Emergence of Time Semicrystals in Holographic Driven-Dissipative Systems

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To be submitted, in cooperation with Yu-Qi Lei, Yu Tian and Shao-Feng Wu.

Outline

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Holographic Approach (AdS/CFT)

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Why Holography?

Rooted in macroscopic brane geometries[1], holographic duality[2] provides a **powerful framework** for strongly coupled systems.

The bulk black hole naturally provides a thermal bath, introducing **dissipation** to the boundary quantum system[3].

Ideal for exploring the long-time evolution of **non-equilibrium dynamics** and driven-dissipative systems[4].

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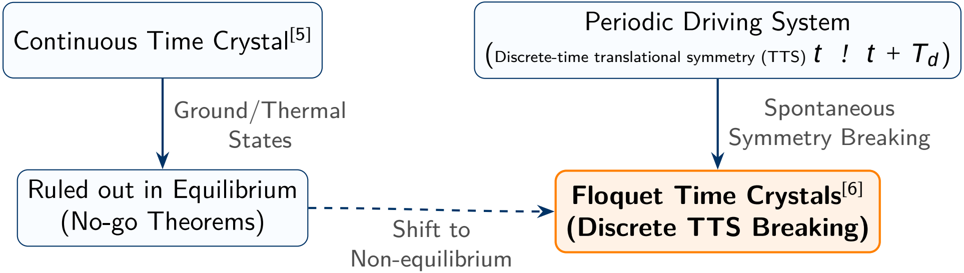
[2] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231-252 (1998).
[3] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, JHEP **12**, 015 (2008).
[4] H. Liu and J. Sonner, Rept. Prog. Phys. **83**, no.1, 016001 (2019).

Origin of Time Crystals



Signature of DTC: Subharmonic response $(T = nT_d) \; !$ a stable temporal order.

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[5] F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012).
[6] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).

Non-equilibrium Dynamic phase

The research of DTC in holography: Holographic dissipative space-time supersolids.[7]

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More non-equilibrium phases in driven quantum manybody systems:

- Higher-order and fractional driven time crystal[8;9]
- Time quasicrystal[10]
- Topological time crystals[11]

[7] P. Yang, M. Baggioli, Z. Cai, Y. Tian and H.B. Zhang. Phys. Rev. Lett. 131, 221601 (2023).
[8] A. Pizzi, J. Knolle and A. Nunnenkamp, Nature Commun. **12**, 2341 (2021).
[9] B. Liu, L. H. Zhang, Q. F. Wang, Y. Ma, T. Y. Han, J. Zhang, Z. Y. Zhang, S. Y. Shao, Q. Li and H. C. Chen, *et al.*, Nature Commun. **15**, no.1, 9730 (2024).
[10] S. Autti, V. B. Eltsov and G. E. Volovik, Phys. Rev. Lett. **120**, no.21, 215301 (2018).
[11] K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski and K. Sacha, New J. Phys. **21**, no.5, 052003 (2019).

From Order to Disorder: The Melt of Time Crystals

Time crystals eventually melt into thermalized chaos through dynamical phase transitions[12 14].

[12] N. Y. Yao, A. C. Potter, I. D. Potirniche and A. Vishwanath, Phys. Rev. Lett. **118**, no.3, 030401 (2017).
[13] P. Frey and S. Rachel, Sci. Adv. **8**, no.9, abm7652 (2022).
[14]R. Yousefjani, K. Sacha and A. Bayat, Phys. Rev. B **111**, no.12, 125159 (2025).

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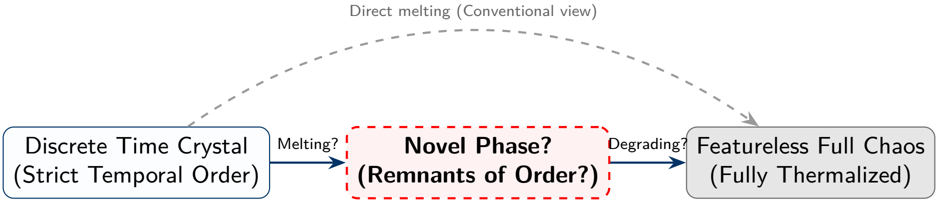

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The Time Semi-Crystal Phase

Physical Essence

Order & Chaos Coexistence: Temporal order "melted" but not destroyed.

Residual "periodic skeleton" embedded in a **chaotic background** (> 0).

