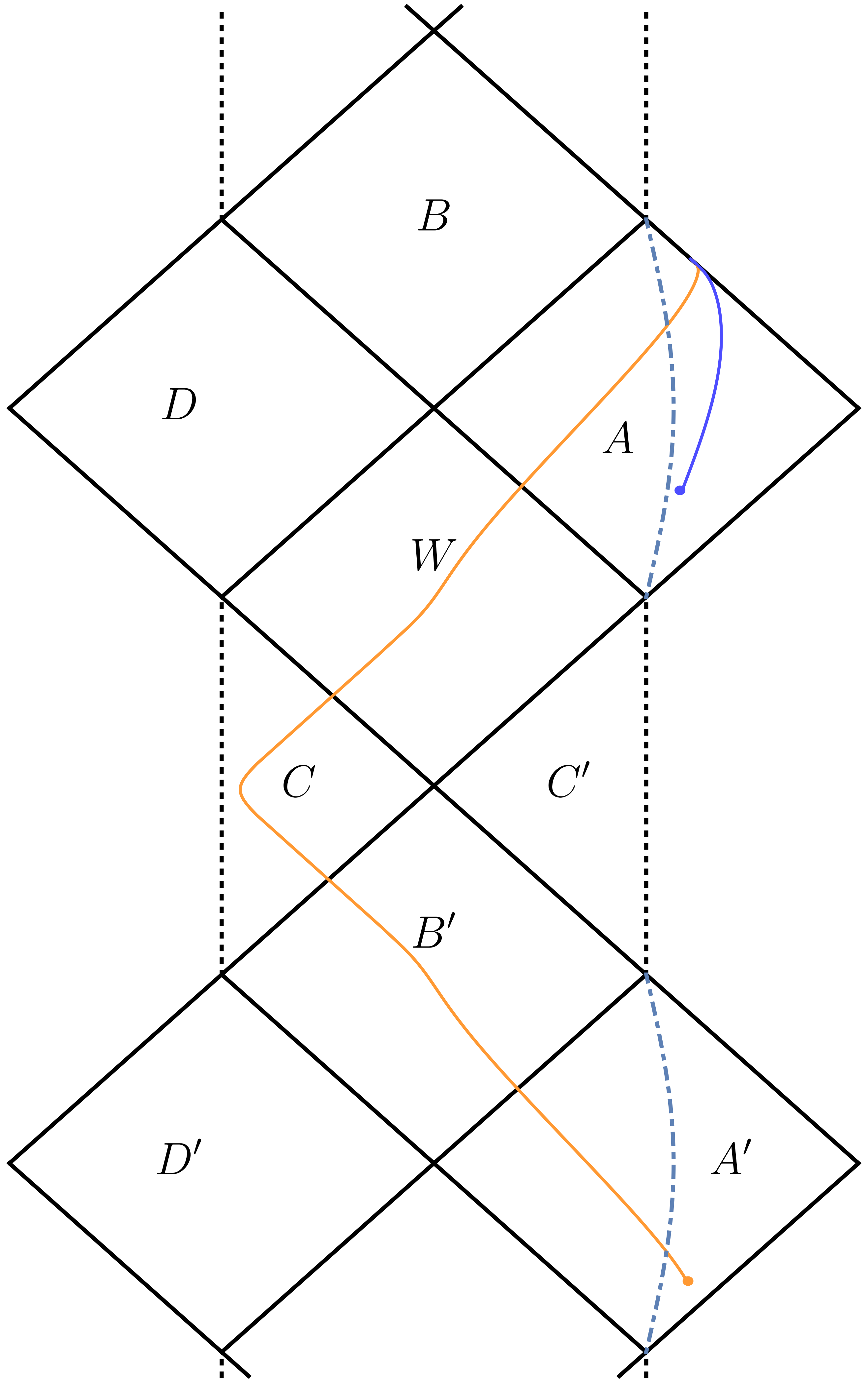