Mathematical Signature (PSD):

$$P(f) = \underbrace{\sum_k A_k \, (f \quad f_k)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}} \; ;$$

where $f_k = \frac{q}{n} f_d$ with coprime integers q and n.

Fig. 1. 3-Time semi-crystal.

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The *n*-Time semi-crystal: Dominant subharmonic peak at $\boldsymbol{f_d = n}$, coexisting with continuous broadband spectrum.

In This Work

Periodically driven holographic system with **spontaneous Z_2 symmetry breaking**.

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Dynamical Phase Transitions

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How temporal order changes from time crystals to full chaos?

Outline

A minimal holographic model described by the Einstein-scalar action[15]

$$S = \int d^4x \sqrt{\ \ g}\ \ R + \frac{6}{L^2}\ \ (\nabla\ \ )^2\ \ V(\ \ )\ \ : \tag{1}$$

where AdS radius $L = 1$. The real scalar field   has the nonlinear potential $V(\ \ ) = m^2\ \ ^2 + \frac{\ \ ^4}{6}$.

[15] T. Faulkner, G. T. Horowitz and M. M. Roberts. JHEP. 2011, 51 (2011).

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$$S=\int d^4x\sqrt{-g}\left[R+\frac{6}{L^2}-(\nabla\phi)^2-V(\phi)\right]. \tag{1}$$

where AdS radius $L=1$. The real scalar field ϕ has the nonlinear potential $V(\phi)=m^2\phi^2+\frac{\phi^4}{6}$.

Work in the probe limit, and fix the background as the Schwarzschild-AdS black hole

$$ds^2=\frac{L^2}{z^2}\left(-h(z)dt^2-2dtdz+dx^2+dy^2\right);\quad h(z)=1-\frac{z^3}{z_h^3}. \tag{2}$$

The black hole horizon $z_h=1$, and the AdS boundary at $z=0$. The Hawking temperature is $T_H=\frac{3}{4\pi z_h}$.

[15] T. Faulkner, G. T. Horowitz and M. M. Roberts. JHEP. 2011, 51 (2011).

The Klein-Gordon equation

$$\nabla^2 \quad \frac{1}{2}\frac{\partial V(\ )}{\partial\ } = 0. \tag{3}$$

Near the AdS boundary ($z \to 0$), the asymptotic behavior of scalar field

$$= z \quad = z\,(A + zB + \quad); \tag{4}$$

where A is the condensate response $\langle O \rangle$ with the *alternative quantization*.

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With the double trace deformation, the action of the dual field can be modified by a scalar operator O

$$S \to S \qquad \int d^3x\, \kappa_d\, O^\dagger O; \tag{5}$$

where $\kappa_d = 2(3 \quad 2 \quad)\kappa$ is the coupling parameter, κ is the rescaled coupling parameter used for convenience later. The Z_2 symmetry can spontaneous breaking.

This deformation imposes a mixed boundary condition to the scalar field near the AdS boundary

$$B = \{ A: \tag{6}$$

Under the isotropy assumption, the equation of motion can be reduced

$$2@_t @_z \quad h\,@_z^2 \quad h'\,@_z \quad + z \quad + \frac{1}{3}\;^3 = 0: \tag{7}$$

With periodic driving $F_d(t)$, the boundary condition generalizes to

$$\begin{aligned} @_t \;\; j_{z=0} &= (@_z \quad \{ \quad + F_d(t))j_{z=0}\,; \\ F_d(t) &= F_0 \;\; \sin(!_d \;\; t): \end{aligned} \tag{8}$$

Here $\{ = \{ = z_h$, and Z_2 symmetry breaking occurs for $\{ \lesssim \;\; 0{:}386$. For definiteness, we fix $\{ = \;\; 2$ and $F_0 = 4$, focusing on the system's response to varying driving frequency $!_d$.

Outline

Characterizing the Dynamical Phases

The distinct dynamical phases of the system based on their spectral and phase-space features.

The Time Semi-Crystal Signature

The Time semi-crystal phase exhibits a unique power spectrum combining order and chaos:

$$P(f) = \underbrace{\sum_k A_k \, (f \quad f_k)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}} ;$$

where $f_k = \frac{q}{n} f_d$ with coprime integers q and n.

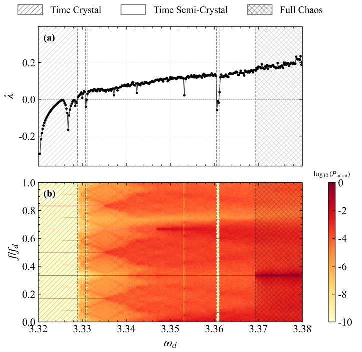


Fig. 2. Dynamical Phase Diagram

Phase Classifications:

Discrete Time Crystal (DTC)

Sharp subharmonic peaks

< 0 (Strict temporal order)

Time Semi-Crystal (TSC)

Subharmonic skeleton + Continuous background

> 0 (Residual order survives)

Full Chaos

Broadband continuous spectrum

> 0 (Order vanishes)

Spectrum & Poincaré Section Features

DTC Phase:

Spectrum: Sharp discrete subharmonic peaks only.
Poincaré section: Discrete points.

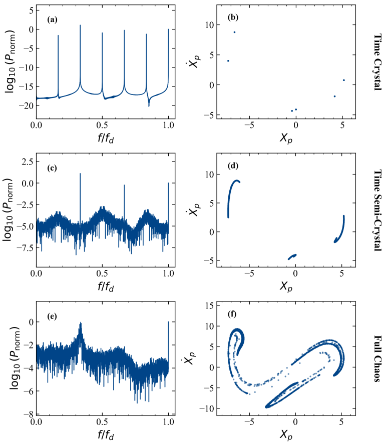
Time Semi-Crystal Phase:

Spectrum: Subharmonic skeleton + continuous background.
Poincaré section: **Finite non-closed curves**.

The coexistence of temporal order and chaos

Full Chaos:

Spectrum: Broadband continuous spectrum.
Poincaré section: Discrete point cloud with fractal structure.



The Melting of Temporal Order

Discrete Time Crystal
Strict Periodicity
< 0

→ Melting →

Time Semi-Crystal (TSC)
Periodic Skeleton + Chaos
> 0

→ Collapse →

Full Chaos
Completely Disorder
> 0

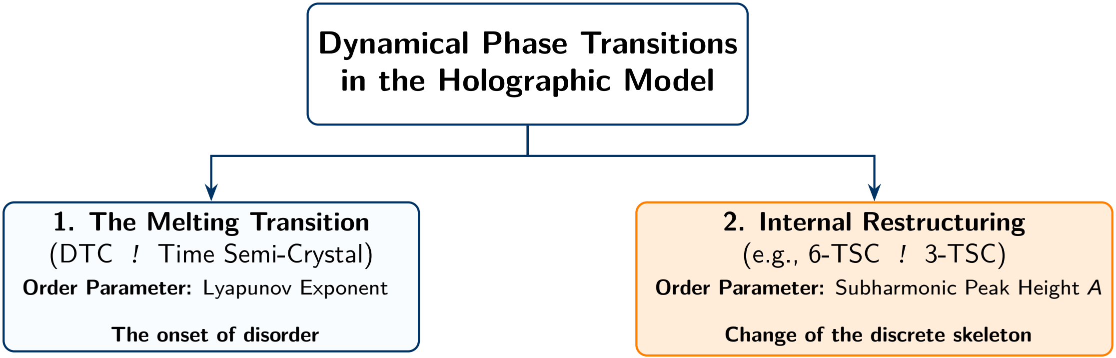
The Time Semi-Crystal Phase

The transition is **not an abrupt collapse** into full chaos.

In the TSC phase, chaotic fluctuations (> 0) coexist with a long-lived **subharmonic skeleton**.

Investigate how this temporal order melts, and how the **residual order evolves** as parameters vary. —**Dynamic phase transition**

Outline



What are the **scaling behaviors** governing these two distinct dynamical transitions?

How **temporal order** changes?

The transition of DTC to Time Semi-Crystal

As $!_d$ approaches a critical value $!_c$, the discrete time crystal begins to melt, giving way to the Time semi-crystal phase.

Order Parameter & Scaling Law

Maximal Lyapunov Exponent ( ): Serves as the order parameter characterizing the onset of chaos.

- < 0: Ordered DTC phase.
- > 0: Time semi-crystal with disorder.

Near the critical point $!_c$, follows a continuous power law:

$$j!_d \quad !_c j \tag{9}$$

Universal Scaling

The critical parameter $!_c$ 3:329. The fitted critical exponent $=$ **0:450** **0:009** is in excellent agreement with the universal value for **period-doubling cascades.**

Melting & Residual Order

Melting of temporal order & **Emergence of temporal disorder** (> 0)

#

Emergence of **residual order embedded in chaos** (Time semi-crystal)

Internal Restructuring: 6-Time semi-crystal to 3-Time semi-crystal

Inside the Time semi-crystal phase, the discrete "skeleton" undergoes dynamical restructuring (e.g., the melting of f_d=6 subharmonic peak).