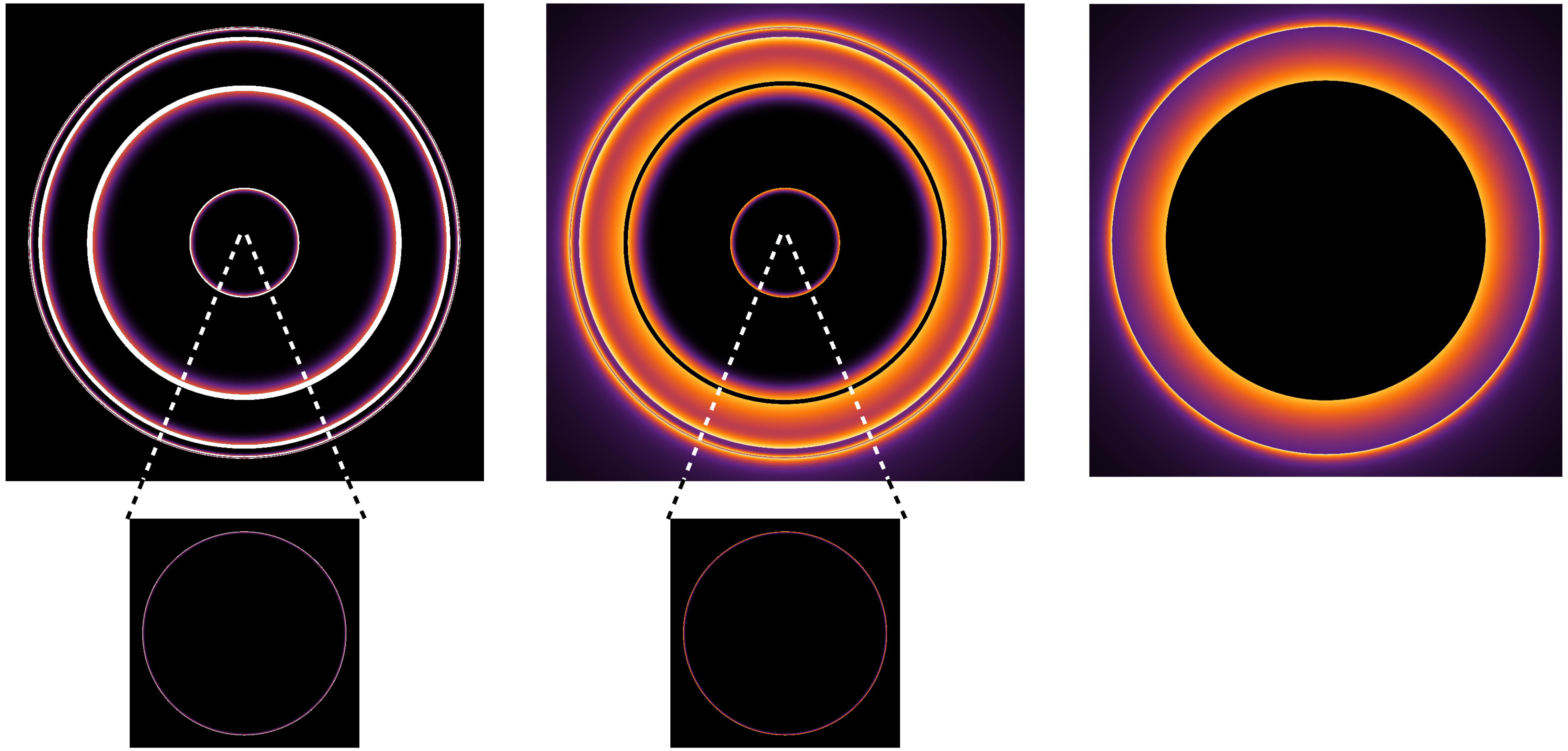


FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles. In each row, the first two panels show the emission intensity I_{em}/I_0 and observational intensity I_{obs}/I_0, normalized to the maximum value I_0, of a thin disk near the quantum-corrected BH (blue) compared to those of the Schwarzschild BH (red), and the third panel depicts the density plot of I_{obs}/I_0 of a thin disk near the quantum-corrected BH. The parameters are $R_s = 2$, $\gamma = 1$ and $\Delta = 0.1$.

Observing effect of LQBH

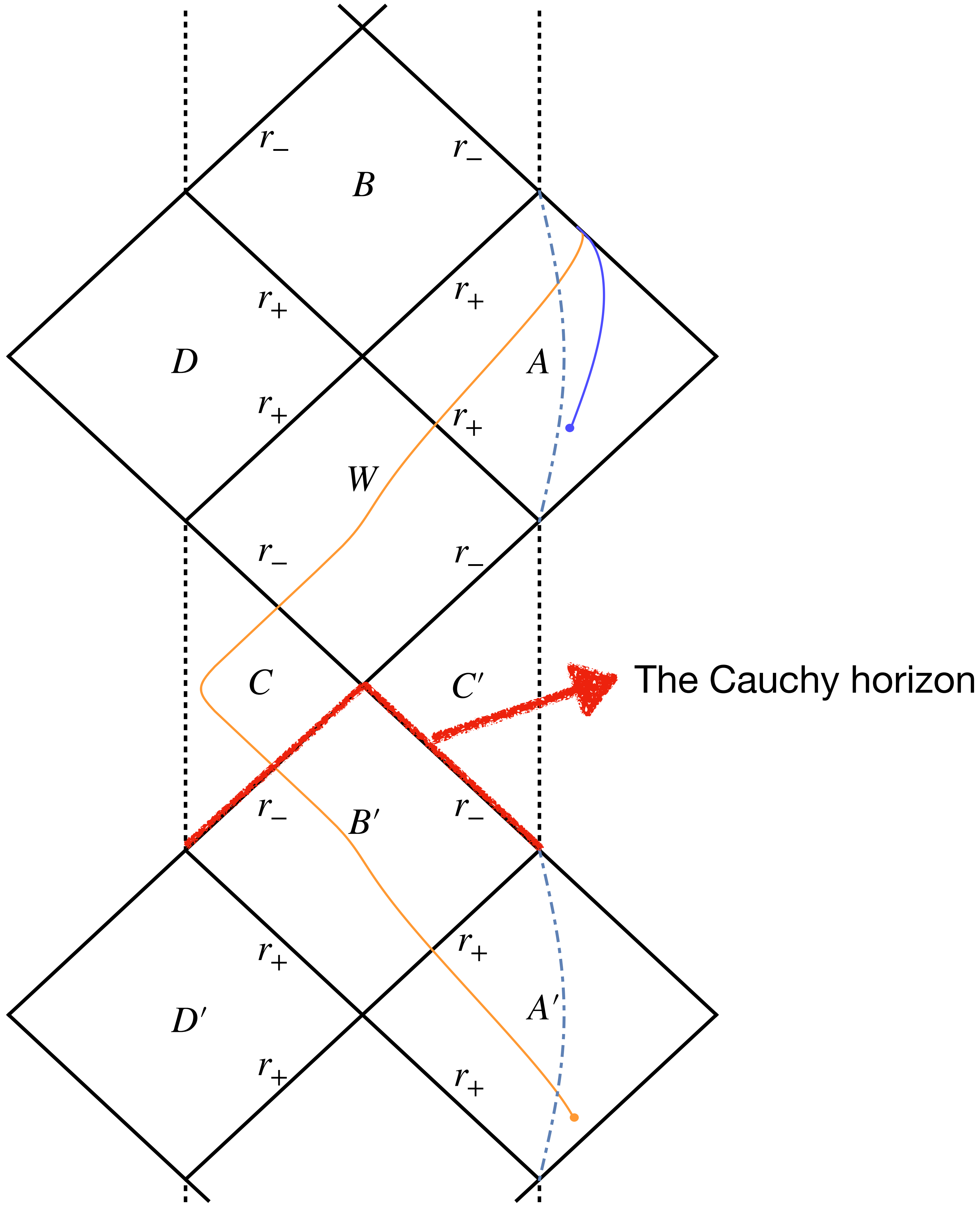


[CZ, Ma, Yang 23']