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Order Parameter

To quantify this internal transition, we define the order parameter A as the normalized height of the **f_d=6 subharmonic peak**:

$$A \qquad \frac{\int_{f_d=6 \quad f}^{f_d=6+ \quad f} P_{\mathrm{norm}}(f)\,df}{\int_{0}^{f_d} P_{\mathrm{norm}}(f)\,df} \tag{10}$$

$P_{\mathrm{norm}}(f)$: Normalized power spectral density.

The decay of A dynamically tracks the “melting” of the 6-TSC structure.

Log-Periodic Power Law (LPPL)

Scaling near Criticality:

The decay of A deviates from a simple power law, exhibiting **log-periodic oscillations**:

1. Trend:

$$A_{\text{trend}} \quad (!_s \quad !_d)$$

2. LPPL Modulation:

$$A = A_{\text{trend}}\,[1 + \quad \cos(\quad \ln(!_s \quad !_d) + \quad {}_0)]$$

= 0:360 0:017: Critical exponent.

= 16:40 0:71: Log-periodic frequency.

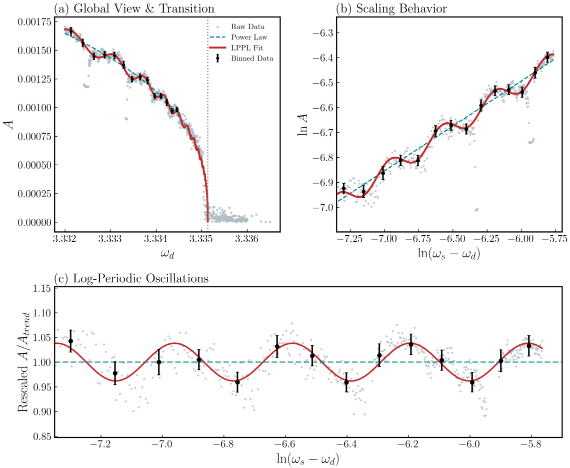

Fig. 3. Transition from 6-TSC to 3-TSC.

Log-Periodic Power Law (LPPL)

Scaling near Criticality: The decay of A deviates from a simple power law, exhibiting **log-periodic oscillations**:

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$$A_{\text{trend}} = C(!_s \quad !_d)$$

2. **LPPL Modulation:**

$$A = A_{\text{trend}}[1 + \quad \cos(\quad \ln(!_s \quad !_d) + \quad _0)]$$

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Decoding the Internal Structure

1. **Hierarchical Skeleton Phase Transition**

   The Time semi-crystal phase is **not a featureless chaotic state**.

   The transition of 6-TSC ! 3-TSC implies the subharmonic skeleton "melts" hierarchically.

This reveals a deeply **nested, hierarchical temporal structure** embedded within the chaos.

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2. **Discrete Scale Invariance (DSI)**

   The log-periodic correction ( ) is a direct physical signature of DSI.

Outline

Summary

1. **Phase Characterization**

   PSD reveals a hybrid phase:

$$P(f) = \underbrace{\sum_n A_n \delta(f - f_d/n)}_{\text{Discrete Skeleton}} + \underbrace{P_{\text{continuous}}(f)}_{\text{Chaotic Background}}$$

Residual temporal order within chaos.

2. **Dynamical Phase Transitions**

   Onset: Melting of Time Crystal $\to$ Emergence of Time semi-crystal.

   Internal: Restructuring of the residual skeleton $\to$ **Log-periodic corrections**.

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2. Dynamical Phase Transitions

Onset: Melting of Time Crystal $\to$ Emergence of Time semi-crystal.

Internal: Restructuring of the residual skeleton $\to$ **Log-periodic corrections**.

The Time semi-crystal is **not a featureless chaotic state**. Its hierarchical residual order holds fundamental physical significance.

Outlook

1. **Theoretical Exploration**

   Investigate the **universality classes** of these novel dynamical transitions.

   Deepen the physical understanding of **Discrete scale invariance** in chaotic environments.

2. **Experimental Verification**

   Connect our holographic model results with quantum many-body experiments, such as ferroelectric/ferromagnetic phase transitions and Rydberg atom simulations.

Thank you!