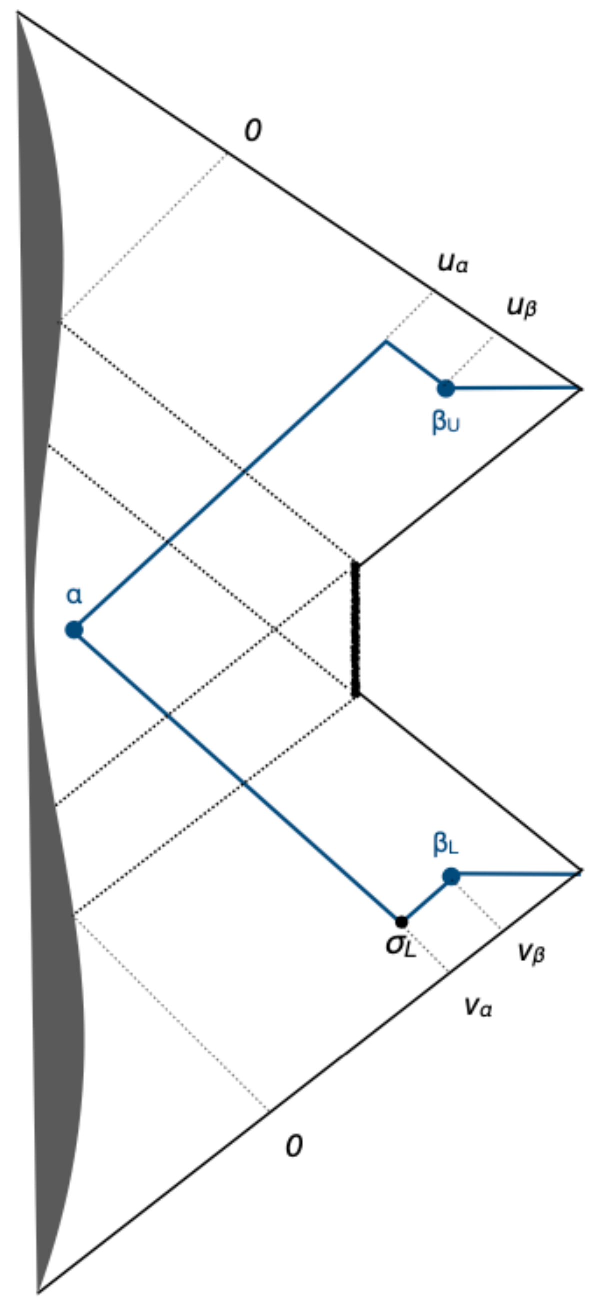
[see Cao, Li, Liu, Zhou 24' for similar work in regular BH]

A new model



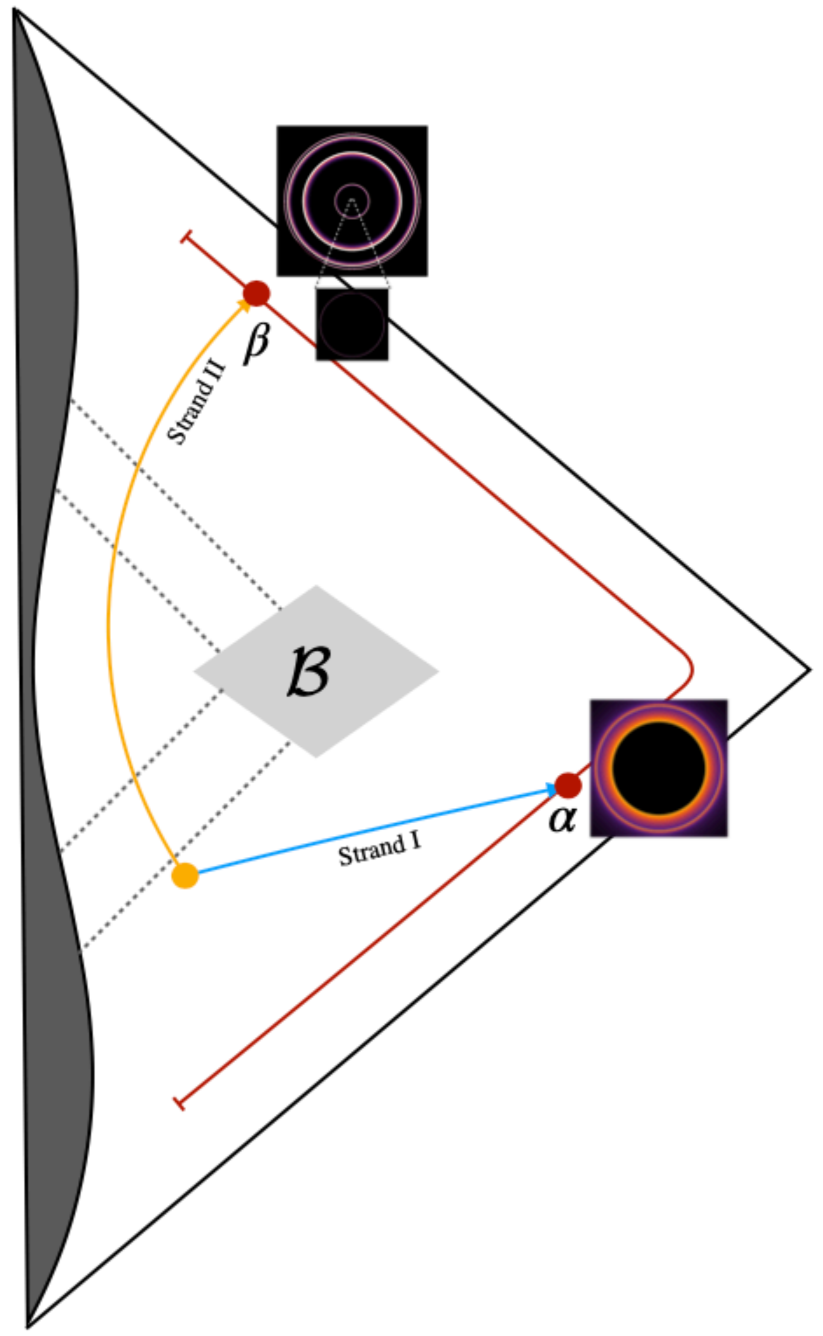
While the spacetime offers distinct advantages, it is not without debates: The existence of Cauchy horizon implies that the spacetime could be unstable under perturbation [Cao, Li et.al. 23' and 24', Shao, CZ, et.al. (2023)]

A new model



Consider the QG effect at the end of evaporation

[Han, Rovelli, Soltani 23']



In a new work, we compute the SF transition amplitude in the B-Region, and study the tunneling of geometry in SF model.

$\det(e_+) = -\det(e_-)$, tunneling between opposite orientations [ArXiv: 2404.02796]

Summary

We briefly review the loop quantum BH modes:

- **The first type is constructed in virtue of the homogeneity of Schwarzschild interior;**
- **The second type considers the interior and exterior as a whole and does quantization;**
- **The third type considers matter collapsing.**

In these models, we have some exciting results, like BH-WH transition, discrete mass spectrum, Nariai limit of BH evolution etc. However, our answer on loop quantum BH hasn't formed a unique picture.

We come up with an observable effects to test quantum BH.

Thank you for your attention !